**Algebra I Rich Task**
**The Border Problem**

### Task Overview
The Border Problem Task is a traditional problem in which students are asked to write an expression to calculate the number of tiles needed to border a square pool. Students work in partners to make sense, analyze and record their conjectures they have about the problem. A full class discussion takes place at the conclusion of the lesson with the goal of having students make connections between an expression and a visual representation.

### Mathematical Big Ideas:
***Variable*** – Students explore what a variable represents when used in an expression (e.g. the side length of the pool). In the border problem, the variable is the side length of the square pool and the expression calculates the number of tiles needed to surround the pool. This expression will work for any possible dimension of a square pool.

> Common partial understanding – For most students, their initial experiences with variables are as a specific unknown. Students have typically solved an equation to determine the value of the variable. In this case, students may try to (incorrectly) solve for variable.

***Equivalence*** – Any quantity or expression can be expressed in an infinite number of ways. Some ways may be more useful than others, depending on the context and purpose. In this task, students will see several different but equivalent expressions for calculating the number of tiles in the border. In many cases, **the expression tells a story about the structure** of the context or how the mathematician sees the context. Some forms may provide a simpler way to do the calculation.

### Standards for Mathematical Practice
Lead Practice
- SMP #7: Look for and make use of structure

Supporting Practices
- SMP #3: Construct viable arguments and critique the reasoning of others
- SMP #4: Model with mathematics

### Cognitive Demand:
This task requires students to make connections between and among mathematical ideas. Research has shown that using high cognitive demand tasks in ways that support rigor will lead to increases in student learning. A critical component of a high cognitive task is that students are invited to explain their thinking, make new connections, describe their process, and/or critique the ideas of others. Make sure the focus is on connections and sense making and not solely on answer getting.

### Expectations for Students as They Complete the Task
- Students will work on the task in pairs.
- Students will record their model and answer(s) to question posed.
- Students will have the freedom to approach the problem any way that makes sense.

### The Border Problem Task

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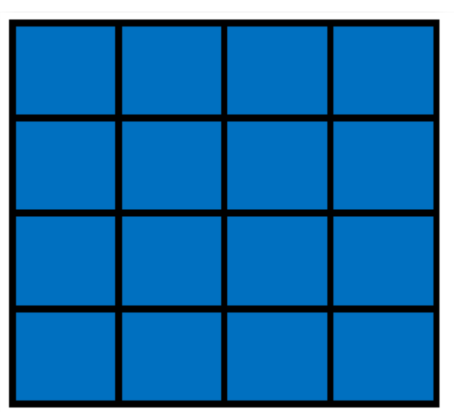
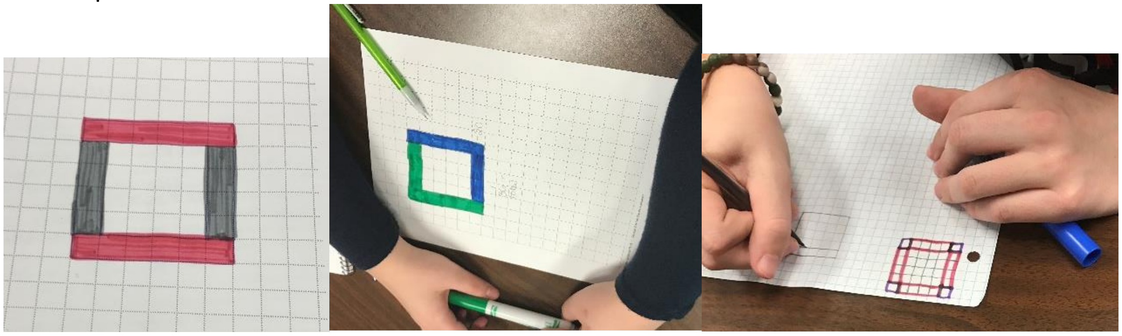
|  | Show an image similar to the one on the left. Ask “What do you notice?” This is an open-ended exploration. Allow for individual think time. Record all noticings, colors, shapes, etc. The Noticing Routine provides access for all students to begin thinking about the problem.<br><br>Introduce (or build on one of the student’s noticings) the idea that this is a pool. Ask students to work in pairs to determine “How many tiles would it take to make a border around the pool?” The focus of the partner work is to provide students with practice in explaining their thinking and actively work to understand their partner’s thinking.<br><br>Give students graph paper. Ask them to use colors to show the border and find an efficient way to count the number of tiles. |

Student work examples



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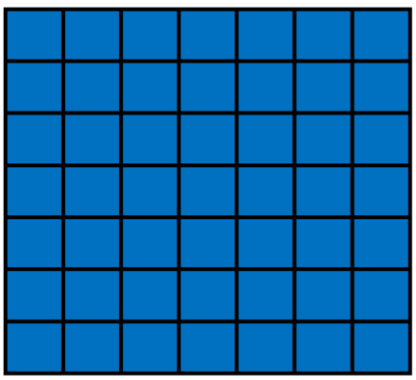
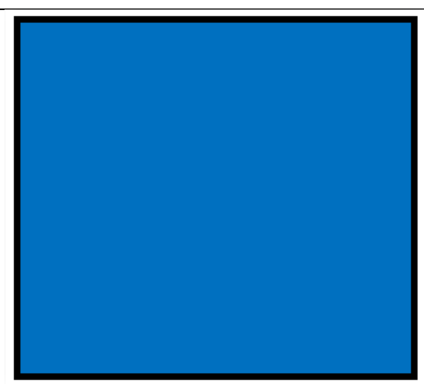
|  | Ask students to now consider a 7x7 pool (pictured to the left). Ask them to determine the total number of tiles it would take to make a border. Encourage them to look for an efficient way to count and show the structure using color.<br><br>Depending on your students' experience, you may additional ask students to predict (or calculate) the number of tiles needed for a 100x100 pool. Intermediate pool sizes may also be assigned. |
|  | Next, ask student to consider a general way for determining the number of tiles for any pool. (pictured to the left). How might they figure out how many tiles are needed to create a border?<br><br>Some students may struggle to represent the length of a pool with a variable. After some work time, discuss student work in which a student has used a variable to represent the edge length of the pool.<br><br>Collect student work. |

**Class Discussion - Connecting Representations**

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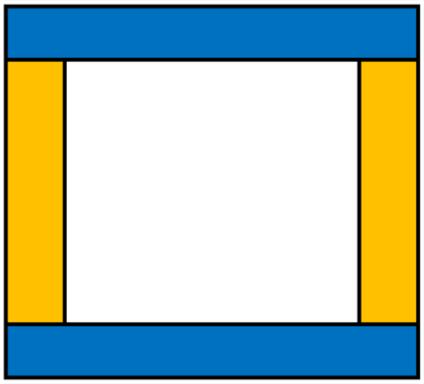
| $2n + 2(n+2)$<br>$4n + 4$<br>$4(n+1)$<br>$2(2n+2)$<br>$4(n+2) - 4$<br>$(n+2)^2 - n^2$ | The class discussion is held the following day. This gives the teacher time to analyze the student work and make instructional decisions about how to orchestrate the discussion.<br><br>As you look over the work, create a list of expressions similar to the list on the left based on your students' work, adding in additional expressions as needed to support the planned discussion.<br><br>Display the list of expressions. Establish with the class that these expressions, or expressions like these, were written by the class during when determining the total number of tiles needed to surround an $n \times n$ pool. |

<table>
<tr>
<td></td>
<td>Show a colored representation such as the one to the left. (For this discussion and all that follow, you are encouraged to use actual student drawings from the previous day’s work.) Say something similar to “Yesterday a student created this drawing. They also wrote one of these expressions. Which expression do you think he or she might have written? Or, if you don’t see one that you think works, you can also write your own.”<br><br>Likely response:<br>$$2n + 2(n + 2)$$<br>Students might also write something like this:<br>$$(n + 2)\ +\ 2n\ +\ (n + 2)$$</td>
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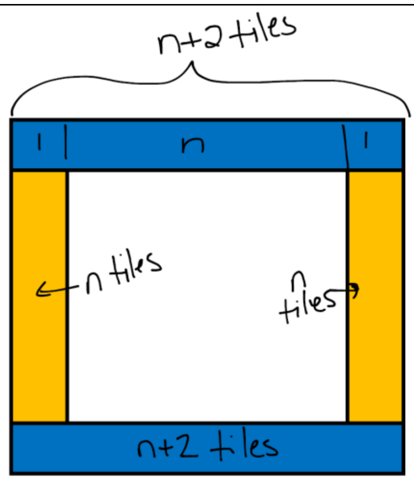
<td>

$$2n\ +\ 2(n + 2)$$
(orange tiles) + (blue tiles)</td>
<td>Annotate the drawing and the expression to capture the students’ explanations and the connections between the two.<br><br>Annotations can include labeling the original drawing, color coding the expression and/or drawing arrows between the expression and the figure. An example is shown to the left.</td>
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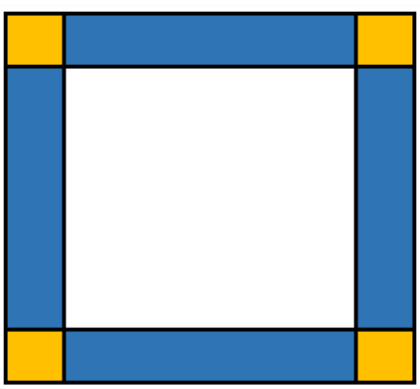
<td></td>
<td>Repeat the above process for another figure such as the one to the left.<br><br>Ideally you will ultimately have at least two correct solutions. They include<br>$$4n + 4$$<br>and<br>$$4(n + 2) - 4$$</td>
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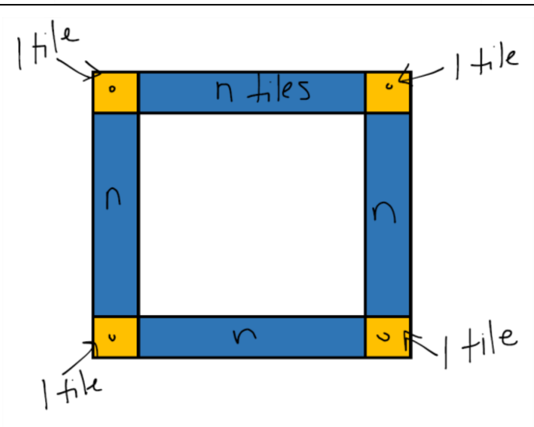
<td>
</td>
<td>Again annotate the figure to make connections between the expression and figure explicit.<br><br>The figure on the left shows a possible annotation corresponding to the expression $4n + 4$.<br><br>It may be helpful to annotate the figure for all suggested expressions. Alternatively, ask students to discuss with each other why one or both are correct.</td>
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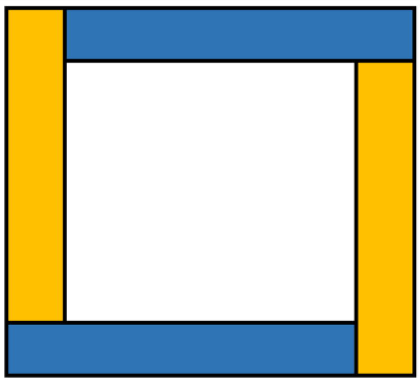
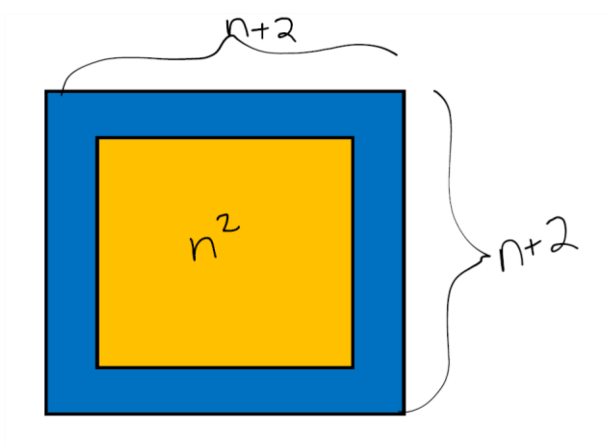
| $4n + 4 = 4(n + 2) - 4$ | Name the idea that these expressions, though written differently, are **equivalent**. One idea of equivalence is that there are an infinite number of ways to write any expression (or any number). However, some forms are more useful or give information about the structure of the problem. |
|  | Choose an unmatched expression, such as $4(n + 1)$<br><br>Provide students with paper and markers or colored pencils. Ask students to create a visual that might match the expression. Select student visuals to discuss with the class, annotating the connections to the expression as before. A possible illustration of $4(n + 1)$ is show here. |
|  | Extension<br><br>Ask students to explain the thinking that gives the expression $(n + 2)^2 - n^2$<br><br>This one might challenge students, in part because it uses “negative space”. |


References:

Kelemanik, Grace, Amy Lucenta, Susan Janssen. Creighton, and Magdalene Lampert. *Routines for Reasoning: Fostering the Mathematical Practices in All Students.* Portsmouth, NH: Heinemann, 2016. Print.