The Kentucky Numeracy Project

Session 4 – Counting and Early Addition

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The Kentucky Numeracy Project

CLOSING

KNP TASK GROUP 304

KNP TASK GROUP 340

INTRODUCTION
## Stages of Literacy Development

**New Mexico Reading Initiative**

http://reta.nmsu.edu/reading/willows/stages/index.html

<table>
<thead>
<tr>
<th>Stage 0: Preliteracy Stage</th>
<th>Initial Phases of Numeracy Development</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Emergent Literacy</strong></td>
<td><strong>Phase 0: Emergent Numeracy</strong></td>
</tr>
<tr>
<td>• Enjoys being read to</td>
<td>• May be able to say number words in sequence up to and beyond ten</td>
</tr>
<tr>
<td>• Pretends to read familiar books</td>
<td>• Attempts to coordinate number words and items in a collection.</td>
</tr>
<tr>
<td>• Names letters of alphabet</td>
<td>• Recognizes numerals 1-5 but may confuse certain numerals in the range of 1-10 (i.e., 6 &amp; 9).</td>
</tr>
<tr>
<td>• Recognizes some signs</td>
<td>• Attempts to count the individual dots when presented with dominos or dice but may be able to immediately recognize some patterns</td>
</tr>
<tr>
<td>• Plays with pencils and paper</td>
<td>• Can represent numbers up to 5 on their fingers but needs to raise each finger one at a time</td>
</tr>
<tr>
<td>• Interest in printing own name</td>
<td>• Number is attached to one object or position (ordinal)</td>
</tr>
<tr>
<td>• Begins developing phonologic awareness</td>
<td><strong>Phase 1: Perceptual / Exactive Numeracy</strong></td>
</tr>
<tr>
<td><strong>Stage 1: Beginning literacy</strong></td>
<td>• Can accurately produce verbal number sequences from 1-20 forward, 10-1 backwards, and identify and sequence numerals from 1-10</td>
</tr>
<tr>
<td>• Develops phonemic awareness</td>
<td>• Can accurately add collections of visible items by counting from one to construct each collection and counting from one to construct the sum.</td>
</tr>
<tr>
<td>• Associates letters with sounds</td>
<td>• May begin to raise fingers simultaneously (rather than one at a time) to solve addition tasks when both addends are less than five</td>
</tr>
<tr>
<td>• Prints letters and numbers</td>
<td>• Number exists as nested quantities (cardinal)</td>
</tr>
<tr>
<td>• Recognizes high-frequency words “by sight”</td>
<td><strong>Phase 2: Figurative / Iconic Numeracy</strong></td>
</tr>
<tr>
<td>• Sounds out regularly spelled words</td>
<td>• Can accurately and correctly construct the count at 1 and continue the count even beyond finger range (&gt; 10)</td>
</tr>
<tr>
<td>• Uses contextual and picture clues</td>
<td>• Often drops back when asked to say the number that comes before (i.e., 12: “nine, ten, eleven, twelve . . . Eleven!”)</td>
</tr>
<tr>
<td>• Writes using inventive spelling</td>
<td>• Has difficulty making sense of subtractive tasks</td>
</tr>
<tr>
<td><strong>Stage 2: Beginning Fluency</strong></td>
<td>• Can accurately identify numerals from 1-30 but commonly reverses numerals above 30 (i.e., 46 is named “sixty four”)</td>
</tr>
<tr>
<td>• Consolidates “sight” vocabulary</td>
<td>• Numbers begin to take on a composite form (i.e., 8 may be described as two 4’s)</td>
</tr>
<tr>
<td>• Expands letter-sound knowledge</td>
<td><strong>Phase 3: Initial Number sequence / Counting on stage</strong></td>
</tr>
<tr>
<td>• Reads simple familiar stories independently</td>
<td>• Can accurately add quantities without relying on physical collections</td>
</tr>
<tr>
<td>• Practices using repeated and partner reading</td>
<td>• Can anticipate counts and keep track of counting internally</td>
</tr>
<tr>
<td>• Develops reading fluency (speed &amp; accuracy)</td>
<td>• Can count forward and backward by 10 on the decade (10, 20, 30,) but not off the decade (22, 32, 42)</td>
</tr>
<tr>
<td>• Writes and spells with less effort</td>
<td><strong>Phase 4: Intermediate/Facile Number sequence / Symbolic Numeracy</strong></td>
</tr>
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<td><strong>Stage 3: Literacy for Growth</strong></td>
<td>• Has developed a deeper understanding of quantitative and operational relationships - number sense (i.e., 20 = 4fives = 2tens; subtraction is the inverse of addition)</td>
</tr>
<tr>
<td>• Reads “for pleasure”</td>
<td>• Numbers exists as composite objects that may be easily partitioned and reassembled to explore solutions – Mental Math Strategies (subtracts e-</td>
</tr>
<tr>
<td>• Reads to gain new knowledge</td>
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</tr>
</tbody>
</table>
Student Development

Abstract: Memorization, Mental Computation, Robust Automaticity with Flexibility, Efficiency, Accuracy, Fluency

Unitizing Visualizing
SAGE Publications 2006 (reprinted 2009),
Cardinality

How many?

“one” “two” “three” “four” “five”

“The number of objects in a set or group (considered as a property of that grouping).”

-Princeton WordNet
The Learning Framework in Number

Unitary Strategies
- Addition and Subtraction
- Arithmetical Strategies Concepts
- Construct (0 - 3)

Composite Strategies
- Addition and Subtraction
- Arithmetical Strategies Concepts
- Construct (4 - 5)
- Multiplication
- and Division
- Arithmetical Strategies Concepts
- Construct (0 - 5)
- Place Value:
- Base-Ten
- Arithmetical Strategies Concepts
- Construct (0 - 5)

Forward and Backwards Number Word Sequences by Ones (Levels 0 – 5)
Forward and Backwards Number Word Sequences related to Composite Units
Written Labels: Numerals (Levels 0 – 5) and Numeral Sequences, Recording Symbols
Structuring Numbers (Levels 0 – 5): Finger Patterns / Spatial Patterns / Combining and Partitioning

US Math Recovery Council, Add+Vantage MR Program
The Learning Framework in Number

- Learning Framework
- The Learning Framework in Number
- Strategies for solving arithmetical problems
- Subitising
- Counting sequences and grouping

Click on one of the tabs below to find out more about that section of the framework.
The Learning Framework in Number

Building addition and subtraction through counting by ones

- Emergent counting
- Perceptual counting
- Figurative counting
- Counting-on-and-back
The Learning Framework in Number

Building addition and subtraction through counting by ones

- Emergent counting
- Perceptual counting
- Figurative counting
- Counting-on-and-back
The Learning Framework in Number

Building addition and subtraction through counting by ones

2-Figurative counting
The Learning Framework in Number

Building addition and subtraction through counting by one

- Emergent counting
- Perceptual counting
- Figurative counting
- Counting-on-and-back
- An example
The Learning Framework in Number

Building addition and subtraction through counting by ones

- Emergent counting
- Perceptual counting
- Figurative counting
- Counting-on and back
- An example
What is 14 + 3?

“14...”
“15, 16, 17”
2 Reason abstractly and quantitatively.

Quantitative reasoning entails ... attending to the meaning of quantities, not just how to compute them ....
The Kentucky Numeracy Project

INTRODUCTION
A = Addition and Subtraction

300 series also indicates *Addition and Subtraction* strand
<table>
<thead>
<tr>
<th>Task Group A340</th>
<th>Kentucky Common Core Academic Standard (KCAS) (*see glossary)</th>
<th>KCAS Domain</th>
<th>KCAS Cluster</th>
<th>Setting (situation &amp; materials)</th>
<th>Activities: Exemplary Learning Experiences (*see glossary)</th>
<th>&quot;I CAN ......&quot; (*see glossary)</th>
<th>Assessment for Learning</th>
<th>Teacher Notes</th>
</tr>
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<tr>
<td>A 340.1</td>
<td>K.CC 5 Count* to answer &quot;how many?&quot; questions about as many as 20 things arranged in a line, a rectangular array, or a circle or as many as 10</td>
<td>Counting and Cardinality</td>
<td>Count to tell the number of items</td>
<td>Two panel dot cards level 2 (see link)</td>
<td>Choose a 2 panel dot card. Open both doors and leave the doors open. Student determines the total number of dots.</td>
<td>Tell the sum of two collection s.</td>
<td>Put out a card with up to 20 dots. Open both doors and, leaving the doors open, ask student &quot;how many dots?&quot;</td>
<td>Use the print link to print the two panel dot cards. The same cards are used for A340.1 and A340.2. Fold the card in half lengthwise at the dotted line. Cut a slit in the top panel to create doors. The total may be written on the back for students</td>
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<tr>
<td>A 340.2</td>
<td>1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6</td>
<td>Operations and Algebraic Thinking</td>
<td>Add and subtract</td>
<td>Two panel dot cards level 2 (see link)</td>
<td>Choose a 2 panel dot card. Open the door on the left, determine the number of black dots and then close the</td>
<td>Tell the sum of two collection s when screened</td>
<td>Put out a card with 8 dots on the left and 5 dots (arranged irregularly) on the right. Flash the dots under the left door</td>
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<td>Choose a 2 panel dot card. Open the door on the left, determine the number of black dots and then close the</td>
<td>Tell the sum by counting on</td>
<td>Put out a card with 17 dots on the left and 5 dots (arranged irregularly) on the right. Flash the dots under the left door</td>
<td>Use the print link to print the two panel dot cards. Fold the card in half lengthwise at the dotted line. Cut a slit in the top panel to create doors. Students may additionally be asked to write the corresponding equation. The sum or the</td>
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# KCAS

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<td>K.CC.5 Count* to answer &quot;how many?&quot; questions about as many as 20 things arranged in a line, a rectangular array, or a circle or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.</td>
<td>Counting and Cardinality</td>
<td>Count to tell the number of objects</td>
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<td>A 340.2</td>
<td>1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).</td>
<td>Operations and Algebraic Thinking</td>
<td>Add and subtract within 20</td>
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<td>A 340.3</td>
<td>Operations and Algebraic Thinking Add and subtract within 20</td>
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Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.
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<th>Setting (situation &amp; materials)</th>
<th>Activities: Exemplary Learning Experiences (*see glossary)</th>
<th>Numeracy Strand (construct/level)</th>
<th>&quot;I CAN ......&quot; (*see glossary)</th>
<th>Assessment for Learning</th>
<th>Student Video Link</th>
<th>Print Link</th>
<th>Interactive Reference</th>
<th>Teacher Notes</th>
</tr>
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<tr>
<td>Counting and Cardinality</td>
<td>Count to find the number of objects</td>
<td>Two panel dot cards - level 2 (see link)</td>
<td>Choose a 2 panel dot card. Open both doors and leave the doors open. Student determines the total number of dots.</td>
<td>Addition and Subtraction</td>
<td>0 to 1 RED</td>
<td>Perceptual counting</td>
<td>... tell the sum of two collections.</td>
<td>Put out a card with up to 20 dots. Open both doors and, leaving the doors open, ask student &quot;how many dots?&quot; Give student permission to touch dots if student is hesitant.</td>
<td><img src="http://www.kentuckymathematics.org/intervention" alt="Link" /></td>
<td></td>
</tr>
</tbody>
</table>

Use the print link to print the two panel dot cards. The sheets are used for A340.1 and A340.2. Fold the card in half lengthwise and cut along the dotted line. Cut a slit in panel to create doors. Text may be written on the front of the card. Students use a checklist for sequencing, it may be helpful for some of the Nf activities that supporting skill. If student is struggling with the number sequence, it may be helpful to some of the Nf activities that supporting skill. If student is hesitant with the number sequence, it may be helpful to some of the Nf activities that supporting skill.
Printable includes 45 two panel dot cards like these.

1. Print.
2. Fold on the dotted line.
3. Cut along top vertical line to make doors.
Examples of 2 panel dot cards

First addend arranged in rows. The first addend is in the range 5 to 15.
Examples of 2 panel dot cards

The second addend is shown in grey. The amount is never more than 6. Sometimes the amount is shown in a regular pattern, other times it appears random.
### A340.1 - Perceptual counting

<table>
<thead>
<tr>
<th>KCAS Domain</th>
<th>KCAS Cluster</th>
<th>Setting (situation &amp; materials)</th>
<th>Activities: Exemplary Learning Experiences (*see glossary)</th>
<th>Numeracy Strand</th>
<th>Level</th>
<th>Numeracy Target</th>
<th>&quot;I CAN ......&quot; (*see glossary)</th>
<th>Assessment for Learning</th>
<th>Student Video Link</th>
<th>Print Link</th>
<th>Interactive</th>
<th>Reference</th>
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</tr>
</thead>
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<td></td>
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<td></td>
<td></td>
<td>Use the print link to print dot cards. The students are used for A340.1 and the Teacher Notes. Fold the card in half lengthwise along the dotted line. Cut a slit along one of the long edges to create doors. May be written on the student dot cards. If student is struggling with the number sequence, it may be helpful to use some of the Nf activities that supporting skill. If necessary, the student can move the dots with the teacher.</td>
</tr>
</tbody>
</table>
“How many dots?”
I can tell the sum of two collections

What does a student need to understand and be able to do to be successful?
I can tell the sum of two collections

What does a student need to understand and be able to do to be successful?

• Forward Number Word Sequence
I can tell the sum of two collections

What does a student need to understand and be able to be successful?

- Forward Number Word Sequence
- One to One correspondence
I can tell the sum of two collections

What does a student need to understand and be able to be successful?

- Forward Number Word Sequence
- One to One correspondence
- Use row structure to know which dots have been counted
I can tell the sum of two collections.

Choose a card. Open both doors. Tell the sum of the dots.
### A340.2 - Figurative counting

<table>
<thead>
<tr>
<th>KNP Entry</th>
<th>Setting (situation &amp; materials)</th>
<th>Activities: Exemplary Learning Experiences (*see glossary)</th>
<th>Numeral Strand</th>
<th>Numeral Target</th>
<th>&quot;I CAN ......&quot; (*see glossary)</th>
<th>Assessment for Learning</th>
<th>Teacher Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.340.2</td>
<td>Two panel dot cards - level 2 (see link)</td>
<td>Choose a 2 panel dot card. Open the door on the left, determine the number of black dots and then close the door. Open briefly (approximately 1 second) the door on the right and determine how many grey dots are under the door. Determine the number of dots altogether.</td>
<td>1 to 2 BLUE Figurative counting</td>
<td>... tell the sum of two collections when screened</td>
<td>Put out a card with 8 dots on the left and 5 dots (arranged irregularly) on the right. Flash the dots under the left door saying &quot;Here are 8 black dots&quot;. Flash the dots under the right door saying &quot;Here are 5 grey dots. How many dots are there altogether?&quot; Ask student to explain strategy. Repeat with other two panel dot cards.</td>
<td>Use the print link to print the two panel dot cards. The same cards are used for A340.1 and A340.2. Fold the card in half lengthwise at the dotted line. Cut a slit in the top panel to create doors. The total may be written on the back for students to use a check. If a student is struggling, leave one of doors open. Start with dot cards where the second addend is no more than 2 or 3. Gradually build up to cards where the second addend is 5 or 6 and student can find the sum with no more than a quick glance under each door. If a teacher is presenting the task, the teacher may say &quot;There are ____ black dots under here, there are ____ grey dots under here... how many dots altogether?&quot; while briefly flashing the dots behind the doors. Students at this construct are starting to use a mental understanding of the quantity to add rather than depending on physical objects. Some may use fingers to keep track others may...</td>
<td></td>
</tr>
</tbody>
</table>
Look briefly behind the first door.

Look briefly behind the second door.

Determine how many dots in all.
I can tell the sum of two collections

when screened
“Distancing the Setting”

Except from the Teacher Notes:
“If a student is struggling, leave one of doors open. Start with dot cards where the second addend is no more than 2 or 3. Gradually build up to cards where the second addend is 5 or 6 and student can find the sum with no more than a quick glance under each door.”
One addend screened

Leave one of the doors open as student solves the task.

Here we see the student use his fingers to “keep track” of the amount under the door.

Door is opened so that student can verify his own work.
Both addends screened

"Here are 4 grey dots. How many altogether"?

"Here are 10 black dots"

Student matched her fingers to the arrangement of the dots

Student uses counting to verify her answer and her strategy.
### A340.3 - Counting on

<table>
<thead>
<tr>
<th>NLP Entry</th>
<th>Kentucky Common Core</th>
<th>Setting (situation &amp; materials)</th>
<th>Activities: Exemplary Learning Experiences (*see glossary)</th>
<th>Numeracy</th>
<th>Constructivism</th>
<th>Initial Number Sequence</th>
<th>Assessment for Learning</th>
<th>Student Video Link</th>
<th>Print Link</th>
<th>Internet Reference</th>
<th>Teacher Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 340.3</td>
<td>Add and subtract within 20, demonstrating fluency for addition and subtraction, and fluency for multiplication and division using properties of operations and their relationship to measure.</td>
<td>Two panel dot cards - level 3 (see link)</td>
<td>Choose a 2 panel dot card. Open the door on the left, determine the number of black dots and then close the door. Open briefly (approximately 1 second) the door on the right and determine how many grey dots are under the door. Determine the number of dots altogether.</td>
<td>Put out a card with 17 dots on the left and 5 dots (arranged irregularly) on the right. Flash the dots under the left door saying &quot;Here are 17 black dots&quot;. Flash the dots under the right door saying &quot;Here are 5 grey dots. How many dots are there altogether?&quot; Ask student to explain strategy. Repeat with other two panel dot cards.</td>
<td>Use the print link to print the two panel dot cards. Fold the card in half lengthwise at the dotted line. Cut a slit in the top panel to create doors. Students may additionally be asked to write the corresponding equation. The sum of the corresponding equation may be written on the back for students to use a check. If a teacher is presenting the task, the teacher may say &quot;There are ___ black dots under here, there are ___ grey dots under here... how many dots altogether?&quot; while briefly flashing the dots behind the doors. Initially students will likely start the count from one and/or build quantities on their fingers. Students will let go of this redundant behaviour naturally as they develop a deeper understanding of quantity and addition. By using large first addends, this behavior is discouraged. Encourage students to explain their strategies. Observe if students are relying on a visible aid such as a number line, 100 chart or calendar. If so, remove or cover the aid.</td>
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Examples of 2 panel dot cards to encourage counting on

The first addend is large (14 to 25)
“Here are 15 black dots”

“Here are 3 grey dots. How many altogether”?

Student monitored counts for second addend on her fingers “16, 17, 18”

Open second door only. Student uses counting on to verify her answer and her strategy.
# A340.3 - Counting on

<table>
<thead>
<tr>
<th>NRP Entry</th>
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<th>Setting (situation &amp; materials)</th>
<th>Activities: Exemplary Learning Experiences (*see glossary)</th>
<th>Numeracy ConstructIV</th>
<th>Numeracy &quot;I CAN ......&quot; (*see glossary)</th>
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<td>Choose a 2 panel dot card. Open the door on the left, determine the number of black dots and then close the door. Open briefly (approximately 1 second) the door on the right and determine how many grey dots are under the door. Determine the number of dots altogether.</td>
<td>Addition and Subtraction 2 to 3 GREEN Initial Number Sequence</td>
<td>... tell the sum by counting on</td>
<td>Put out a card with 17 dots on the left and 5 dots (arranged irregularly) on the right. Flash the dots under the left door saying “Here are 17 black dots”. Flash the dots under the right door saying “Here are 5 grey dots. How many dots are there altogether?” Ask student to explain strategy. Repeat with other two panel dot cards.</td>
<td><a href="#">Link</a></td>
<td><a href="#">Link</a></td>
<td><a href="#">Link</a></td>
<td>Use the print link to print the two panel dot cards. Fold the card in half lengthwise at the dotted line. Cut a slit in the top panel to create doors. Students may additionally be asked to write the corresponding equation. The sum or the corresponding equation may be written on the back for students to use as a check. If a teacher is presenting the task, the teacher may say “There are ___ black dots under here, there are ___ grey dots under here... how many dots altogether?” while briefly flashing the dots behind the doors. Initially students will likely start the count from one and/or build quantities on their fingers. Students will let go of this redundant behaviour naturally as they develop a deeper understanding of quantity and addition. By using large first addends, this behavior is discouraged. Encourage students to explain their strategies. Observe if students are relying on a visible aid such as a number line, 100 chart or calendar. If so, remove or cover the aid.</td>
</tr>
</tbody>
</table>
The Kentucky Numeracy Project

CLOSING

KNP TASK GROUP 308

KNP TASK GROUP 340

INTRODUCTION
<table>
<thead>
<tr>
<th>KNP Entry</th>
<th>Kentucky Common Core Academic Standard (KCAS) (*see glossary)</th>
<th>KCAS Domain</th>
<th>KCAS Cluster</th>
<th>Setting (situation &amp; materials)</th>
<th>Activities: Exemplary Learning Experiences (*see glossary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A304.1</td>
<td>K.CC.4 Understand relationship between number and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.</td>
<td>Counting and Cardinality</td>
<td>Count to tell the number of objects</td>
<td>Fill 20 Game Board (see link), 20 counters or pennies per player, dot or numeral cube</td>
<td>Each student uses 1 &quot;Fill 20&quot; game board. On a player's turn, the player rolls a cube and adds that many counters to his/her game board. The player should say how many counters he or she has. The first player to fill his/her board wins the game.</td>
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</table>
| A304.1    | K.CC.4 Understand relationship between number and quantities; connect counting to cardinality.  
 a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. | Counting and Cardinality | Count to tell the number of objects | Fill 20 Game Board (see link), 20 counters or pennies per player, dot or numeral cube | Each student uses 1 "Fill 20" game board. On a player's turn, the player rolls a cube and adds that many counters to his/her game board. The player should say how many counters he or she has. The first player to fill his/her board wins the game. |
Each student uses 1 "Fill 20" game board. On a player's turn, the player rolls a cube and adds that many counters to his/her game board. The player should say how many counters he or she has. The first player to fill his/her board wins the game.
### A304.1

<table>
<thead>
<tr>
<th>Numeracy Level</th>
<th>Construct/Level</th>
<th>&quot;I CAN ......&quot; (*see glossary)</th>
<th>Assessment for Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numeracy Strand</td>
<td>Additon &amp; Subtraction</td>
<td>0 to 1 RED Perceptual counting</td>
<td>Put 14 counters or pennies on a game board. Show board to student and ask &quot;How many counters are on this board.&quot; Allow student to point and/or touch the counters.</td>
</tr>
</tbody>
</table>

#### fill 20" game board
- On a game board. The player who rolls a cube and adds that number to the counters he or she has. The player with the most counters on their board wins the game.

#### Fill 20 Game Board
- The cube of students who have played with the game.
<table>
<thead>
<tr>
<th>&quot;I CAN ......&quot; (*see glossary)</th>
<th>Assessment for Learning</th>
<th>Teacher Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;carefully count each item to 20&quot;</td>
<td>Put 14 counters or pennies on a game board. Show board to student and ask &quot;How many counters are on this board.&quot; Allow student to point and/or touch the counters.</td>
<td>The cube can be adjusted to meet the needs for students. To make the game simpler, use a dot cube in the range 1 to 3. To make the game more complex, use a numeral cube.</td>
</tr>
</tbody>
</table>
I can carefully count each item to 20!

Materials: game board, counters, number cube.

**Fill 20**

**Directions:**
1. Get one game board and one bag of counters from the folder.
2. Roll the number cube and put that many counters on your game board.
3. Tell your teacher or partner how many counters you have all together.
4. Take turns with your teacher or partner.
5. Be sure to tell your teacher or partner how many counters you have altogether on your board after each roll.
6. The first player to fill a game board wins.
“How many counters do you have on your board?”
“How many counters do you need to put on your board?”
“How many counters do you have on your board altogether?”
<table>
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<tr>
<th>KNP Entry</th>
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<th>Activities: Exemplary Learning Experiences (*see glossary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A304.2</td>
<td>1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., (8 + 6 = 8 + 2 + 4 = 10 + 4 = 14)); decomposing a number leading to a ten (e.g., (13 - 4 = 13 - 3 - 1 = 10 - 1 = 9)); using the relationship between addition and subtraction (e.g., knowing that (8 + 4 = 12), one knows (12 - 8 = 4)); and creating equivalent but easier or known sums (e.g., adding (6 + 7) by creating the known equivalent (6 + 6 + 1 = 12 + 1 = 13)).</td>
<td>Operations and Algebraic Thinking</td>
<td>Add and subtract within 20</td>
<td>Fill 20 Game Board (see link), 20 counters or pennies per player, numeral cube</td>
<td>Each student uses 1 &quot;Fill 20&quot; game board. On a player's turn, the player rolls a numeral cube. The player mentally determines how many counters he/she will have after adding the rolled amount to his/her game board and say the prediction aloud. The student should then add the counters to his/her game board and determine the total. The first player to fill his/her board wins the game. The teacher may choose to add a rule that if the prediction is incorrect, the player does not add any counters on that turn.</td>
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<td>KNP Entry</td>
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<td>1.OA.6</td>
<td>Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).</td>
<td>Operations and Algebraic Thinking</td>
<td>Add and subtract within 20</td>
<td>Fill 20 Game Board (see link), 20 counters or pennies per player, numeral cube</td>
<td>Each student uses 1 &quot;Fill 20&quot; game board. On a player's turn, the player rolls a numeral cube. The player mentally determines how many counters he/she will have after adding the rolled amount to his/her game board and say the prediction aloud. The student should then add the counters to his/her game board and determine the total. The first player to fill his/her board wins the game. The teacher may choose to add a rule that if the prediction is incorrect, the player does not add any counters on that turn.</td>
</tr>
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<td>Setting (situation &amp; materials)</td>
<td>Activities: Exemplary Learning Experiences (*see glossary)</td>
<td>Numeracy Strand</td>
<td>Construct/Level</td>
<td>Numeracy Target</td>
<td>&quot;I CAN ……” (*see glossary)</td>
</tr>
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<td>-------------------------------</td>
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</tr>
<tr>
<td>Fill 20 Game Board (see link), 20 counters or pennies per player, numeral cube</td>
<td>Each student uses 1 &quot;Fill 20&quot; game board. On a player’s turn, the player rolls a numeral cube. The player mentally determines how many counters he/she will have after adding the rolled amount to his/her game board and say the prediction aloud. The student should then add the counters to his/her game board and determine the total. The first player to fill his/her board wins the game. The teacher may choose to add a rule that if the prediction is incorrect, the player does not add any counters on that turn.</td>
<td>Addition</td>
<td>1 to 2 BLUE</td>
<td>Figurative counting</td>
<td>...add when the first collection is visible and the second collection is covered.</td>
</tr>
</tbody>
</table>
### "I CAN ......" (*see glossary*)

<table>
<thead>
<tr>
<th>Assessment for Learning</th>
<th>Teacher Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show a game board filled with 12 counters for 1-3 seconds and then cover. Say to the student, &quot;If you have a 12 counters and you add 4 counters, how many counters will you have in all?&quot; If desired, repeat for similar quantities.</td>
<td>To make game simpler, use a numeral cube in the range 1 to 3. To increase the challenge, place a paper over the child's game board just before the child rolls for his or her turn. After the child rolls, say &quot;You have ____. What will you have when you add ____?&quot;</td>
</tr>
<tr>
<td>Teacher partner small group</td>
<td>Fill 20 Game Board</td>
</tr>
</tbody>
</table>
A304.2
Fill 20 Game

I can add when the first collection is visible and the second collection is covered!

Materials: game boards, counters, number cube

Fill 20

Directions:
1. Get one game board and one bag of counters from the folder.
2. Roll the number cube.
3. Before putting that many counters on your board, make a prediction of how many counters you will have altogether on your game board.
4. Put your counters on your game board and see if your prediction was correct.
5. Take turns with your teacher or partner.
6. Be sure to tell your teacher or partner your prediction of how many counters you will have altogether on your board after each roll.
7. The first player to fill a game board wins.
“How many counters do you have?”
“How many counters will be on your board after you add 4 more?”

-OR-

“You have 3 counters on your board. How many counters will be on your board after you add 4 more?”
“Let’s check your prediction. Were you correct?”

“How many more counters do you think you’ll need to have 10 altogether?”
<table>
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<tr>
<td>A 304.3</td>
<td>1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 - 12 + 1 - 13$).</td>
<td>Operation and Algebraic Thinking</td>
<td>Add and subtract within 20</td>
<td>Fill 20 Game Board (see link), 20 counters per player, cube labeled &quot;$+2, +3, -1, -2, -3, -4$&quot;</td>
<td>Each student uses 1 &quot;Fill 20&quot; game board. Before starting the game, each student should fill his/her game board with 20 counters. During a turn, the player rolls the cube, mentally determines and says aloud 'how many' counters will result after applying the action indicated on the cube. The player adds or removes the indicated number of counters and determines if the prediction was correct. First student to empty his/her board wins the game. A screen may be placed over the game board to encourage mental imagery and strategies.</td>
</tr>
</tbody>
</table>
### A304.3

<table>
<thead>
<tr>
<th>KCAS Cluster</th>
<th>Setting (situation &amp; materials)</th>
<th>Activities: Exemplary Learning Experiences (*see glossary)</th>
<th>Numeracy Strand</th>
<th>Construct/Level</th>
<th>Numeracy Target</th>
<th>&quot;I CAN .......&quot; (*see glossary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count to tell the number of objects</td>
<td>Fill 20 Game Board (see link), 20 counters per player, cube labeled &quot;+2, +3, 1, -2, -3, -4&quot;</td>
<td>Each student uses 1 &quot;Fill 20&quot; game board. Before starting the game, each student should fill his/her game board with 20 counters. During a turn, the player rolls the cube, mentally determines and says aloud 'how many' counters will result after applying the action indicated on the cube. The player adds or removes the indicated number of counters and determines if the prediction was correct. First student to empty his/her board wins the game. A screen may be placed over the game board to encourage mental imagery and strategies.</td>
<td></td>
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</tr>
</tbody>
</table>

"I CAN ......." statement:
- Add and subtract by counting on or counting back
I can add and subtract by counting on or by counting back!

Fill 20

Directions:
1. Get one game board and one bag of counters from the folder.
2. Fill your game board with 20 counters.
3. Roll the number cube.
4. Use mental math to figure out how many counters you should have on your board after your roll and tell your partner.
5. Put your counters on (or take off) your game board and check yourself.
6. Take turns with your teacher or partner.
7. The first player to empty a game board wins.
“How many counters are on your board?”

“How many counters are in the gray squares?”

“How many counters are in the white squares?”
“What did you roll?”

“What do you need to do?”

“Why?”

“How many will you have left?”
“How did you know it was 17?

“Can you tell me how many counters you’ll have to take off in order to have 10 counters left?”

“What will you need to roll in order to only have 13 counters?”

“How many more counters do you have than your partner?”
### Numeracy Target

<table>
<thead>
<tr>
<th>Initial Number Sequence</th>
<th>&quot;I CAN .......&quot; (*see glossary)</th>
<th>Assessment for Learning</th>
<th>Student Grouping</th>
<th>Video Link</th>
<th>Print Link</th>
<th>Interactive Website</th>
<th>Reference</th>
<th>Teacher Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>...add and subtract by counting on or counting back</td>
<td>Briefly show a game board filled with 16 counters. Say to the student, &quot;If you have a 16 counters and you remove 4 counters, how many counters will you have left?&quot; If desired, repeat for similar quantities.</td>
<td>teacher partner small group</td>
<td>Fill 20 and Fill 30 game boards</td>
<td></td>
<td></td>
<td></td>
<td>To increase the challenge, use the Fill 30 game board and start so that each player has 30 counters. To make game simpler, use a cube with smaller numbers. To make game quicker, use a cube with only subtraction.</td>
</tr>
</tbody>
</table>
The Kentucky Numeracy Project

INTRODUCTION
Upcoming KNP Sessions, 3:30 to 4:30 p.m. ET

- March 31 – Structuring to Twenty
- April 21 – Advanced Addition and Subtraction
- May 12 – Multiplication and Division
- June 2 – Tens and Ones

http://kymath.org/intervention/iKNP.asp
Reflection Questions

1) When looking at the “Development of Literacy and Numeracy” chart, think about which phase is least addressed in US textbooks? What is the impact on students of the missing phase?

2) How might you modify learning activities to provide plenty of opportunities, as needed, for students to count hidden/imaginary quantities when adding or subtracting?

3) Are your students facile at the counting-on stage, as evidenced by a missing addend task in the context of hidden counters as shown on the Count Me in Too website video example?