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Session 8 – Base Ten Arithmetical Strategies

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http://www.irememberjfk.com/mt/mt-tb.cgi/753



Valuing Mental Computation Online

What is mental computation?

Why teach mental computation?

What to do in the classroom

Further readings





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mputation Online

Valuing Mental Computation Onl

Why teach mental computation?

Mental computation is now recognised as an important form of computation. The importance can be summa

- it is the most common form of computation used by adults. ("85% of calculations done by adults in their daily life." McIntosh and Northcote 1998)
- it is used for estimation
- it is needed to check calculator answers
- students see it as an easy way of doing calculations
- it is an excellent way of learning how numbers work
- it is a creative and problem solving approach to number.







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Valuing Mental Computation Online

Jump or Split

The two main groups of strategies used for mental computation of addition and subtraction questions are • the jump methods (Beishuizen, 1993) where students build on from one of the numbers and

• the split methods where students will partition both numbers into tens and ones (Cobb and Wheatley, 1

The use of the jump method or the split method is often a function of the instructional sequence students through. Although weaker students have an observed preference for the split method. Beishuizen and Ar that this is because students initially find it difficult to increment by tens off the decade, such as 37, 47, 5 Nevertheless, when encountering the more difficult 2-digit addition and subtraction questions requiring restudents using the split method are more likely to make errors and to find it difficult to modify their stratec

We need to encourage students to move beyond a count by one method but there is no preference for te students one facile method over another. Rather the value is in students being comfortable to use which is more efficient for the particular question.



52 – 24 =

Let's see how many ways we can find to get the answer....

Please type an explanation for one or more solution strategy(ies) into the "Main Room" text box.

Now think about which method is most efficient?







Subtractive

In this example the student subtracts the units before subtracting the tens. She is subtracting the right side of the numeral before subtracting the left.

52 - 24**= 28**



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In this example the student has reduced both of the numbers within the question by 2 and then used addition to find a solution.

50 is 2 less than 52 and 2 less than 24 is 22

52 - 24 is the same as 50 - 22

22+<mark>28</mark>= 50



Click on the question to see each step.















Tools for the Common Core Standards

News about tools that are being developed to support implementation of the Common Core State Standards



Home About Tools

Draft progression on Counting and Cardinality and Operations and Algebraic Thinking (K–2)

Posted on May 8, 2011 by Bill McCallum

We have combined the Kindergarten Counting and Cardinality Progression with the K-5 Operations and Algebraic Thinking Progression. Here is the first half, <u>ccss progression cc oa K2 2001 05 08</u>, going as far as Grade 2. As usual,



Recent Posts

- Draft progression on Counting and Cardinality and Operations and Algebraic Thinking (K-2)
- Recommendations for initial professional development on the Common Correl

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(b) http://commoncoretools.files.wordpre...

decomposing a ten.^{2.NBT.7}

Strategies and algorithms The Standards distinguish strategies from algorithms.[•] For example, students use strategies for addition and subtraction in Grades K-3, but are expected to fluently add and subtract whole numbers using standard algorithms by the end of Grade 4. Use of the standard algorithms can be viewed as the culmination of a long progression of reasoning about quantities, the base-ten system, and the properties of operations.

This progression distinguishes between two types of computational strategies: special strategies and general methods. For example, a special strategy for computing 398 + 17 is to decompose 17 as 2 + 15, and evaluate (398 + 2) + 15. Special strategies either cannot be extended to all numbers represented in the base-ten system or require considerable modification in order to do so. A more readily generalizable method of computing 398 + 17 is to combine like base-ten units. General methods extend to all numbers represented in the base-ten system. A general method is not necessarily efficient. For example, counting on by ones is a general method that can be easily modified for use with finite decimals. General methods based on place value, however, are more efficient and can be viewed as closely connected with standard algorithms. of 10, using concrete models or drawings and strate on place value, properties of operations, and/or the i between addition and subtraction; relate the strateg ten method and explain the reasoning used. Unde in adding two-digit numbers, one adds tens and tens ones; and sometimes it is necessary to compose a te

a one aigit number, and adding a two aigit number and

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^{2.NBT.7} Add and subtract within 1000, using concrete drawings and strategies based on place value, prope erations, and/or the relationship between addition at tion; relate the strategy to a written method. Unde in adding or subtracting three-digit numbers, one ac tracts hundreds and hundreds, tens and tens, ones and sometimes it is necessary to compose or decomp hundreds.

• Computation algorithm. A set of predefined steps to a class of problems that gives the correct result in when the steps are carried out correctly. See also: c strategy.

Computation strategy. Purposeful manipulations the chosen for specific problems, may not have a fixed may be aimed at converting one problem into another computation algorithm.

Draft, 4/7/2011, comment at commoncoretools.wordpress.com.

ownloading (629.29 KB of 631.09 KB) : http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf



















T = Base Ten Arithmetical Strategies (Tens and Ones)

104.5

500 series also indicates Base Ten Arithmetical Strategies

24.2

CNP #

T 524.1

KNP# with	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy	Strand	Construct/Lev el
T524.1	<u>1.NBT.2</u> . Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones – called a "ten." The numbers from 11	Number & Operations in Base Ten	lace value understanding and properties of operations to add and subtract.	100 bead rack (10 rows of 10) or 100 bead string, writing material	The teacher will create an amount on a 100 bead rack and flash the amount for the students. Students will write the corresponding numeral. Students will share their answers and explain their reasoning.	Baro Ten Arithmation Cturt anion	pase ien Antnmeticat strategies	0 to 1

Mathematics » Grade 1 » Number & Operations in Base Ten

1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

•10 can be thought of as a bundle of ten ones — called a "ten."

The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).





Ten as a "unit" Teen numbers as 10 and some more



100 Bead Rack



•10 rows of 10 beads

•Rows color coded 5 & 5

•Color coded groups of 50

Build-your-own Bead Rack



Materials: •Sewing elastic (either $\frac{1}{8}$ or $\frac{1}{4}$ inch)

•Plastic pony beads in two colors (your choice of colors)

Directions: Thread beads onto elastic, cut elastic to fit and tie elastic to form a loop. Stretch loop(s) around a student dry erase board.

Build-your-own Bead Rack



Use 10 loops

Reverse color order for rows 6 through 10

100 Bead Rack



"Desktop Abacus" from Amazon (ignore or cover place value labels)




"Read" and "Build" quantities to 100



T 524.1

KNP# with	Ken tuc ky Com	KCAS Domain	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy	Construct/Le	Numeracy	"I CAN" (*see glossary)	Assessment for Learning (*see glossary)	Student	Video Link	Print Link	Interactive	Reference
T524.1	1.N BT.2 Und erst and that the two digit s of a two- digit num ber	Number & Operations in Base Ten	Ose place value understanding and properties of (10 rows of 10) or 100 bead string, writing material material	The teacher will create an amount on a 100 bead rack and flash the amount for the students. Students will write the corresponding numeral. Students will share their answers and explain their reasoning.	Base Ten Arithmetical Strategies	0 to 1	tens or ones with materials	connect the written numeral to the quantity in the range 1 to 100.	Briefly show 79 on a bead rack, then cover. Ask student to write the corresponding numeral. Repeat for other amounts in the range 1 to 100.	small group or pairs		http://www.kymath.org/intervention/doc/Numeracy	http://www.dreambox.com/teachertools-guick-	



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Hide Card Show Card Answer

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Tenframe from 4 to 10	
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Tenframe from 21 to 40	
One-Wire Mathrack from 4 to 10	
Two-Wire Mathrack from 11 to 20	
Two-Wire Mathrack from 4 to 20 (displayed as doubles)	
Ten-Wire Mathrack from 20 to 100 (multiples of 10 only)	
Ten-Wire Mathrack from 21 to 50	
Ten-Wire Mathrack from 41 to 100	

Free Dreambox Teacher Tools http://www.dreambox.com/teachertools-quick-images



Extension questions

- •How many beads if we add 10?
- •How many if we take away 10?
- •What if we add 1 bead?
- •What if we take away 1 bead?
- •What if we take away 8 beads?

•What do we add to make 60?



100 Bead String

A 100 Bead String consists of alternating color groups of 10 beads.



100 Bead String



- •How many beads?
- •What if I add/subtract 1?
- •What if I add/subtract 10?
- •How many blue beads are left in the group?
- •What will I have if I move the last blue bead over? •How many beads do I need to have to 100?

T524

#dNM	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	
T524.2	1.NBT.9. Add one-digit numbers to two-digit numbers, and add multiples of 10 to one-digit and two-digit numbers.	Number & Operations in Base Ten	Use place value understanding and properties of operations to add and subtract	
T524.3	2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Number & Operations in Base Ten	Use place value understanding and properties of operations to add and subtract	
T524.4	2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Number & Operations in Base Ten	Use place value understanding and properties of operations to add and subtract	

T524

KNP# with	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster		Setting (sit uatio n क्ष materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy	Strand	Construct/Le vel	Numeracy	Target
T524.2	1.NBT.9. Add one- digit numbers to two- digit numbers, and add multiples of 10 to one-digit and two-	Number & Operations in	Use place value understanding	and properties	Steal the Crown card game (see link), 100 bead rack or 100 bead string	Students play Steal the Crown in partners or small groups. Game cards and directions are included in the print link. Students will use a 100 bead	Base Ten	Arithm etical	1 to 2 RED	tens and ones	with materials
T524.3	2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of	Number & Operations in	Use place value understanding	and properties	Steal the Crown card game (see link of T524.2), 100 bead string or 100 bead	Students play Steal the Crown in partners or small groups. Game cards and directions are included in the print link of entry T524.2. Students may	Base Ten	Arithm etical	2 to 3 BLUE	tens and ones	without
T524.4	2.NBT.5. Fluently add and subtract within 100 using strategies based on place	Number & Operations in	se place value Inderstanding	nd properties	Steal the Crown card game (see link of T524.2) writing material	in partners or small groups. Game and directions are included in the print link or T	Base Ten	Arithm etical	TO 4 GREEN	facile	structuring to



Zeus on the Loose







Steal the Grown! game cards



Copy the "back" image on the back of each page of cards to prevent "see-through"

Steal The Crown!

Materials: Crown card (or a toy or paper crown), addition cards, subtraction cards, specialty cards. Additionally, a 100's chart, 100 bead rack, 100 bead string or sticks & bundles may be used.

- 1. Deal each player 4 cards and put the remaining cards face down on the table.
- 2. The youngest player goes first and begins by putting a card face up on the table.
- 3. The next player will put a card on top of that card and will announce the total. (This stack will be called the Treasury). Each time a numeral card is played, the Treasury is adjusted according to the card played. The Treasury total may be shown on a 100 board, bead rack or bead string or with sticks and bundles.
- 4. All players will draw a card after his or her turn.
- 5. In order to steal the crown, do one of the following:
 - a. Play a specialty card that allows you to steal the crown
 - b. Play a card that makes the treasury equal a multiple of 10.
- 6. When the Treasury equals or exceeds 100, the last card is drawn or the teacher calls the end of time, the player who has the crown wins.

To prepare deck:

Print cards (pages 2-8) on cardstock. The final page can be copied onto the back of the card pages. This will prevent players from being able to see through the cards. Cut apart cards. Cards may be laminated.



Steal the Crown!



To Steal the Crown: •Play a specialty card that allows you to steal the crown.

•Play a card that makes the Treasury equal a multiple of 10.

The winner is the player who has the crown at end of game. Play ends when:
The Treasury reaches or exceeds 100
Last card is drawn
Time is up



KNP# with	Kentucky Common Core Academic	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct/Le vel	Numeracy Target	"I CAN" (*see glossary)	Asse Lea g
T524.2	1.NBT.9. Add one- digit numbers to two- digit numbers, and add multiples of 10 to one-digit and two- digit numbers.	Number & Operations in Base Ten	se place value understanding and properties	Steal the Crown card game (see link), 100 bead rack or 100 bead string	Students play Steal the Crown in partners or small groups. Game cards and directions are included in the print link. <u>Students will use a 100 bead</u> rack or a 100 bead string to keep track of the ongoing total.	Base Ten Arithmetical Strategies	1 to 2 RED	tens and ones with materials	add or subtract a single digit number or 10 from a number in the range 1 to 100 with support of a bead rack or bead string.	Show 65 rack. A beads? \ if I ta Show : "How ma will I ha it?" Co sim

T524.2

Students will use a 100 bead rack to keep track of the Treasury.



T524.2

Observe how students determine what beads to move.

Ask how students determine the new amount.



T524.2

Students can keep track of the Treasury on the 100 bead string.



Ask questions such as: •How can you get the crown? •What would you need to add/subtract?

•What is the nearest multiple of 10?



KNP# with	Kentucky Common Core Academic	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct/Le vel	Numeracy Target	"I CAN" (*see glossary)	
T524.3	2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, propertie s of operation s, and/or the	Number & Operations in Base Ten	Use place value understanding and properties	Steal the Crown card game (see link of T524.2), 100 bead string or 100 bead rack, writing material	Students play Steal the Crown in partners or small groups. Game cards and directions are included in the print link of entry T524.2. Students may initially use a 100 bead rack or 100 bead string as a check but should move away from using materials. Student may be asked to record their thinking using an Empty Number Line or other format.	Base Ten Arithmetical Strategies	2 to 3 BLUE	tens and ones without materials	add or subtract a single digit number or 10 from a number in the range 1 to 100.	As A Co tas to



T524.3

+ 5

21

24

(24)

3031



Students can record the their reasoning on an Empty Number Line

KNP# with	Kentucky Common Core Academic	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct/Le vel	Numeracy Target	"I CAN (*see glos
T524.4	2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, propertie s of operation	Number & Operations in Base Ten	Use place value understanding and	Steal the Crown card game (see link of T524.2), writing material	Students play Steal the Crown in partners or small groups. Game and directions are included in the print link or T 524.2. Student may be asked to record their thinking using an <u>Empty Number Line or other</u> <u>format.</u>	Base Ten Arithmetical Strategies	3 TO 4 GREEN	facile structuring to 100	add subtrac single d number from a nu in the rai to 100 us variety menta strategi









Distancing the Setting



KNP# with	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy	Construct/Le	Numeracy	"I CAN" (*see glossary)	A: L
T524.5	2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: 100 can be thought of as a bundle of ten tens – called a "hundred." The numbers 100, 200, 300, 400, 500, 600, 700,	Number & Operations in Base Ten	Understand place value	Ten 100 Bead Strings	Teacher will show an amount in the range 1 to 1000 using bead strings and students will write the corresponding number. For example, show 425 by placing out 4 complete (100) bead strings and a bead string showing 35 beads. Alternatively, students can be asked to build a number using bead strings. Ask questions such as "What would be 10 more/less? What would be 100 more/less?", then make the corresponding change with the beads.	Base Ten Arithmetical Strategies	4 to 5 P URPLE	facile structuring to 1000	connect the written numeral to the quantity in the range 1 to 1000 and I can add and subtract 10 and 100 without counting by ones.	Shov strir "Ho Wł beads have sim

Mathematics » Grade 2 » Number & Operations in Base Ten

2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

•100 can be thought of as a bundle of ten tens — called a "hundred."

•The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).





KNP# with	Kentucky Common Core Academic	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct/Le vel	Numeracy Target	"I CAN" (*see glossary)	4
T524.5	2.NBT.1. Understa nd that the three digits of a three- digit number represent amounts of hundreds, tens, and ones;	Number & Operations in Base Ten	Under stand pl ace value	Ten 100 Bead Strings	Teacher will show an amount in the range 1 to 1000 using bead strings and students will write the corresponding number. For example, show 425 by placing out 4 complete (100) bead strings and a bead string showing 35 beads. Alternatively, students can be asked to build a number using bead strings. Ask questions such as "What would be 10 more/less? What would be 100 more/less?", then make the corresponding change	Base Ten Arithmetical Strategies	4 to 5 P URPLE	facile structuring to 1000	connect the written numeral to the quantity in the range 1 to 1000 and 1 can add and subtract 10 and 100 without counting by ones.	Sho stri "H W bead hav si



T524.5



See "100" as a unit

T524.5

Extension questions: Start counting from 425 What if I add/subtract 100? What if I add/subtract 10? What do I subtract to get 400? What do I add to get 500?







Alternative option -Use 100 bead racks to build numbers.







T523.1

KNP entry number	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)
Т 523.1	 1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. 	Number & Operations in Base Ten	Use place value understanding and properties of operations to add and subtract.	Delivery Game: 2 truck picture mats and tens or ones customer order cards, one set per pair of students (see link); 20 bundles of ten popsicle sticks, 50 loose sticks and 4 hair-holders per pair; writing space	Delivery Game: Each player starts by placing 5 bundles and 5 loose sticks on his/her truck. The customer order cards are placed face down between the players. Players take turns taking a card and following the instructions. Write a "shipping report" of each stop the addition or subtraction sentence for the truck contents. Play continues until all cards have been used. The player with the greatest number of sticks wins the game.
KNP entry number	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)
------------------	---	---------------------------------	--	---	--
T 523.1	 1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. 	Number & Operations in Base Ten	Use place value understanding and properties of operations to add and subtract.	Delivery Game: 2 truck picture mats and t <u>ens or on</u> es customer order cards, one set per pair of students (see link); 20 bundles of ten popsicle sticks, 50 loose sticks and 4 hair-holders per pair; writing space	Delivery Game: Each player starts by placing 5 bundles and 5 loose sticks on his/her truck. The customer order cards are placed face down between the players. Players take turns taking a card and following the instructions. Write a "shipping report" of each stop the addition or subtraction sentence for the truck contents. Play continues until all cards have been used. The player with the greatest number of sticks wins the game.

Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand (from AVMR)	Construct/Level (from AVMR)	Numeracy Target (from AVMR)	"I CAN" (*see glossary)	Assessment for Learning (*see glossary)	Student Grouping	Video Link	Print Link	Interactive Website	Reference
 ivery Game: Each player starts by cing 5 bundles and 5 loose sticks on s/her truck. The customer order rds are placed face down between players. Players take turns taking ard and following the instructions. te a "shipping report" of each stop> addition or subtraction sentence the truck contents. Play continues itil all cards have been used. The ayer with the greatest number of sticks wins the game. 	Base Ten Arithmetical Strategies	0 to 1 YELLOW	tens or ones	add or subtract ones or tens using materials. I can also writo the matching addition and subtraction sentences.	Show and hide a collection of 14 (1 bundle and 4 loose) sticks. Show and add 4 bundles to the hidden collection. Tell the student that you are adding 4 bundles. Ask the student how many sticks there are in all. Show and add 9 sticks to the hidden colloction and ask how many. Ask the student to write the addition sentence for each action.	partners		http://www.kymdth.org/intervention/doc/Numerac yProject/T523-1-DeliveryGame.docx		

ssment for Learning (*see glossary)	Student Grouping	Video Link	Print Link	Interactive Website	Reference	Teacher Notes
Show and hide a ction of 14 (1 bundle 4 loose) sticks. Show add 4 bundles to the den collection. Tell student that you are og 4 bundles. Ask the ent how many sticks are in all. Show and ent in all. Show and sticks to the hidden lection and ask how . Ask the student to vrite the addition ence for each action.	partners		<pre>ttp://www.kymath.org/intervention/doc/Numerac yProject/T523-1-DeliveryG.me.docx</pre>			Some students may not yet be able to conceive of a group of ten as 1 thing to be counted. Those students may need more experience with building numbers using sticks and bundles with arrow cards and with counting items in groups. The standard here refers to operating with an addend/subtrahend that is either a 1-digit number or a multiple of 10. Activity T 523.2 is more difficult, because the cards include 2-digit numbers that are not multiples of 10. Use toy trucks instead of the printed truck pictures, if desired. Instead of sticks and bundles, you may wish to use bundles of coffee stirrers and other base ten materials. Note that actual bundles and loose items of the thing to be counted (i.e. sticks and bundles) may be more supportive of student conceptual understanding than a piece of plastic marked with 9 dividing lines (i.e. base ten long blocks). Encourage students to talk about the how many-ness of the collections (including the number of bundles, the number of loose sticks and 1 ten are the same thing is a very abstract concept.

<image/> <image/> <image/> <image/> <image/> <image/> <image/>	Printables for T523.1 (each is 8 ½ x 11 when printed)	Deliver 30 sticks Deliver 3 sticks Deliver 2 sticks	Deliver 10 sticks Deliver 8 sticks Deliver 40 sticks	Deliver 20 sticks Deliver 5 sticks Deliver 7 sticks
<image/> <image/> <image/> <image/> <image/> <image/> <image/>		Pick up 30 sticks Pick up 2 sticks Pick up 7 sticks	Pick up 10 sticks Pick up 3 sticks Pick up 8 sticks	Pick up 20 sticks Pick up 4 sticks Pick up 9 sticks



A "bundle" = 10 sticks

Consider using sticks instead of base ten blocks. Students can physically break apart a bundle of 10 to have a concrete model of regrouping!





Materials for T523.1







Students take turns drawing a card and either "pick up" or "deliver" the appropriate amount of sticks. Students each put 5 bundles and 5 sticks on their truck "How much is that altogether?"





"How many sticks do you have altogether in your truck?"

"Could you show what you just did with numbers?"







KNP entry number	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand
T 523.2	2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Number & Operations in Base Ten	Use place value understanding and properties of operations to add and subtract.	Delivery Game: 2 truck picture mats and tens and ones customer order cards, one set per pair of students (see link); 20 bundles of ten popsicle sticks, 50 loose sticks and 4 hair-holders per pair; writing space	Delivery Game: Each player starts by placing 5 bundles and 5 loose sticks on his/her truck. The customer order cards are placed face down between the players. Players take turns taking a card and following the instructions. Write a "shipping report" of each stopthe addition or subtraction sentence for the truck contents. Play continues until all cards have been used. The player with the greatest number of sticks wins the game.	Rase Ten Arithmetical Strateoies

" CAN" (*see glossary)	Assessment for Learning (*see glossary)	Student Grouping	Video Link	Print Link	Interactive Website	<u>Reference</u>	Teacher Notes
add or subtract ones and/or tens using materials. I can also write the matching addition and subtraction sentences.	Show and hide a collection of 14 (1 bundle and 4 loose) sticks. Show and add 11 (1 bundle and 1 stick) to the hidden collection. Ask how many in all Show and add 27 to the hidden collection. Ask how many in all.	partners		http://www.kymath.org/intervention/doc/Numerac yProject/T-523-2-DeliveryGame.docx			Also assess student thinking for subtracting from a hidden collection, including "easy" questions that do not require unbundling/decomposing and "hard" questions that do. <u>Carefully select the tasks</u> in an order that allows students to think about related facts. Note that this activity uses cards containing numbers made of tens <i>and</i> ones, whereas the previous version used cards that were either multiples of ten or single digit numbers.

<image/> <image/> <image/> <image/> <image/> <image/> <image/>	Printables for T523.2 (each is 8 ½ x 11 when printed)	Deliver 7 sticks Deliver 5 sticks Deliver 9 sticks	Deliver 10 sticks Deliver 20 sticks Deliver 10 sticks	Deliver 22 sticks Deliver 18 sticks Deliver 14 sticks	
<image/>		Pick up 4 sticks Pick up 6 sticks Pick up 8 sticks	Pick up 10 sticks Pick up 20 sticks Pick up 11 sticks	Pick up 24 sticks Pick up 26 sticks Pick up 19 sticks	



Materials for T523.2









"I can get a bundle of 10 and just take out 1 stick."

"I can break a bundle of 10 apart and get the sticks I need to have 9." "What do you need to do?"

"What can you do if you don't have enough sticks?"

"What's the easiest way to get 9 sticks?"



KNP entry number	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand (from AVMR)	Construct/Level (from AVMR)	Numeracy Target (from AVMR)	
Τ 523.3	2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Number & Operations in Base Ten	Use place value understanding and properties of operations to add and subtract.	Delivery Game: 2 truck picture mats and numeral customer order cards, one set per pair of students (see link); marker; writing space	Delivery Game: Each player starts by writing 55 (as the imagined number of sticks inside) on the truck. The customer order cards are placed face down between the players. Players take turns taking a card and following the instructions. Mark on the truck the change in the number of sticks after each turn and the new total. Play continues until all cards have been used. The player with the greatest number at the end wins the game.	Base Ten Arithmetical Strategies	2 to 3 BLUE	tens and ones without materials	 sı qı əx

"I CAN" (*see glossary)	Assessment for Learning (*see glossary)	Student Grouping	Video Link	Print Link	Interactive Website	<u>Reference</u>	Teacher Notes
mentally add and subtract imagined quantities of sticks and bundles and explain my thinking.	Ask a student to tell you how much is 4 bundles and 6 sticks. Then, ask the student to tell how much there would be if you added 1 bundle and 2 sticks. Finally, ask how much if you added another bundle and 7 loose sticks.	partners		http://www.kymath.org/intervention/doc/numeracyFr oiect/T-523-3-DelivervGame.docx			Also assess student thinking for subtracting 2- digit numbers. Note whether or not the student is able to mentally "unbundle" tens as needed to subtract. Support students in using whatever notation makes sense to them (i.e. writing the number of sticks and bundles, using an empty number line, writing equations) while playing the game.

<image/> <image/> <image/> <image/> <image/> <image/> <image/>	Printables for T523.3	Deliver 12 Deliver 18 Deliver 11	Deliver 20 Deliver 24 Deliver 19	Deliver 33 Deliver 13 Deliver 21
<image/> <image/> <image/> <image/> <image/> <image/> <image/>		Pick up 19 Pick up 16 Pick up 13	Pick up 18 Pick up 15 Pick up 12	Pick up 17 Pick up 14 Pick up 11

Materials for T523.3







Students write numbers on their delivery truck instead of building the number with bundles and sticks.





Students draw a card, either add to or subtract from the total "in" their delivery truck, and explain or model their thinking.







"I know it's 44!"



"How did you work it out so quickly?"



KNP entry number	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand (from AVMR)	Construct/Level (from AVMR)	Numeracy Target (from AVMR)	
T 523.4	2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Number & Operations in Base Ten	Use place value understanding and properties of operations to add and subtract.	Addition and subtraction cards (see link); writing space	One student will take an addition and subtraction card (in the sequence specified on the cards) and read it to the other student who will tell the answer and explain his/her thinking.	Base Ten Arithmetical Strategies	3 TO 4 GREEN	facile structuring to 100	mei for al

					T	5	2:	3.4
Construct/Level (from AVMR)	Numeracy Target (from AVMR)	"I CAN" (*see glossary)	Student Grouping	Video Link	Print Link	Interactive Website	<u>Reference</u>	Teacher Notes
3 TO 4 GREEN	facile structuring to 100	mentally add and subtract for all sums and differences within 100.	partners		http://www.kymath.org/intervention/doc/NumeracyProj ect/T-523-4-DeliveryGame.docx			Present addition and subtraction problems as a string of related problems to allow students to consider using related facts as they develop efficient mental strategies for operations with 2 digit numbers. Although this learning experience appears to be different from the others in this task group, the learning objective (of becoming facile with conceptual place value and fluency with mental opertaions) is consistent. You may wish to provide each student with a picture of the truck to aid in using mental imagery of the quantities as they calculate. Be cautious about limiting student understanding by showing students early on how to operate on digits in a place rather than thinking about the quantities involved.

20 + 10	20 + 11	21 + 11				
21 + 12	29 + 10	29 + 11				
29 + 12	30 + 15	29 + 15				

30 + 12 set.cett	31 + 11 5418.54482	32 + 10
50 + 18	49 + 18	48 + 18
60 + 17	59 + 17	58 + 17

Printables for T523.4

60 — 30 Historia	59 – 30	59 – 29 secceda 57 – 20 secceda			
60 - 31 	61 - 32				
57 — 19 Historie	58 – 20	58 – 21 seconte			

100 – 30	90 – 30 sea.com	90 – 29 set, celta
90 - 31	70 – 40	70 – 39 Sith Contra
70 – 41	98 – 20	98 – 19 50.000



Materials for T523.4









Student draws a card and reads it to her partner

Partner tells the answer and explains/records their thinking







"I know it's 29! I just pretended the 31 was a 30 and subtracted it from the 60 and then I took away 1 more and I got 29."



KNP entry numbe	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand (from AVMR)
T 523.5	2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	Number & Operations in Ease Ten	e place value understanding and properties of operations to add and subtract.	<i>Delivery Game:</i> 2 truck picture mats, big bundles of 100, bundles of 10 and stick pictures, and customer order cards to 1000, one set per pair of students (see link); writing space	Delivery Game: Each player starts by placing pictures of 5 big-bundles (of ten tens or one hundred), 5 bundles (of ten), and 5 loose sticks on or in his/her truck. The customer order cards are placed face down between the players. Players take turns taking a card and following the instructions. Players may make equal trades (such as trading a big- bundle of 100 for ten bundles of 10) as needed to follow the directions on the card. Write a "shipping report" for each stopthe addition or subtraction sentence of the changes in the truck amounts. Play continues until all cards have been used. The player with the greatest number of sticks wins the game.	Base Ten Arithmetical Strategies

"I CAN" (*see glossary)	Assessment for Learning (*see glossary)	Student Grouping	Video Link	Print Link	Interactive Website	Reference	Teacher Note
add or subtract ones, tens and/or hundreds using materials. I can also write the matching addition and subtraction sentences.	Show and hide a collection of 382 sticks represented by pictures of 3 big-bundles, 8 bundles, and 2 loose sticks. Show and add 213 (represented by pictures of 2 big-bundles, 1 bundle, and 3 loose sticks) to the hidden collection. Ask how many in all. Finally, show and add another 157 (represented by pictures of 1 big-bundle, 5 bundles and 7 loose sticks) to the hidden collection. Ask how many in all.	partners		/www.kymath.org/intervention/doc/NumeracyProject/T- 523-5-DeliveryGame.docx			The easier version of this game is the cards face up on the truck, Encourage more sophisticated students put the cards "in" the picture). Also assess the development strategies, similar to the Assess task provided. Ask "easy" questio unbundling/decomposing and "h do.



." 'y)	Assessment for Learning (*see glossary)	Student Grouping	Video Link	Print Link	Interactive Website	<u>Reference</u>	Teacher Notes
nes, tens ; using lso write tion and ences.	Show and hide a collection of 382 sticks represented by pictures of 3 big-bundles, 8 bundles, and 2 loose sticks. Show and add 213 (represented by pictures of 2 big-bundles, 1 bundle, and 3 loose sticks) to the hidden collection. Ask how many in all. Finally, show and add another 157 (represented by pictures of 1 big-bundle, 5 bundles and 7 loose sticks) to the hidden collection. Ask how many in all.	partners		<pre>www.kymath.org/intervention/doc/NumeracyProject/T- 523-5-DeliveryGame.docx</pre>			The easier version of this game is where students put the cards face up on the truck, so they are visible. Encourage more sophisticated thinking by having students put the cards "in" the truck (under the picture). Also assess the development of subtraction strategies, similar to the Assessment for Learning task provided. Ask "easy" questions that don't require unbundling/decomposing and "hard" questions that do.

T				
	Printables for T523.5	Deliver 169 sticks	Deliver 288 sticks	Deliver 322 sticks
		Deliver 178 sticks	Deliver 115 sticks	Deliver 209 sticks
Best Delivery Service We are fast and accurate!		Deliver 250 sticks	Deliver 180 sticks	Deliver 211 sticks
		Pick up 180 sticks	Pick up 210 sticks	Pick up 175 sticks
		Pick up 293 sticks	Pick up 268 sticks	Pick up 310 sticks
Best Delivery Service		Pick up 106 sticks	Pick up 129 sticks	Pick up 301 sticks
We are fast and accurate!				





Materials for T523.5







For a more challenging version, have students turn the cards face down or put them under the truck





"I only have 5 stick cards."



"I can trade a bundle of 10 card for 10 stick cards and complete my delivery."












Reflection Questions

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1)How can you be sure students are thinking about the relative values (quantities) of the numbers involved in an addition or subtraction situation?

2)What are some different ways to think about finding the difference of 150 – 98? Which of the ways is most efficient? Which ways do you think best support student understanding and application of the base ten structure? Explain your reasoning.

3)Why is it important to have students develop and communicate their strategies used in doing advanced mental computation?

4)What is the difference of memorization and meaningful memorization?

5)Why do you think the authors of the new standards delayed teaching of the traditional algorithm for addition and subtraction until fourth grade?