CHAPTER 3:

FUNCTIONS IN 3-D

3.1 DEFINITION OF A FUNCTION OF TWO VARIABLES

A function of two variables is a relation that assigns to every ordered pair of input values (x, y) a unique output value denoted $z = f(x, y)$. Unless otherwise stated, we will assume that the variables x and y in the input (x, y) as well as the corresponding output value $z = f(x, y)$ represent real numbers.

The intuition for this definition is that for every ordered pair of numbers (x_0, y_0) that can be entered into the function, there can be only one associated output returned by the function. If we can find any (x_0, y_0) such that $f(x_0, y_0) = a$ and $f(x_0, y_0) = b$, and a is not equal to b, then f is not a function.

Example Exercise 3.1.1: If x, y, and z are defined below, determine whether f is a function.

x = length of a rectangle in meters.
y = width of a rectangle in meters.
z = area of the rectangle with length x and width y in square meters.

To understand the situation lets take a specific input, say (2, 3). Then we have $f(2, 3) = 6$ and can only be equal to 6. It doesn't matter how many times (2, 3) is input into the function, the result will always be 6. In general for any input (x_0, y_0) whatsoever we must have $f(x_0, y_0) = x_0y_0$ and that is the only possible output. Hence every ordered pair (x, y) has only one associated output and thus f is a function. In this case, we can also say that *z is a function of x and y*.

Example Exercise 3.1.2: In California, a worker pays 25% of his salary in taxes and in Mississippi, a worker pays 10% of his salary in taxes. If x, y, and z are defined below, determine whether f is a function.

x = salary in dollars per hour.

y = hours worked
z = money that remains after taxes

To understand the situation lets take a specific input, say *(x,y)* = (20,30). In California, \$600 are earned and \$150 dollars are taken in taxes hence $z = \$450$. In Mississippi, \$600 dollars are earned and \$60 are taken in taxes hence $z = 540$. As the state is not included in the input, there are at least two possible outputs for $(x,y) = (20,30)$ hence z is not a function of x and y.

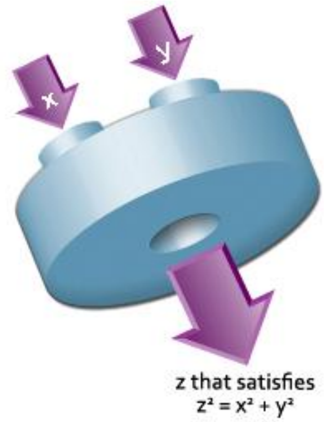
Example Exercise 3.1.3: If x, y, and z satisfy the relation $z = 5x + y^2$, determine whether z is a function of x and y.

To visualize this in the form of a function, we can write the relation in the following form:

If we put into f is (2, 3) the output is $z = 5(2) + (3)^2 = 19$. There is no alternative way to do the computation. We always get $z = 19$. Similarly, for any other input (x_0, y_0), the output must be $z = 5x_0 + y_0^2$ and can only be equal to this value. Hence $z = 5x + y^2$ represents a relation where z is a function of x and y.

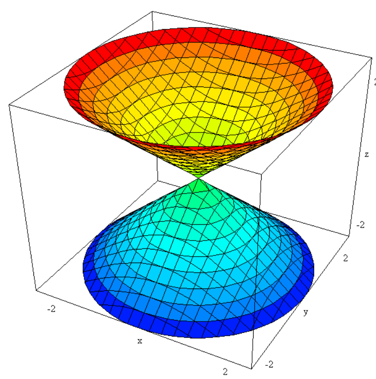
Example Exercise 3.1.4: If x, y, and z satisfy the relation $z^2 = x^2 + y^2$, determine whether z is a function of x and y.

To visualize this in the form of a function, we can write the relation in the following form:



Given the input (2, 0), we know that the z must satisfy that $z^2 = 2^2 + 0^2 = 4$. Hence when the input (2, 0) is entered into the function, both $z = -2$ and $z = 2$ are legitimate outputs. Hence we have an input that produces two possible outputs therefore z is not a function of x and y.

Example Exercise 3.1.5: Assume that x, y, and z satisfy the relation given by the points in the following surface:

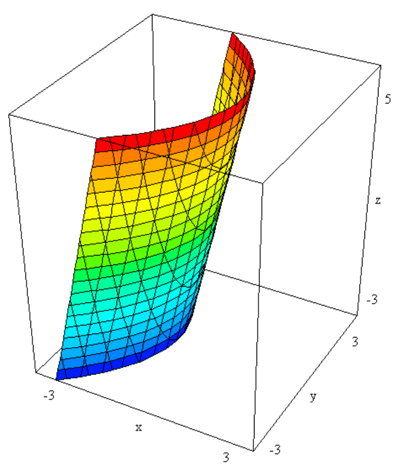


Determine whether z is a function of x and y.

To visualize this in the form of a function, we can write the relation in the following form:

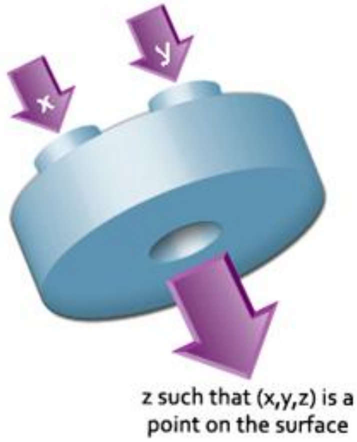
Given the input (2, 0) we know that the output *z* must be a point on the indicated surface (2, 0, *z*). Observing the surface, there are two such points (2, 0, 2) and (2, 0, -2). Hence when the input (*x*, *y*) = (2, 0) is entered into the function, both *z* = -2 and *z* = 2 are legitimate outputs. Hence for (*x*, *y*) = (2, 0) there are two possible *z* values that satisfy the relation and therefore *z* is not a function of *x* and *y*.

Example Exercise 3.1.6: Assume that *x*, *y*, and *z* satisfy the relation given by the following graph:



Determine whether *z* is a function of *x* and *y*.

To visualize this in the form of a function, we can write the relation in the following form:



Given the input (1, 0) we know that the output must be the z - coordinate of the point (1, 0, z) on the indicated surface. Viewing the graph, we can obtain only one such point (1, 0, 5). So, when the input is (1, 0) there is only one output, $z = 5$. Similarly, for any other input (x_0, y_0), we look above and below the representation of the input on the xy plane and find that there is at most one point of the form (x_0, y_0, z) on the surface. Viewing the surface, we can see that for each input, there is only one z value associated. Hence the surface represents a relation where z is a function of x and y.

3.2 REPRESENTATIONS OF A FUNCTION OF TWO VARIABLES

In this section, we start with a situation and explore how we can represent the same situation with a formula, a table, a contour, and a surface. We explore the strong and the weak points of each representation. We continually emphasize that they all are trying to represent the same set of points that satisfy the given situation. And we attempt to incorporate geometric visualization whenever it is appropriate.

THE SITUATION

Assume that two parents work. The father earns \$5.00 an hour and the mother earns \$10.00 an hour. Let

x = Number of hours that mom works in a week
y = Number of hours that dad works in a week, and
$z = f(x, y)$ = Weekly salary for the family

If we know how many hours mom and dad worked during a given week, we would like to find a means to obtain the family salary for the week given the number of hours that mom has worked and the number of hours that dad has worked. To do this we are going to use a formula, a table, a contour diagram, and a surface.

REPRESENTATIONS OF FUNCTIONS WITH A FORMULA

To find a formula for the family salary, we can divide the salary into two parts.

Mom's Salary = (Mom's hours)(10) = $10x$
Dad's Salary = (Dad's hours)(5) = $5y$

In this manner we can conclude that the family salary, z, is related to the weekly salaries of mom and dad by the formula $z = f(x, y) = 10x + 5y$

REPRESENTING FUNCTIONS WITH A TABLE

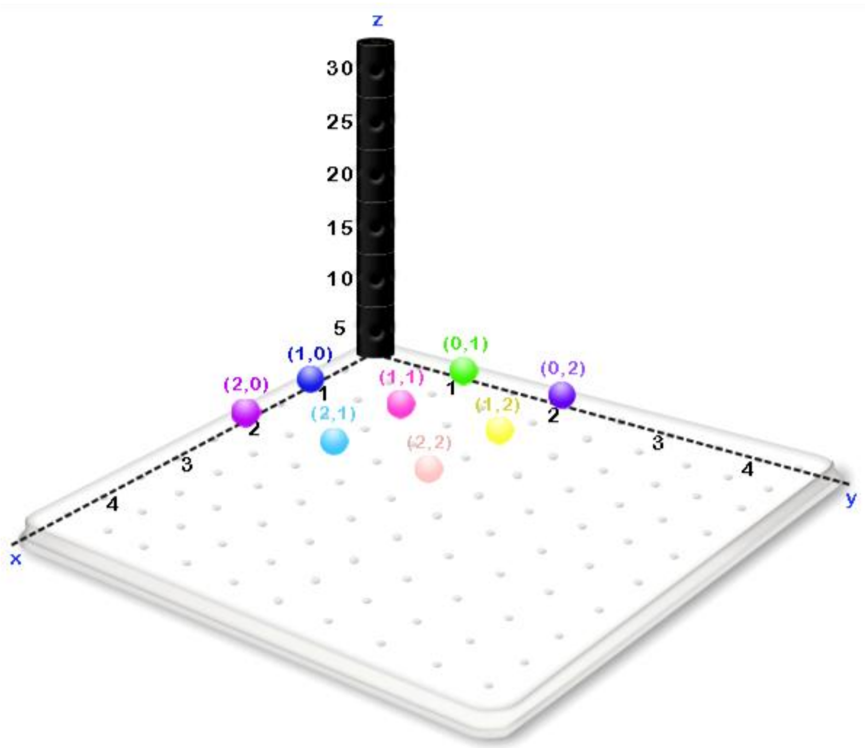
To represent the family salary associated with x and y using a table, we can start by selecting various values of x and y.

	$y = 0$	$y = 1$	$y = 2$
$x = 0$			
$x = 1$			
$x = 2$			

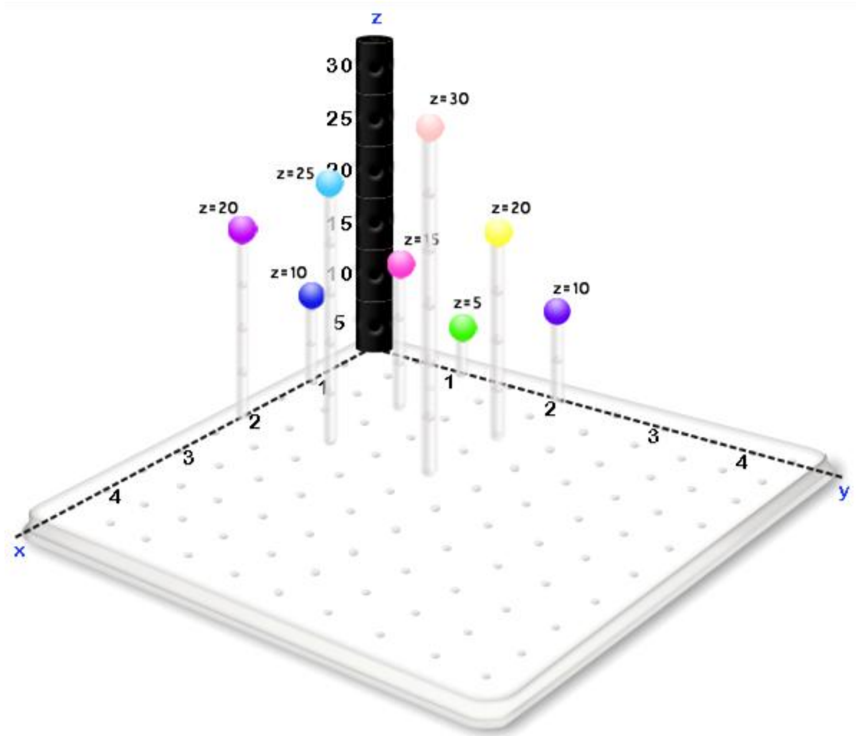
For each x and y value, we can obtain the family salary associated. For example, if mom works 1 hour and dad works 2 hours the family salary is twenty dollars. Upon finding the value for z associated with all the combinations of x and y. we can complete the table.

	$y = 0$	$y = 1$	$y = 2$
$x = 0$	0	5	10
$x = 1$	10	15	20
$x = 2$	20	25	30

Geometrically, we can see the information contained in the table by first placing a point for each (x, y) in the table on the xy plane of our 3-D space

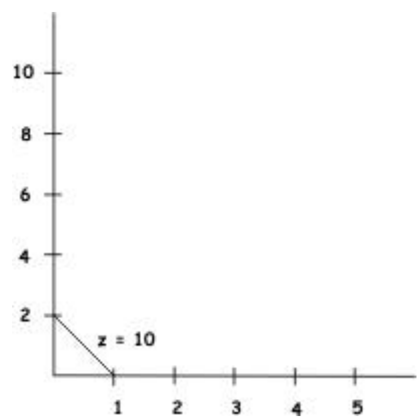


Then we can raise each point to its appropriate z value (height) in 3 dimensions.

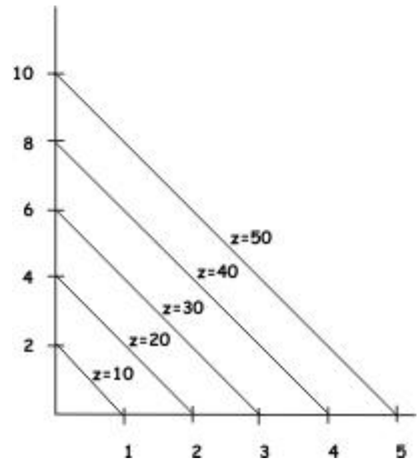


REPRESENTING FUNCTIONS WITH A CONTOUR DIAGRAM

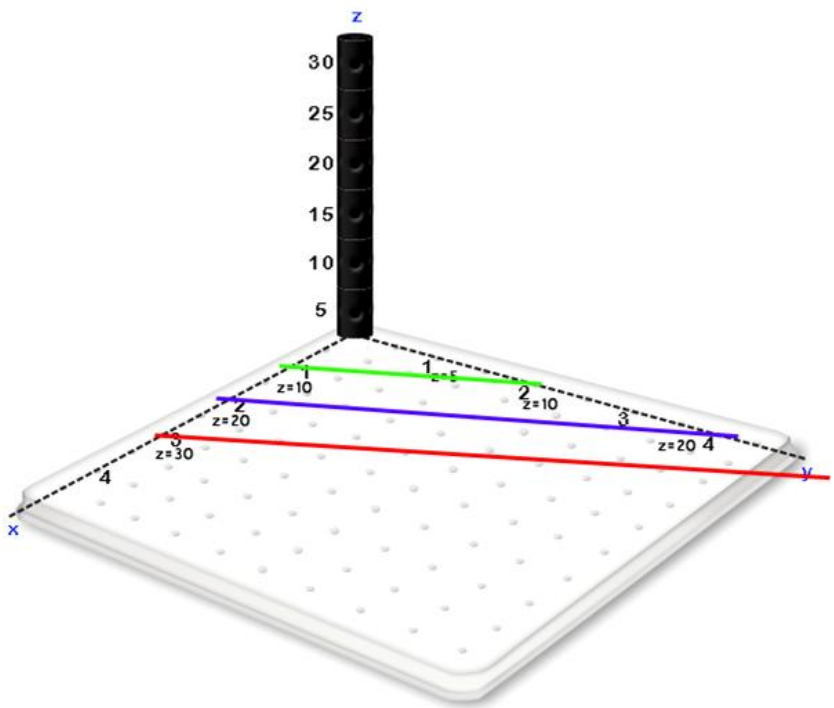
To obtain a contour diagram for the situation, we start by looking for ways in which we can obtain a family of $10. A few ways of obtaining this are (0, 2), (.5, 1), and (1, 0). With a little work, we can conclude that all (*x*, *y*), which can give us a family salary of $10, are contained in the following graph:



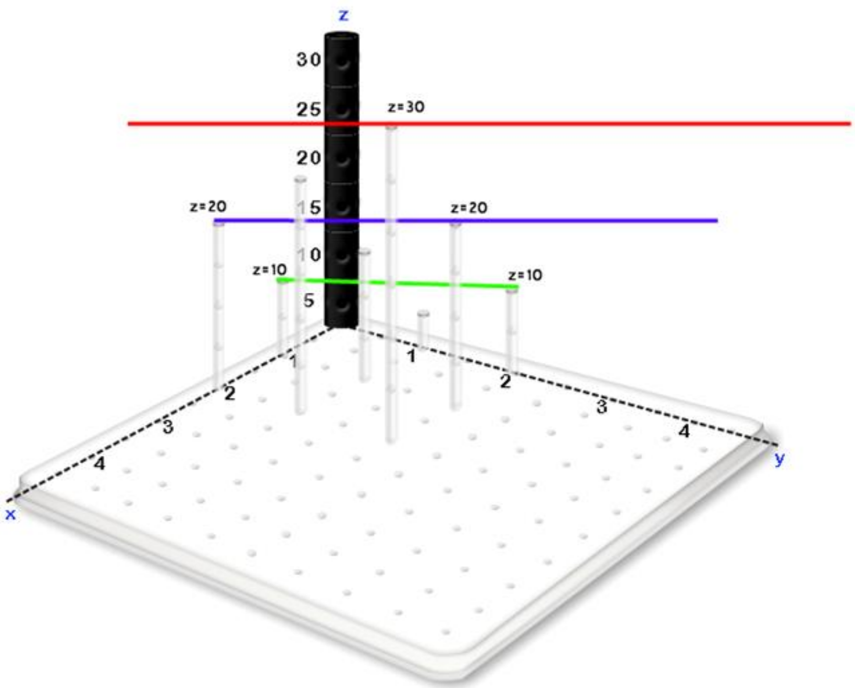
We then look for all values of (*x*, *y*) which can give us a family salary of $20, $30, $40, and $50 in a similar manner. When all these graphs are obtained, we put them together to form the following graph which is called a contour diagram.



Geometrically, we can see the information contained in the table by first placing each curve on the (x, y) plane of our 3-D space, clearly labeling each curve with its associated height.

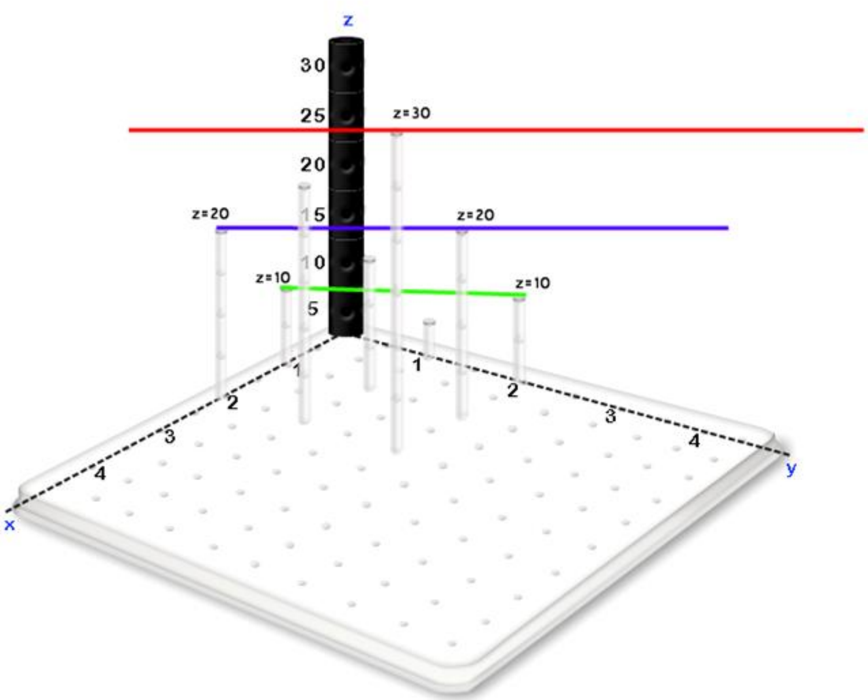


We can then raise each curve to its appropriate height in 3 dimensions to visualize the contour in three dimensions.

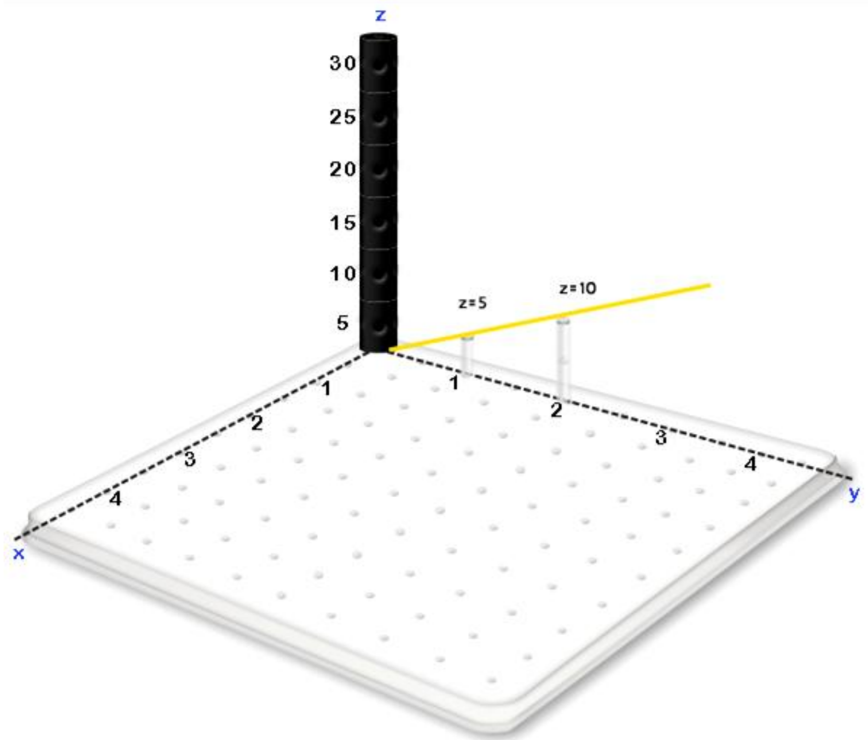


REPRESENTING FUNCTIONS WITH A SURFACE

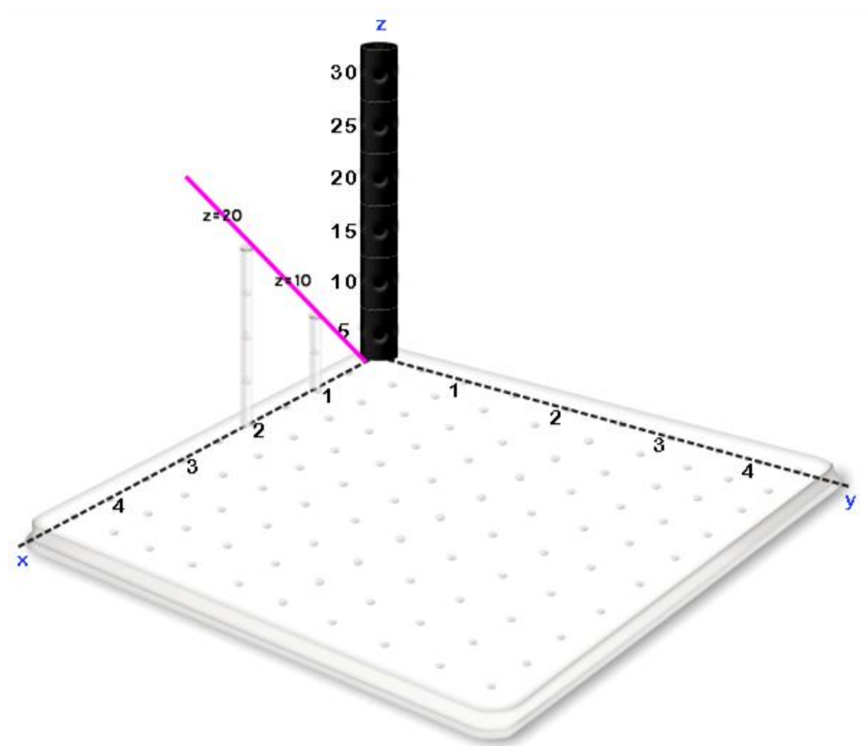
To obtain a surface, which contains all the points associated with the situation, we wish to obtain enough points to guess the rest. The best way to do this is to start with the contours we have already obtained, The following points in 3 - space we know satisfy our situation.



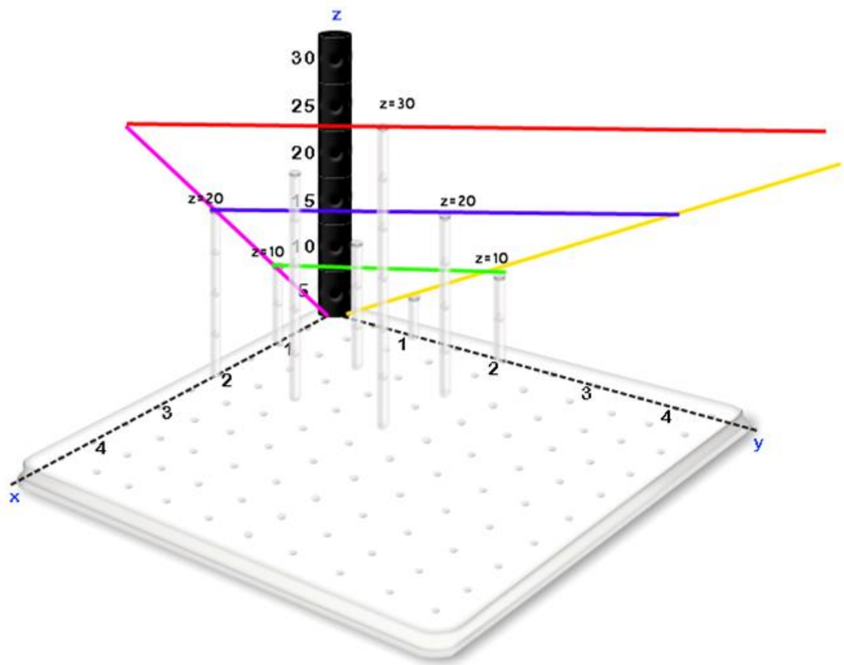
If we wish more points before guessing the surface, we can consider all the points in 3 space where mom does not work $x = 0$.



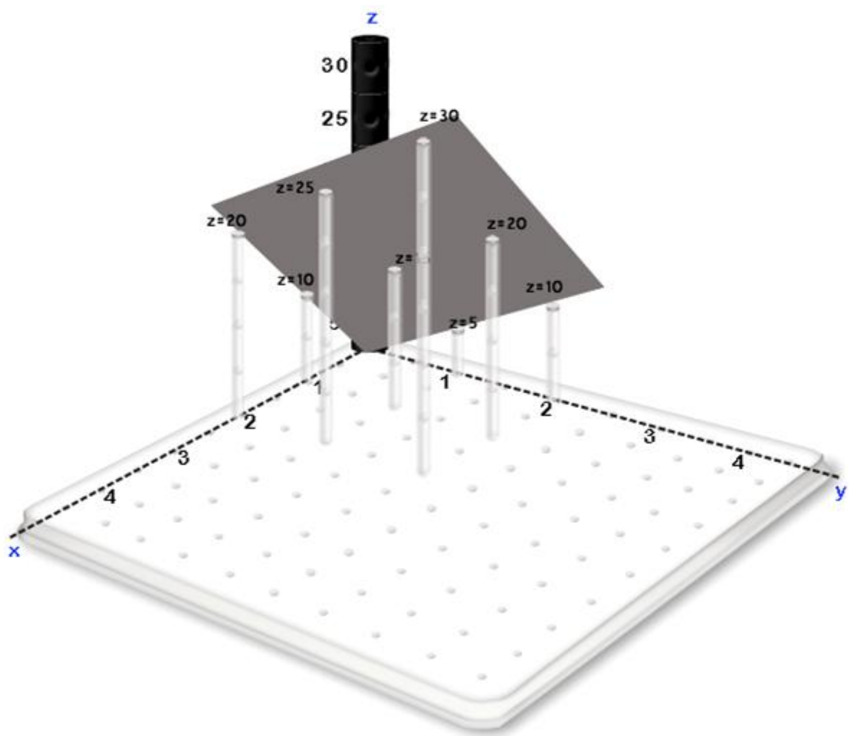
We can also consider all the points in 3 - space where dad does not work $y = 0$.



Placing all of these points that we have determined will satisfy the situation together in our 3 dimensional systems, we obtain the following:



Considering this skeleton of a surface, it is not difficult to guess the remaining points and place a surface that is consistent with these data.



3.3 DOMAIN AND RANGE OF A FUNCTION OF TWO VARIABLES

DEFINITION OF DOMAIN AND RANGE

We learned in the first section of this chapter that a function of two variables is a relation that assigns a unique output value *z* to every ordered pair of input values (*x*, *y*).

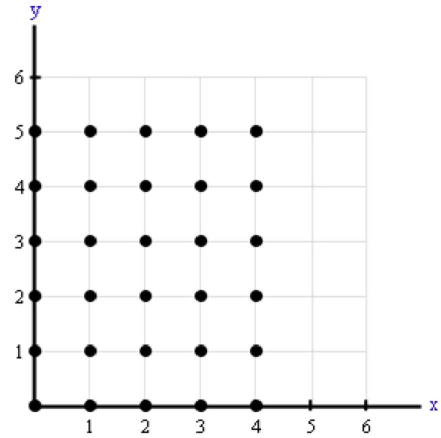
The set of all input values *(x,y)* that can be entered into a function *f* is called the domain of *f*. The set of all output values that the function f can return are called the range of *f*.

DOMAIN AND RANGE WITH SITUATIONS

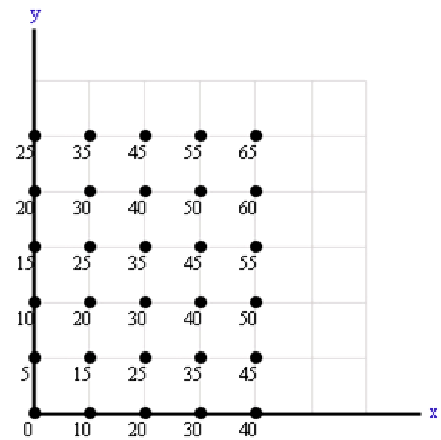
Example Exercise 3.3.1: A store has 4 apples and 5 oranges in stock and sells apples for 10 cents each and oranges for 5 cents each. A function *f* receives *x* = *number of apples* and *y* = *number of oranges* as input and returns *z* = *cost of x apples and y oranges*. If the store does not permit customers to buy fractions of a fruit, what are the domain and the range of the function?

Solution:

A customer can buy 0, 1, 2, 3 or 4 apples and 0, 1, 2, 3, 4 or 5 oranges. If *x* = *number of apples* and *y* = *number of oranges*, the following points on the *xy* – plane represent the possible *(x,y)* values that can be entered into *f*.



From this diagram, it can be seen that the domain of *f* consists of the ordered integer pairs from *(0,0)* to *(4,5)*. To determine the range, we can write the output value that *f* will return next to each possible ordered pair in the domain of *f*:



By observing all of the possible outputs from all of the possible inputs, we can see that the range of *f* is *{0, 5, 10, 15, 20, 25,…, 65}*.

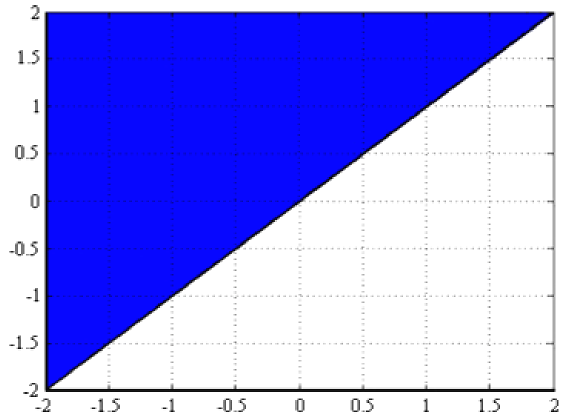
DOMAIN AND RANGE WITH FORMULAS

Example Exercise 3.3.2: A function *f* has input *(x,y)* and output $f(x,y) = \sqrt{y-x}$. What are the domain and range of *f?*

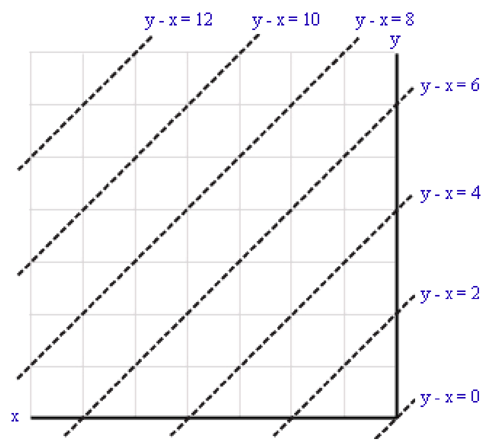
Solution:

The domain of the function consists of all *(x,y)* such that $y - x > 0$. The ordered pairs *(x,y)* that satisfy this are shaded blue in the *xy*-plane below. This shaded region represents the domain of *f*. The surface that represents the function *f* will reside over this blue shaded region.

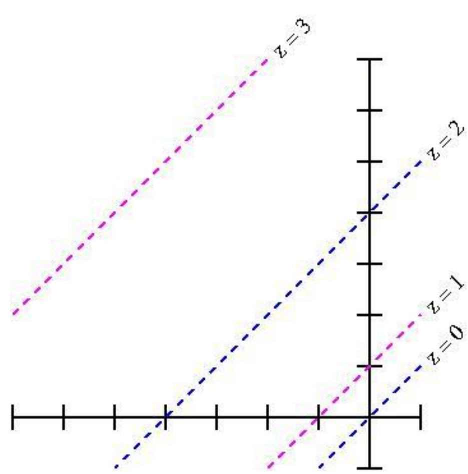
There will be no z values associated with the unshaded region. For example, the point *(1,0)* lies in the unshaded region. When the point *(1,0)* is entered into the function *f*, *f* returns $f(1,0) = \sqrt{0-1} = \sqrt{-1}$ which is undefined.



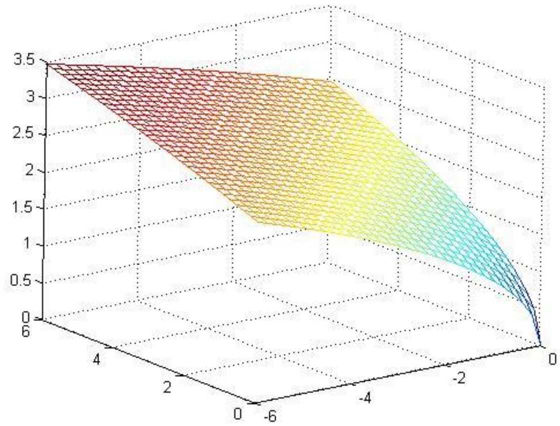
To determine the range we can observe the behavior of the lines $y - x = k$ where k is constant.



These lines will produce the following contour diagram for the surface



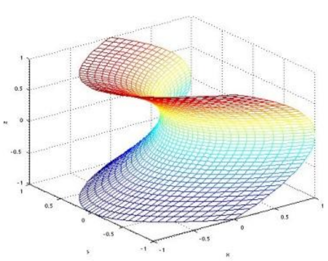
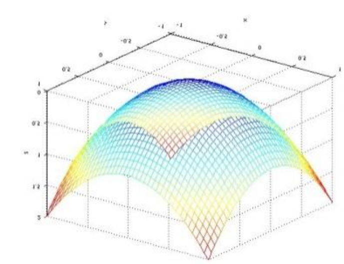
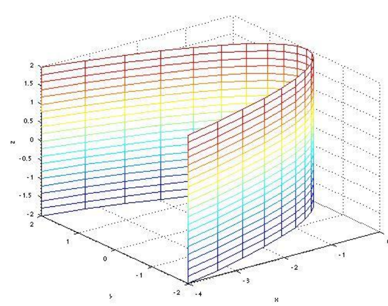
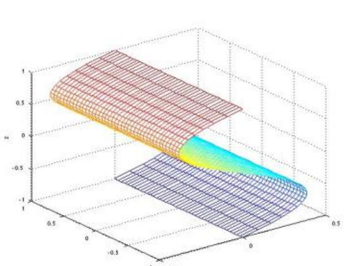
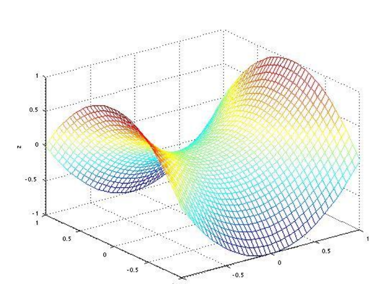
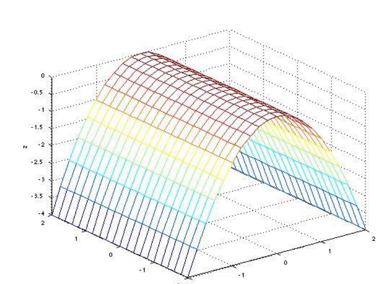
The surface associated with *f* will look like the following:



Hence, as $y - x$ approaches ∞, the output of f which is $\sqrt{y-x}$ will also approach ∞. The range of f is $0 \leq z \leq \infty$.

EXERCISE PROBLEMS:

1) Indicate which of the following surfaces could represent a functional relation between *x*,*y* and *z*.

A. 	B. 	C. 
D. 	E. 	F. 

2) Indicate which of the following formulas could represent a functional relation between *x,y* and *z*. If it could represent a functional relation, shade the domain of the function in the *xy* plane and indicate the range of the function in the form $a \leq z \leq b$.

a) $x + y + z = 4$
b) $x^2 + y^2 + z^2 = 1$
c) $\sqrt{x^2 + y^2 + z^2} = 3$
d) $x + y + z^2 = 5$
e) $x + y + z^3 = 2$
f) $\sqrt{x^2 + y^2 + z} = 3$

3) For each of the following functions that are represented by a formula, do the following:

(i) Create a table which contains the output from the function for all integer values of *x* and *y* between -2 and 2.
(ii) Construct a contour diagram of the function for all integer values of *z* between 0 and 4.
(iii) Draw the graphs of the cross sections *x=-2, x=0, x=2, y=-2, y=0* and *y=2.*
(iv) Place sufficient curves from parts (ii) and (iii) on your 3D kit to guess the nature of the surface.
(v) Use a graphing utility to verify that the surface you guessed does represent function.

a. $z = 3x + 4$
b. $z = 2x + 3y + 2$
c. $z = x^2 + y^2 + 1$
d. $z = 4x^2 + y^2$
e. $z = x - y^2 - 1$
f. $z = x^2 - y^2$
g. $z = \sqrt{4x^2 + y^2}$

PRACTICE PROBLEMS

PP 3.1.1

The axis places z in a corner of the base of the 3D Kit. Suppose that *f* is given by Table 7.2.2. Use your 3D Kit to represent the graph of *f*.

x/y	1	3	5
1	2	4	6
3	3	5	7
5	4	6	8

Table 7.2.2

PP 3.1.2

The axis places z in a corner of the base of 3D Kit. Let *f* be defined by $f(x) = 3x - 2y + 2$. Use your 3D Kit to represent at least 9 points of the graph of *f*.

PP 3.1.3

Given the relationship $x + y + z = 8$.

a. Use your 3D Kit and place the points $(x, y) = (2, 2), (2, 3), (3, 2)$ and $(3, 3)$ on the xy plane.

b. Next to each point label the *z* values associated with the point. How many z values are there for each point?

c. Does the relation define z as a function of x and y? If not, find a contradiction (an input (x, y) that has two outputs), if so, show clearly that for every (x_0, y_0) there is only one possible z value.

d. Raise the points placed on the base of your 3D Kit to the height of their associated z values.

e. These are points on the graph of what function?

PP 3.1.4

Given the relationship $x + y + z^2 = -2$.

a. Use your 3D Kit and place the points $(x, y) = (1, 1), (5, 6), (6, 5)$ and $(3, 3)$ on the xy plane.

b. Next to each point label the *z* values associated with the point. How many z values are there for each point?

c. Does the relation define z as a function of x and y? If not, find a contradiction, if so, show clearly that for every (x_0, y_0) there is only one possible z value.

d. Raise the points placed on the base of your 3D Kit to the height of their associated *z* values.

e. These are points on the graph of what function?

PP 3.2.1

For travel on interstates, there is a limit to the total weight of a vehicle on the road. If x = weight of a truck in tons, y = weight of a truck in tons, and $z = f(x, y)$ = total weight of the truck in tons.

a. Find a formula to represent f given inputs x and y.

b. Let x and y be integers such that $0 \le x \le 3$, and $0 \le y \le 3$. Construct a table to represent f given inputs x and y.

c. If $z = f(x, y) = 2$, find the graph of all x and y that can produce a total weight of 3 tons.
If $z = f(x, y) = 3$, find the graph of all x and y that can produce a total weight of 2 tons.
If $z = f(x, y) = 4$, find the graph of all x and y that can produce a total weight of 4 tons.

d. Place all these graphs together in a contour diagram, carefully labeling the z value associated with each contour.

e. Draw the contour diagram on the base of your 3D Kit. Place a line over each contour and rise each contour to its associated z value.

f. If $x = 0$ (a weightless truck which has yet to be invented!), find the graph of all y and z.
If $y = 0$ (no cargo), find the graph of all x and z.

g. Place the surface that represents the function f and use a graphing utility to draw a 3D graph of this function.

PP 3.2.2

A function f can be represented with the formula $f(x, y) = \sqrt{x^2 + y^2}$. In this lab we will find the graph of the surface that can also represent the function f.

a. Draw the contours corresponding to $z = 0$, $z = 2$, $z = 3$ and $z = 4$ on the xy plane of your 3D Kit, labeling each curve with its corresponding z value. Raise each curve to its corresponding height. What can you say about the contour corresponding to $z = -2$? What do you think the surface will look like?.

b. Represent on your 3D Kit the cross-section corresponding to $y = 0$. How does this fit with the contour curves you had previously found?

c. Represent on your 3D Kit the cross-sections corresponding to $x = 0$ and $y = 0$ together with the contour curves found in part (a). What do you think the surface will look like?

d. Draw the graph of the surface that can be used to represent f.

e. Use a graphing utility to verify your graph of the surface.

PP 3.2.3

In this problem you will use the 3D Kit to represent the graph of $f(x, y) = x + y^2$.

a. Represent the contours corresponding to $z = 0$, $z = 1$, $z = 2$ and $z = 3$ on the xy plane of your 3D Kit, labeling each curve with its corresponding z value. Now raise each curve to its corresponding height. What can you say about the contour curve corresponding to $z = -1$? What do you think the surface will look like?.

b. Now put down all the contours for a while and let's work with cross sections. Represent on your 3D Kit the cross-section corresponding to $y = 0$. How does this fit with the contour curves you had previously found?

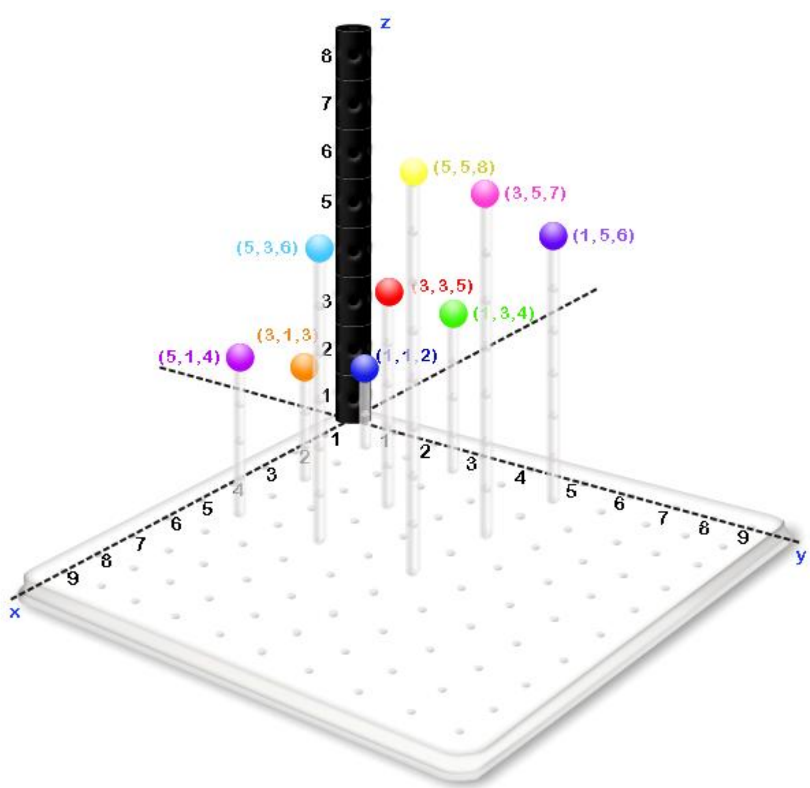
c. Represent on your 3D Kit the cross-sections corresponding to $x = 0$ and $y = 0$ together with the contour curves found in part (a). What do you think the surface will look like?

d. Draw the graph of the surface.

e. Use a graphing utility to verify your graph of the surface.

SOLUTIONS:

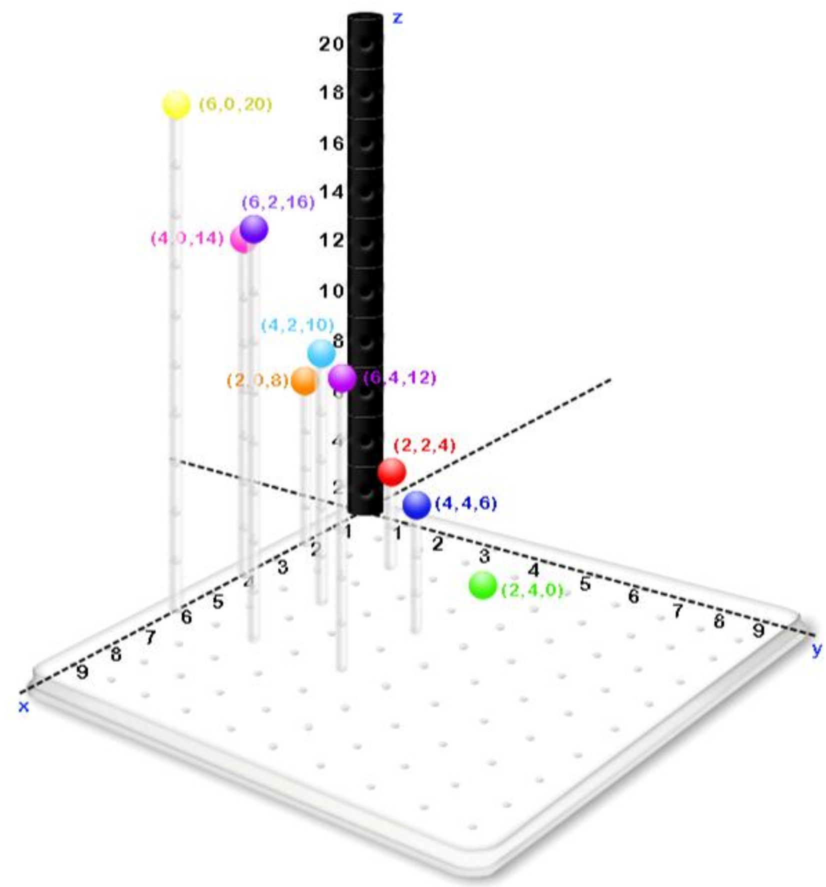
PP 3.1.1



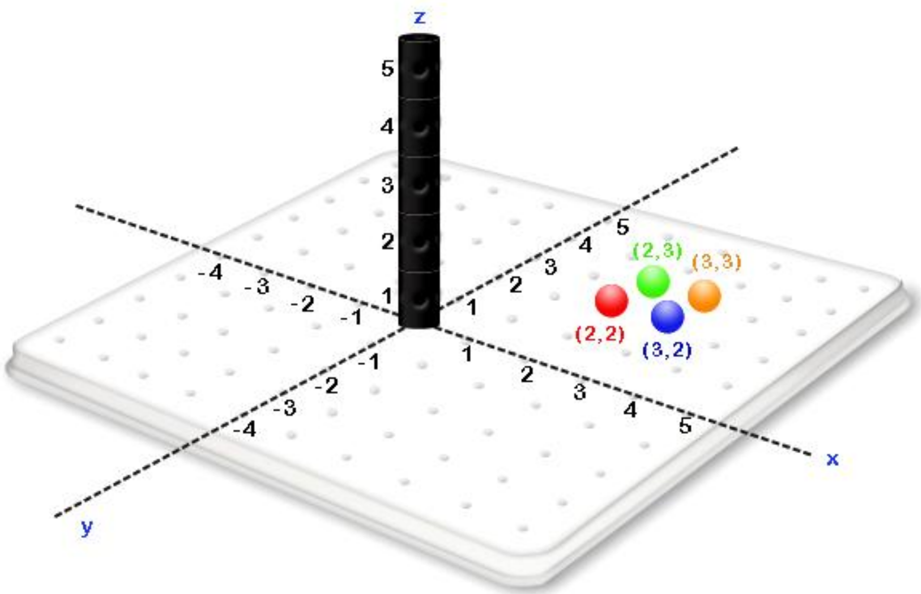
PP 3.1.2

x/y	0	2	4
2	8	4	0
4	14	10	6
6	20	16	12

To consider the points in the axis Z of 2 in 2.



PP 3.1.3



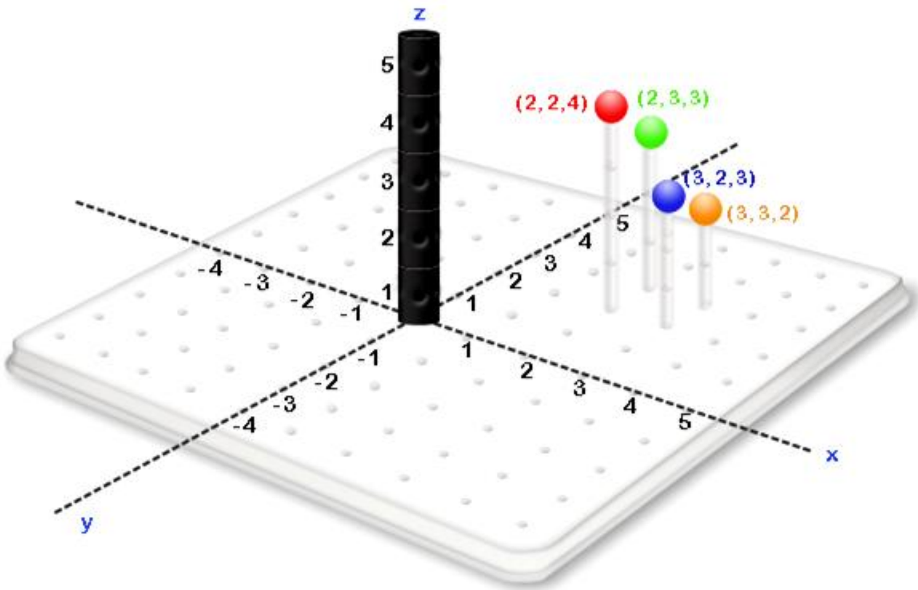
a.

b. (x, y) = (2, 2) ⇒ 2 + 2 + z = 8 ⇒ 4 + z = 8 ⇒ z = 8 - 4 ⇒ z = 4.
(x, y) = (2, 3) ⇒ 2 + 3 + z = 8 ⇒ 5 + z = 8 ⇒ z = 8 - 5 ⇒ z = 3.
(x , y) = (3, 2) ⇒ 3 + 2 + z = 8 ⇒ 5 + z = 8 ⇒ z = 8 - 5 ⇒ z = 3.
(x, y) = (3, 3) ⇒ 3 + 3 + z = 8 ⇒ 6 + z = 8 ⇒ z = 8 - 6 ⇒ z = 2.

There is only one value for each point.

c. x + y + z = 8 ⇒ z = 8 - x - y.

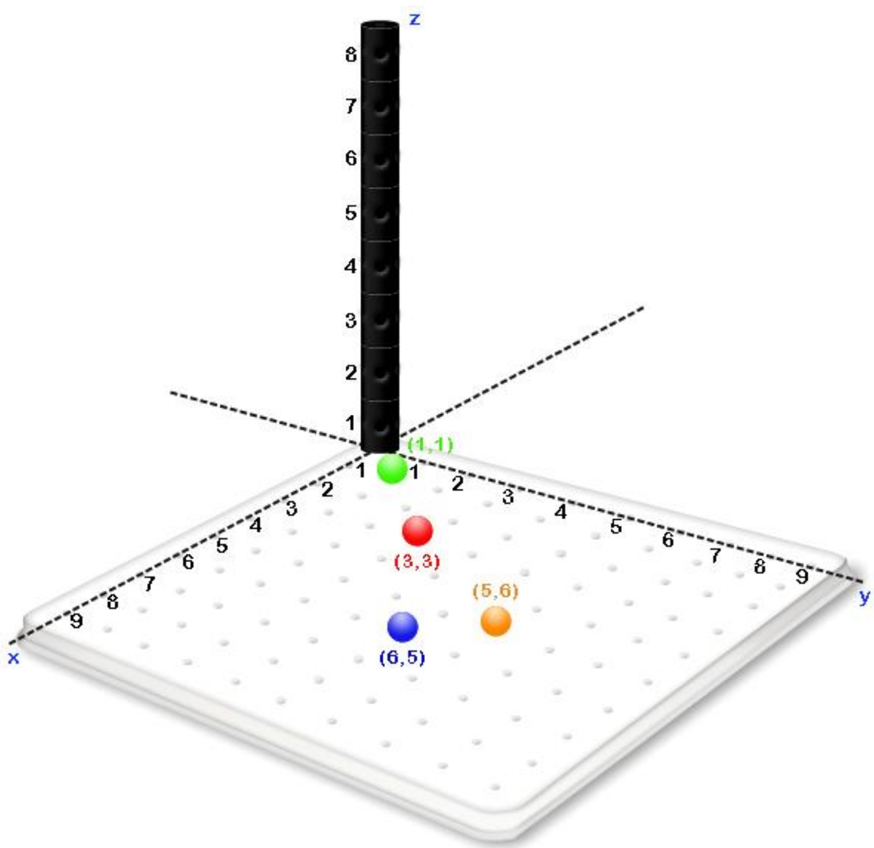
Observe that for every input (x0, y0) there is only one output z = 8 - x0 - y0.



d.

e. These points are on the graph of the function $z = f(x, y) = 8 - x - y$.

PP 3.1.4



a.

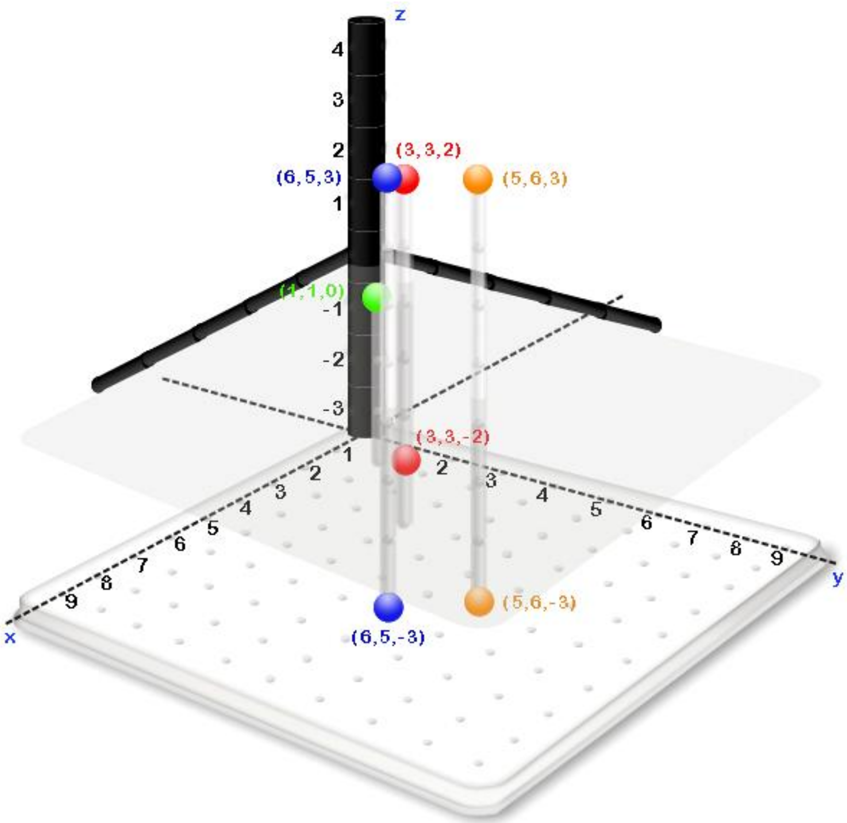
b. $(x, y) = (1, 1) \Rightarrow 1 + 1 - z^2 = 2 \Rightarrow 2 - z^2 = 2 \Rightarrow - z^2 = 2 - 2 \Rightarrow z^2 = 0 \Rightarrow z = 0$.
$(x, y) = (3, 3) \Rightarrow 3 + 3 - z^2 = 2 \Rightarrow 6 - z^2 = 2 \Rightarrow - z^2 = 2 - 6 \Rightarrow - z^2 = - 4 \Rightarrow z^2 = 4 \Rightarrow z = \pm 2$.
$(x, y) = (5, 6) \Rightarrow 5 + 6 - z^2 = 2 \Rightarrow 11 - z^2 = 2 \Rightarrow - z^2 = 2 - 11 \Rightarrow - z^2 = - 9 \Rightarrow z^2 = 9 \Rightarrow z = \pm 3$.
$(z, z) = (6, 5) \Rightarrow 6 + 5 - z^2 = 2 \Rightarrow 11 - z^2 = 2 \Rightarrow - z^2 = 2 - 11 \Rightarrow - z^2 = - 9 \Rightarrow z^2 = 9 \Rightarrow z = \pm 3$.

There is two value for each point.

$x + y - z^2 = 2 \Rightarrow x + y - 2 = z^2 \Rightarrow \pm (x + y - 2)^{1/2} = z$.

The previous relation does not represent a function, since there are inputs (x0, y0) that have more than one output, for example $z = (x0 + y0 - 2)^{1/2}$ and $z = - (x0 + y0 - 2)^{1/2}$.

c. Considers the base of 3D Kit as the plane *z* = - 3.



d.

e. Two functions might be had, $z = (x + y - 2)^{1/2}$ and $z = - (x + y - 2)^{1/2}$.

PP 3.2.1

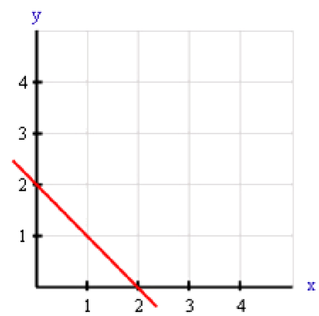
a. $z = f(x, y) = x + y$.

b. $0 \leq x \leq 3, 0 \leq y \leq 3.$

x/y	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

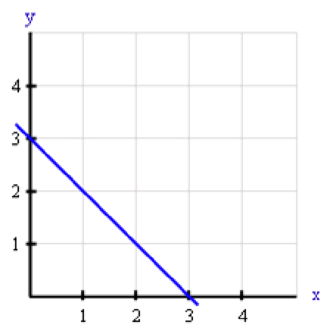
c. $z = 2 \Rightarrow x + y = 2.$

x	y
0	2
2	0



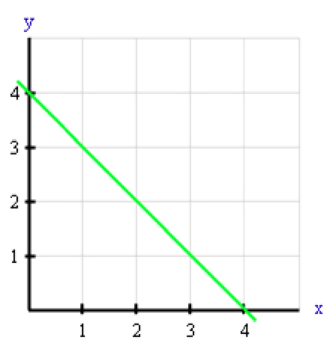
$z = 3 \Rightarrow x + y = 3.$

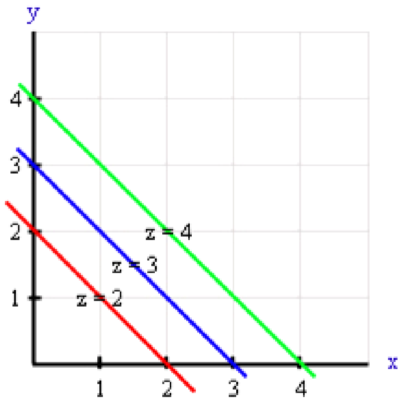
x	y
0	3
3	0

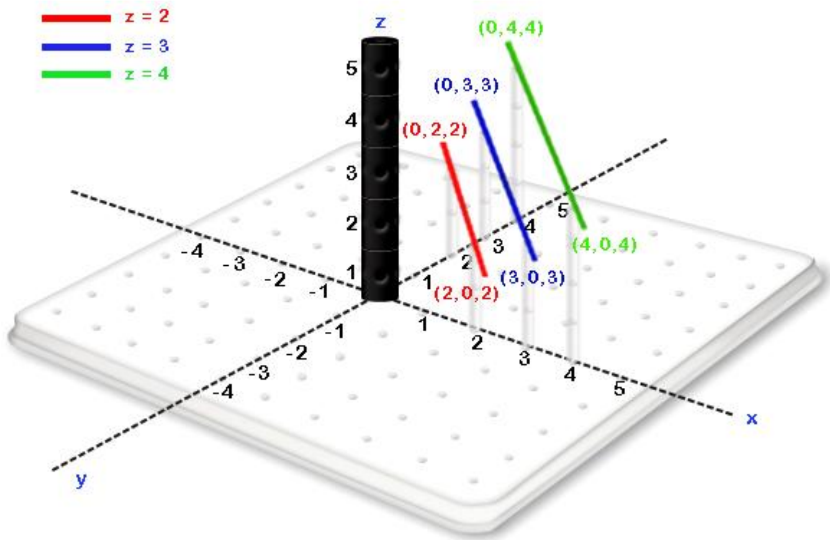


$z = 4 \Rightarrow x + y = 2.$

x	y
0	4
4	0



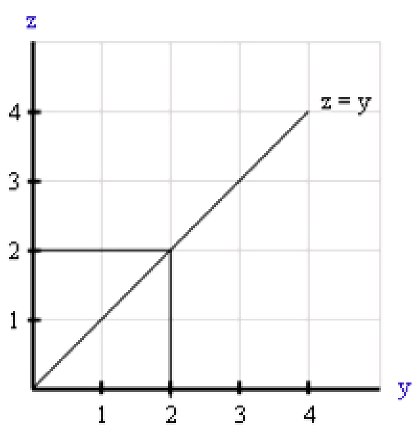
d. 

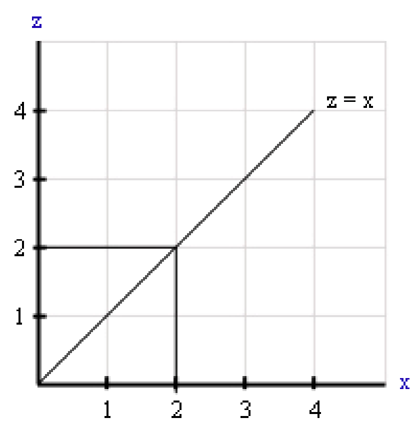
e. 

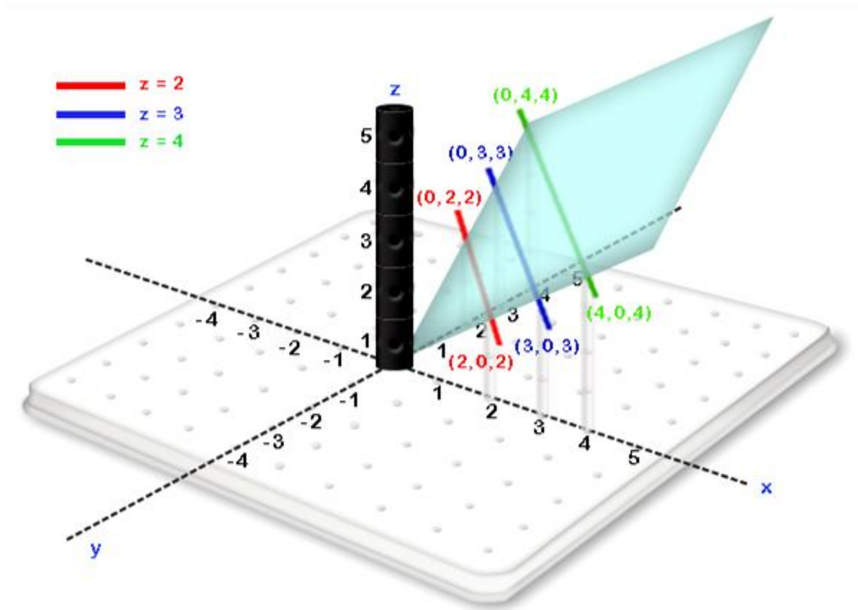
f.

$x = 0 \Rightarrow z = y$

$y = 0 \Rightarrow z = x$





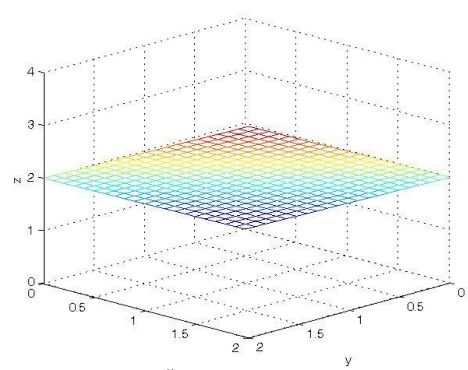


g.

With the following commands in MATLAB or OCTAVE,

```
[x,y] = meshgrid(0:.1:2, 0:.1:2);
z = x  + y;
mesh(x,y,z)
```

we obtain the following graph.



PP 3.2.2

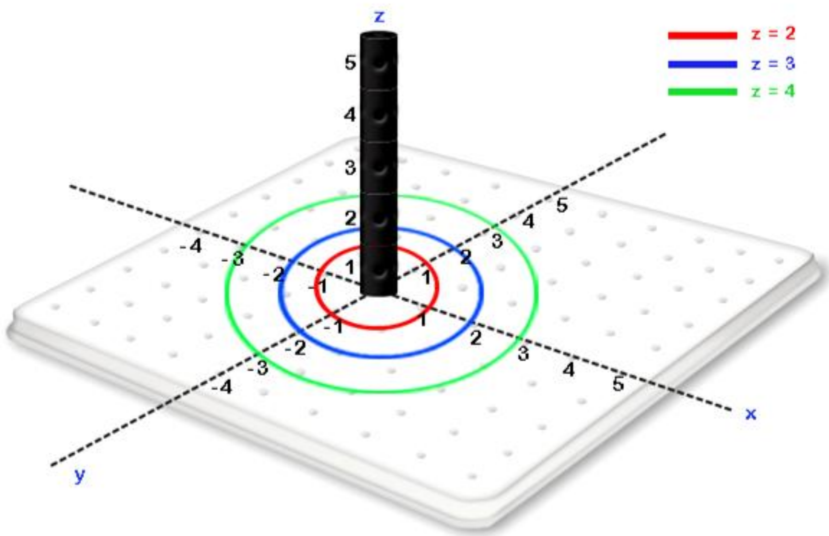
a. The contours are:

z = 0 ⇒ $0 = \sqrt{x^2 + y^2}$ ⇒ $x^2 + y^2 = 0$ ⇒ x = 0 and y = 0 ⇒ (0, 0).

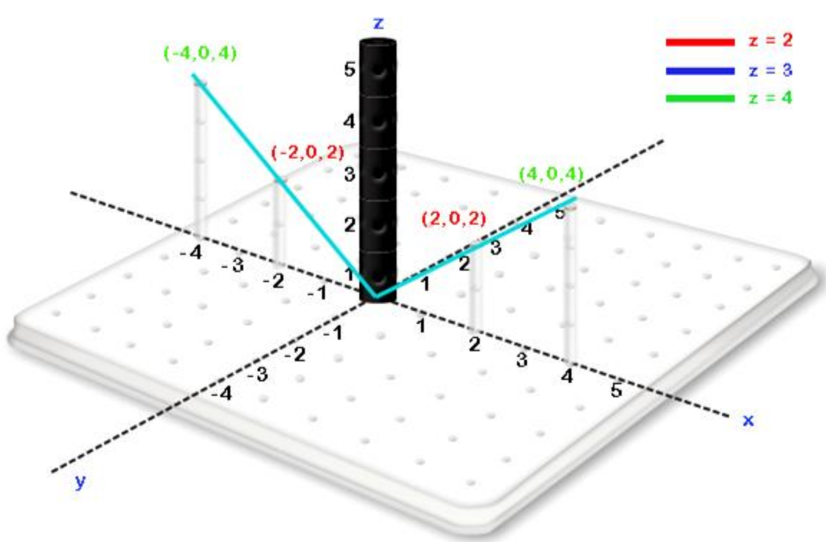
z = 2 ⇒ $2 = \sqrt{x^2 + y^2}$ ⇒ $x^2 + y^2 = 4$ ⇒ Equation of a circle of radius 2 with center in (0, 0).

z = 3 ⇒ $3 = \sqrt{x^2 + y^2}$ ⇒ $x^2 + y^2 = 9$ ⇒ Equation of a circle of radius 3 with center in (0, 0).

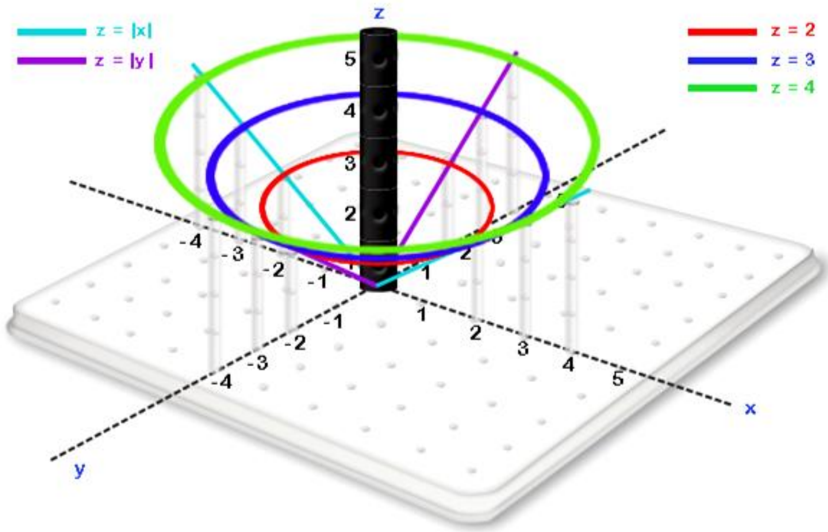
z = 4 ⇒ $4 = \sqrt{x^2 + y^2}$ ⇒ $x^2 + y^2 = 16$ ⇒ Equation of a circle of radius 4 with center in (0, 0).



b. y = 0 ⇒ $z = \sqrt{x^2 + 0^2}$. This is $z = |x|$ so we know (0,0,0), (-2,0,2), (2,0,2), (-4,0,4) and (4,0,4) are some of the points that satisfy $z = |x|$ when y = 0. The cross section y = 0 which includes these points and others is shown on the kit below.



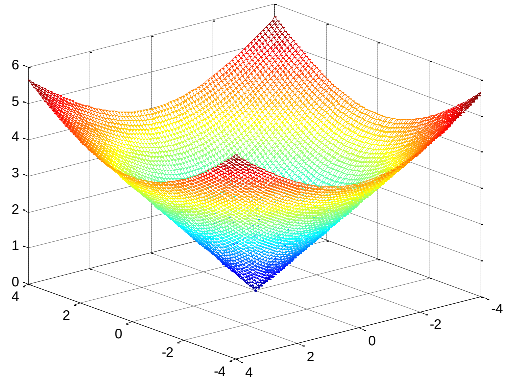
c. x = 0 ⇒ $z = \sqrt{0^2 + y^2}$. This is $z = |y|$. So we know (0,0,0), (0,-2,2), (0,2,2), (0,-4,4) and (0,-4,4) are some of the points satisfying $z = |y|$ when x = 0. The cross section x = 0 as well as y = 0 and the contours are shown below.



d & e. With the following commands in MATLAB or OCTAVE,

```
[x,y] = meshgrid(-2:.1:2, -2:.1:2);
z = sqrt(x.*x  + y.*y);
 mesh(x,y,z)
```

we obtain the following graph which demonstrates the computer image for the surface representation of the function $f(x, y) = \sqrt{x^2 + y^2}$.



PP 3.2.3

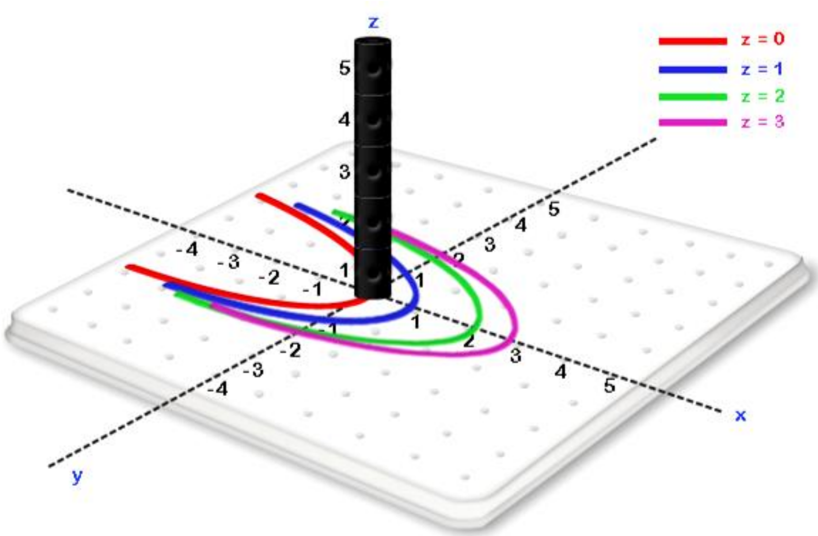
a. The contours are:

1. $z = 0 \Rightarrow x + y^2 = 0 \Rightarrow x = -y^2$. This is the equation of a parabola that open toward them $x < 0$ with vertex (0, 0).

2. $z = 1 \Rightarrow x + y^2 = 1 \Rightarrow x = 1 - y^2$. This is the equation of a parabola that open toward them $x < 1$ with vertex (1, 0).

3. $z = 2 \Rightarrow x + y^2 = 2 \Rightarrow x = 2 - y^2$. This is the equation of a parabola that open toward them $x < 2$ with vertex (2, 0).

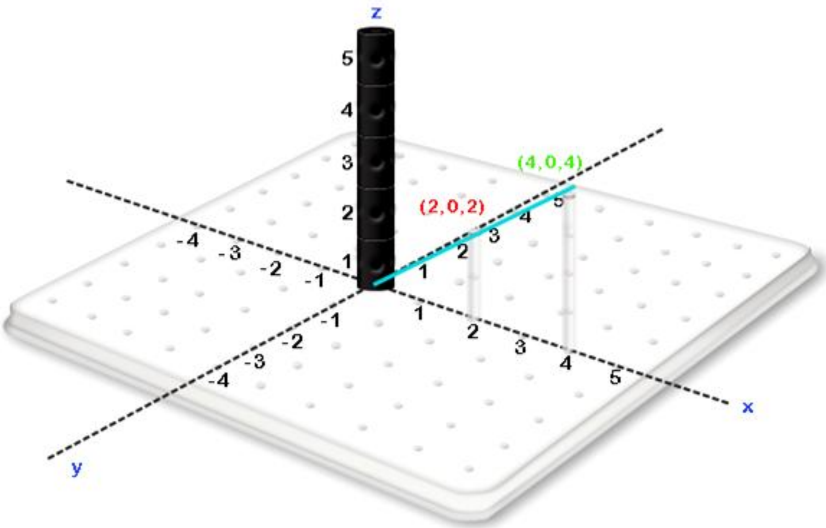
4. $z = 3 \Rightarrow x + y^2 = 3 \Rightarrow x = 3 - y^2$. This is the equation of a parabola that open toward them $x < 3$ with vertex (3, 0).



Now, if $z = -1 \Rightarrow x + y^2 = -1 \Rightarrow x = -1 - y^2$. This is the equation of a parabola that open toward them $x < -1$ with vertex (-1, 0).

b. If $y = 0 \Rightarrow z = f(x, 0) = x + 0^2 = x$. This is $z = x$.

x	z
2	2
4	4

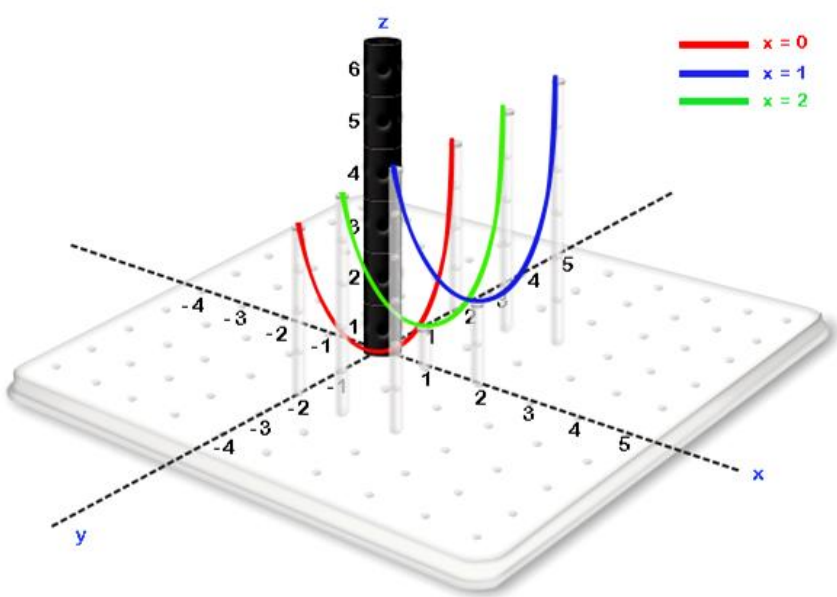


c.

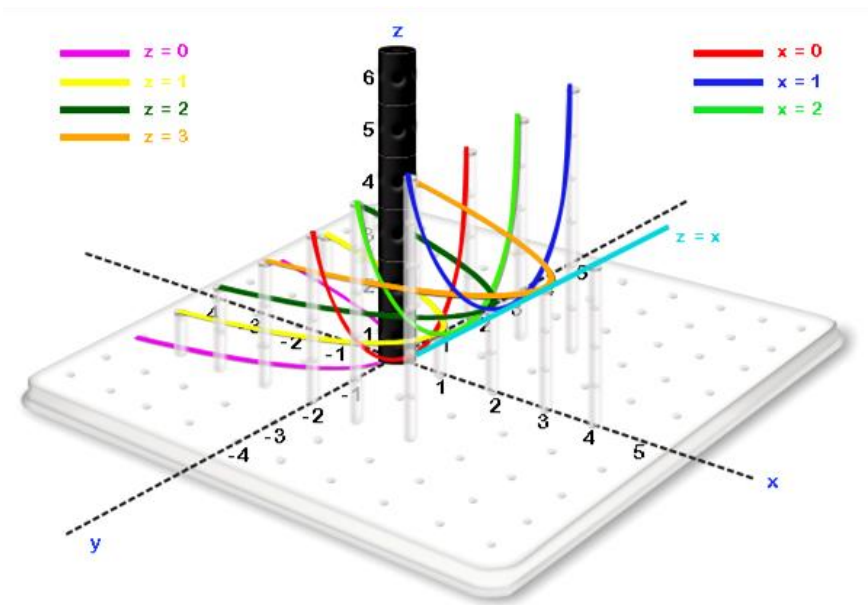
d. If x = 0 ⇒ $z = 0 + y^2 = y^2$. This is the equation of a parabola that open toward them z > 0 with vertex (0, 0) on the yz plane.

If x = 1 ⇒ $z = 1 + y^2$.This is the equation of a parabola that open toward them z > 1 with vertex (0, 1) on the yz plane.

If x = 2 ⇒ $z = 2 + y^2$.This is the equation of a parabola that open toward them z > 2 with vertex (0, 2) on the yz plane.



Then, for (a) and (b) we obtain that:



e. With the following commands in MATLAB or OCTAVE,

```
[x,y] = meshgrid(-2:.1:2, -2:.1:2);
z = x  + y.*y;
mesh(x,y,z)
```

we obtain the following graph which represents the computer image for the surface representation of the function $f(x, y) = x + y^2$.