The Kentucky Numeracy Project

Session 7 – Multiplication and Division

May 12, 2011

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The Kentucky Numeracy Project
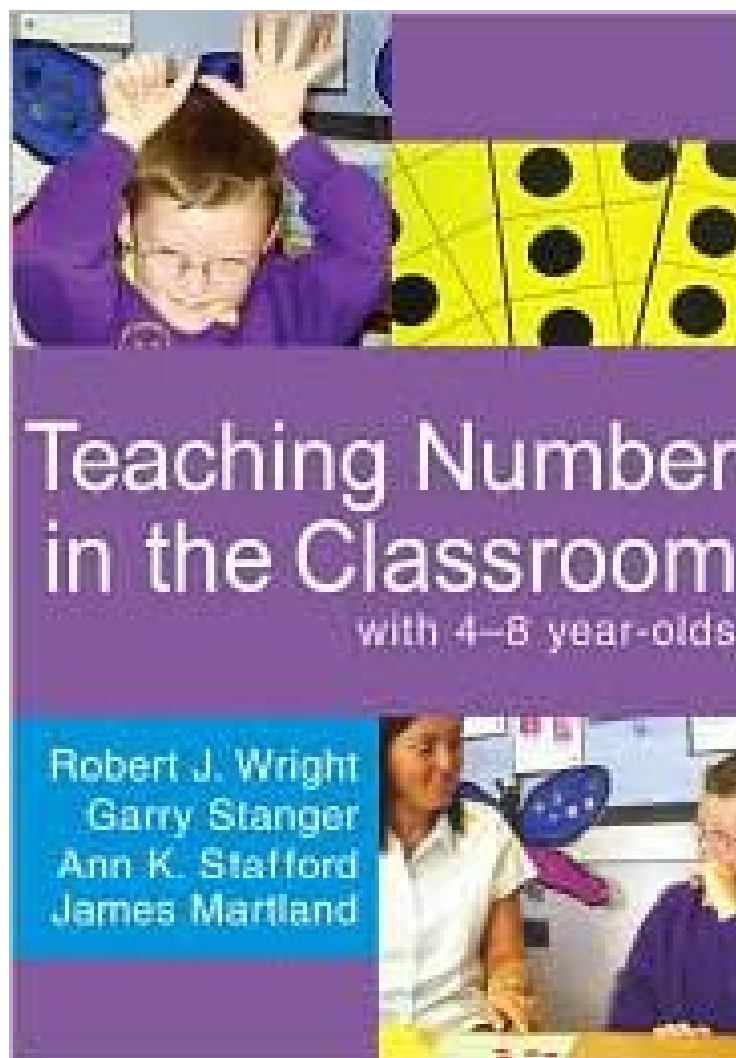
CLOSING

KNP TASK GROUP 435

KNP TASK GROUP 436

INTRODUCTION

SAGE Publications 2006 (reprinted 2009),
ISBN 978-1-4129-0758-3

One day they ran into each other on the road. The Hare couldn't understand how the slow-moving Tortoise could accomplish anything.
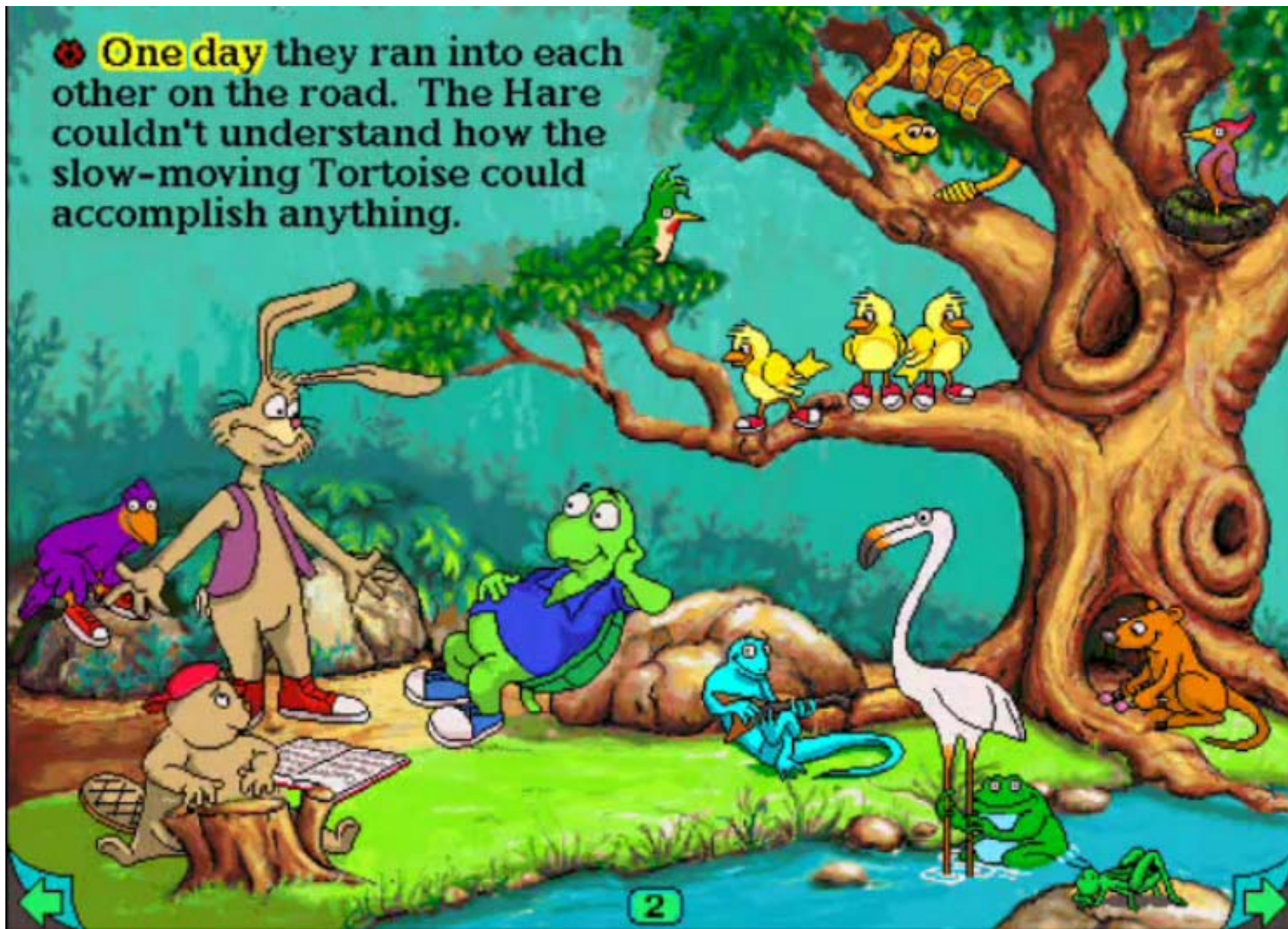
Everyone was excited on the day of the big race.
The Hare was busy making sure everyone noticed him.
The Tortoise could hardly believe that he was going to race.

4

After eating, the Hare felt very tired. So he decided to rest just for a little while. He immediately fell asleep. The Tortoise was surprised to find the Hare sleeping.

The Tortoise was nearing the finish line when the Hare awoke. He sprinted, trying to catch the Tortoise, but the Hare was too late. The Tortoise won.

KCAS for Mathematical Content - Grade Level Introductions

Mathematics | Grade 1
In Grade 1, instructional time should focus on four critical areas: (1) developing **understanding** of addition, subtraction, and **strategies** for addition and subtraction within 20; …

Mathematics | Grade 2
In Grade 2, instructional time should focus on four critical areas: … (2) **building fluency** with addition and subtraction; …

KCAS for Mathematical Content - Grade Level Introductions

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing **understanding** of multiplication and division and **strategies** for multiplication and division within 100; …

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing **understanding and fluency** with multi-digit multiplication, and developing **understanding** of dividing to find quotients involving multi-digit dividends; …

"understanding"

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

Students who **lack understanding** of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to: **(1)** consider analogous problems; **(2)** represent problems coherently; **(3)** justify conclusions, **(4)** apply the mathematics to practical situations, **(5)** use technology mindfully to work with the mathematics, **(6)** explain the mathematics accurately to other students, **(7)** step back for an overview, or **(8)** deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

Color Tiles
Classroom Basics Kit
Set of 2,000
ETA 42832

/enumeracy.com/files/February%201999_%20Battista.pdf - Windows Internet Explorer

http://enumeracy.com/files/February%201999_%20Battista.pdf

orites | The Kentucky Center for M... | Kentucky Department of E... | Northern Kentucky Univers... | Welcome to Math Recove... | Common Core State Stan... | weightwatchers.com eTools
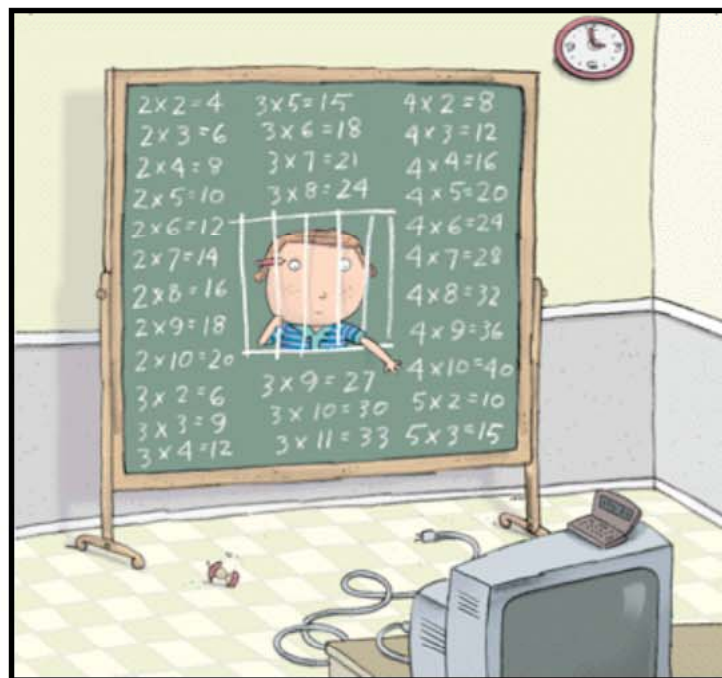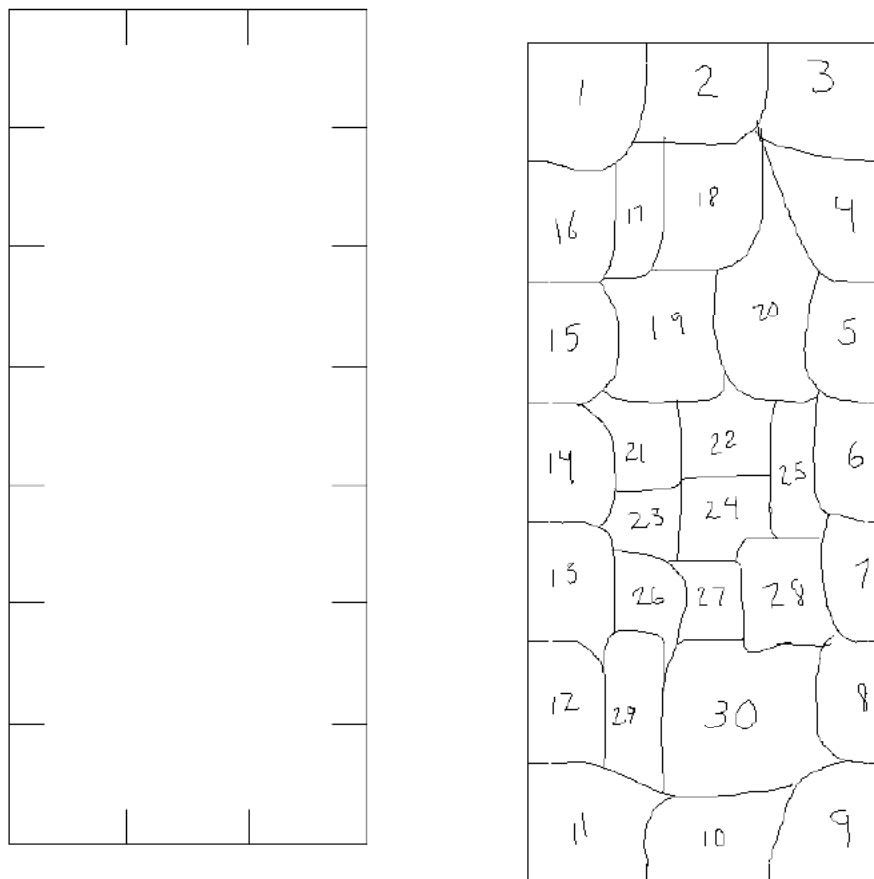
o://enumeracy.com/files/February...

ONLINE ARTICLE

Kappan Home
PDK Home

Illustration © 1999 by Jem Sullivan

The Mathematical Miseducation
Of America's Youth

By Michael T. Battista

http://enumeracy.com/files/February%201999_%20Battista.pdf - Windows Internet Explorer

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Favorites   The Kentucky Center for M...   Kentucky Department of E...   Northern Kentucky Univers...   Welcome to Math Recove...   Common Core State Stan...   weightwatchers.com eToo

http://enumeracy.com/files/February...

FIGURE 2.

CS was then asked to predict how many of the plastic squares it would take to completely cover the rectangle. She drew squares where she thought they would go and counted 30, as shown in Figure 2.

splatsq100 - Windows Internet Explorer

http://www.primarygames.co.uk/pg2/splat/splatsq100.html

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splatsq100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

CLEAR

PRINT

MAIN

Done

| Teachers | Parents | Children | Research | **Learning Framework** | Assessment | FAQ | About |

>> Home / Learning Framework / The Learning Framework in Number

The Learning Framework in Number

- Learning Framework
- **The Learning Framework in Number**
- Strategies for solving arithmetical problems
- Subitising
- Counting sequences and grouping

Click on one of the tabs below to find out more about that section of the framework.

Building place value through grouping

Building fractions through equal sharing

Building multiplication and division through equal grouping and counting

Building addition and subtraction through grouping

Building addition and subtraction through counting by ones

Verbal and written labels
Forward number words and sequences (0-5)
Backward number words and sequences (0-5)
Counting by 10s and 100s (1-3)
Number of identification (0-5)
Finger patterns
Recording symbols

The Learning Framework in Number was developed for the Count Me In Too project in 1996 by Professor Bob Wright. It draws on a range of theories from research projects conducted over the last fifteen years. These projects investigated the formation of number concepts in children in Australia and overseas.

Teachers   **Parents**   **Children**   **Research**   **Learning Framework**   **Assessment**   **FAQ**   **About**

- Learning Framework
- **The Learning Framework in Number**
- Strategies for solving arithmetical problems
- Subitising
- Counting sequences and grouping

The Learning Framework in Number

Building multiplication and division through equal grouping and counting

PAUSED   0:00:37.120

Building multiplication and Division through grouping and counting

- Forming equal groups
- Perceptual multiples
- Figurative units
- Repeated abstract composite units
- Multiplication and division as operations

Count Me In Too

>> Home / Learning Framework / The Learning Framework in Number

The Learning Framework in Number

Building multiplication and division through equal grouping and counting

Nine.

PAUSED ▮▮▮▮▮▮▮▮▮▮▯▯▯▯▯▯▯▯▯▯ 0:01:01.412

▶ |◀ ▶| 🔈 ──🔘─ 🔊

Building multiplication and Division through grouping and counting

- Forming equal groups
- Perceptual multiples
- Figurative units
- Repeated abstract composite units
- Multiplication and division as operations

>> Home / Learning Framework / The Learning Framework in Number

The Learning Framework in Number

Building multiplication and division through equal grouping and counting

I said "Three, six, nine"

PAUSED   0:01:23.748

Count Me In Too

>> Home / Learning Framework / The Learning Framework in Number

- Learning Framework
- **The Learning Framework in Number**
- Strategies for solving arithmetical problems
- Subitising
- Counting sequences and grouping

The Learning Framework in Number

Building multiplication and division through equal grouping and counting

equals twelve.

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Building multiplication and Division through grouping and counting

- Forming equal groups
- Perceptual multiples
- Figurative units
- Repeated abstract composite units
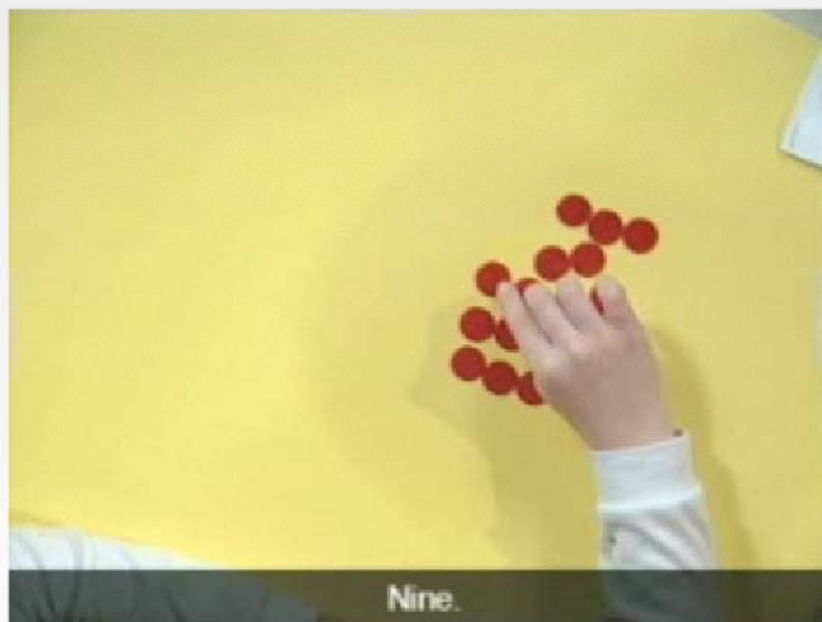- Multiplication and division as operations

Count Me In Too

>> Home / Learning Framework / The Learning Framework in Number

The Learning Framework in Number

Building multiplication and division through equal grouping and counting

Building multiplication and Division through grouping and counting

- Forming equal groups
- Perceptual multiples
- Figurative units
- Repeated abstract composite units
- Multiplication and division as operations

Because eight times four is the opposite of

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Internet | Protected Mode: Off

M = Multiplication and Division

KNP Entry

M 435.2

M 435.3

M 435.4

M 435.5

M 435.6

400 series also indicates *Multiplication and Division*

Task group M435

KNP	Kentucky Common Core Academic Standard (KCAS)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)
M	3.OA.1. Interpret products of whole numbers (e.g.,	Operati ons and Represe	nt & solve problems	Tree backgrounds	Roll the die. Place out that number of trees. Roll the die
M	3.OA.1. Interpret products of whole numbers (e.g.,	Operati ons and Represe	nt & solve problems	Multiplicative images (see	Place out a picture showing items in equal groups. Say
M	3.OA.1. Interpret products of whole numbers (e.g.,	Operati ons and Represe	nt & solve problems	Multiplication images with	The teacher or another student will flash the
M	3.OA.1. Interpret products of whole numbers (e.g.,	Operati ons and Represe	nt & solve problems	Multiplicative quick images	The teacher or another student will flash the
M	3.OA.4. Determine the unknown whole number in	Operati ons and Represe	nt and solve problems	Multiplicative quick images,	One student or the teacher will describe a quick image

Mathematics » Grade 3 » Introduction

Students develop an **understanding of the meanings of multiplication and division** of whole numbers through activities and problems involving **equal-sized groups, arrays, and area models**; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. (continued...)

Mathematics » Grade 3 » Introduction

Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors.

By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

M 435.1

KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct/Level (from AVMR)	Numeracy Target	"I CAN ......" (*see glossary)	
M 435.1	3.OA.1. Interpret products of whole numbers (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects ea., i.e. describe a context in which a total number of objects can be expressed as 5×7	Operations and Algebraic Thinking	Represent & solve problems involving multiplication/division	Tree backgrounds (6), small bird pictures or other counters (36), dot die, writing surface	Roll the die. Place out that number of trees. Roll the die again. Place that number of birds on each tree Tell your partner how many groups you have and how many are in each group. Tell your partner how many birds there are in all. Record an equation that matches.	Multiplication and Division	0 to 1 RED	Initial grouping	... make equal groups and determine how many in all.	A As o a n 'l i

M 435.1

Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct/Level (from AVMR) Numeracy	Target	"I CAN ......" (*see glossary)	Assessment for Learning	Student	Video Link	Print Link	Interactive	Reference	Teach
Roll the die. Place out that number of trees. Roll the die again. Place that number of birds on each tree Tell your partner how many groups you have and how many are in each group. Tell your partner how many birds there are in all. Record an equation that matches.	Multiplication and Division	0 to 1 RED	Initial grouping	... make equal groups and determine how many in all.	Ask student to get 4 trees. Ask student to place 6 birds on each tree. Say "Tell me about that." Ask follow-up questions such as "How many groups?", "How many are in each group?" and "How many birds are there in all?" Ask student to write a matching equation.	Partners		http://www.kentuckymathematics.org/inte			Other backgr may be used. house backg counters, lily p small frogs, o circle counter Assist students language w groups and ite recording of initially be o introduce stu both an multiplica

M 435.1 print link

M 435.1
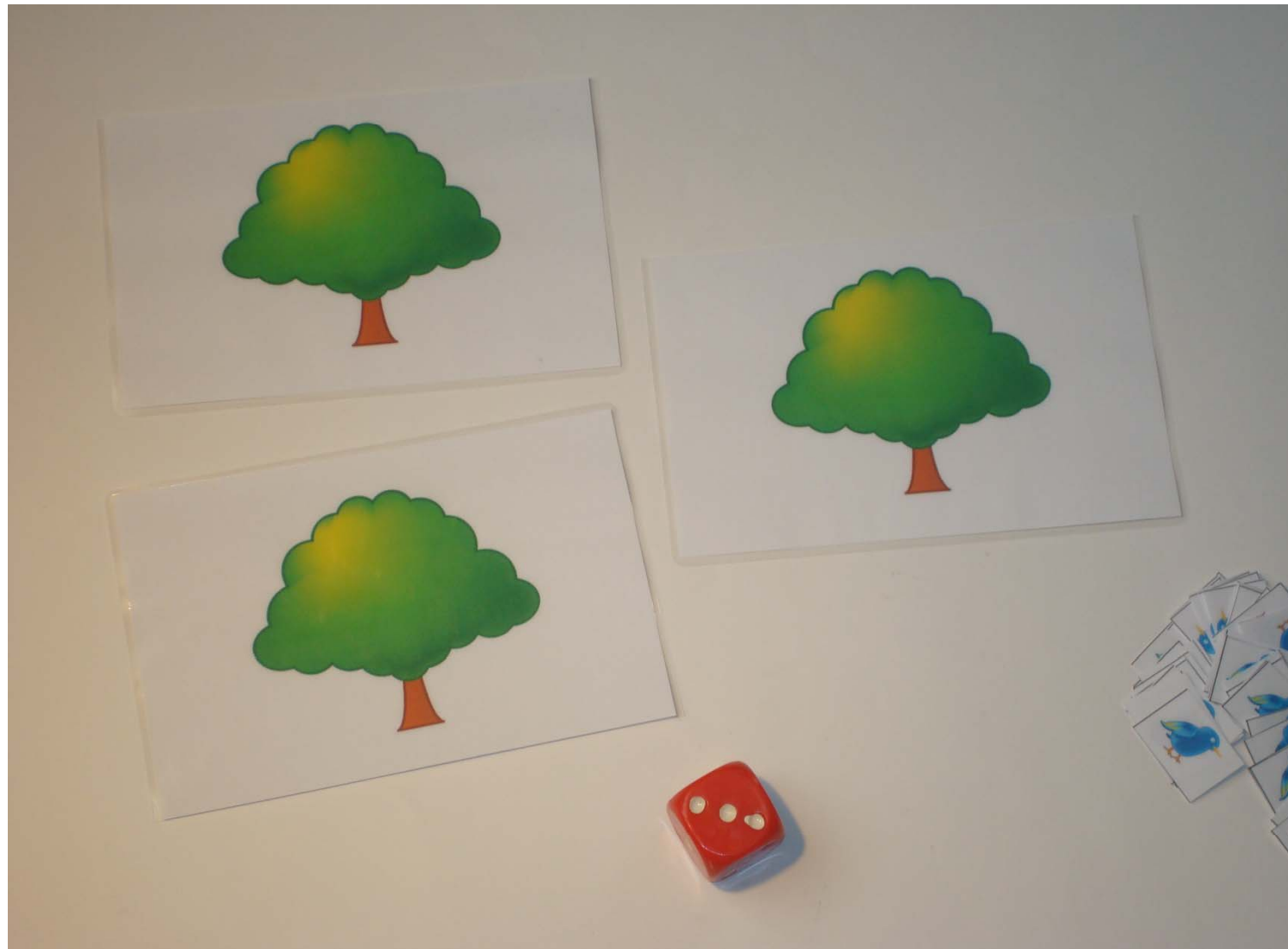
M 435.1

3 groups of 6

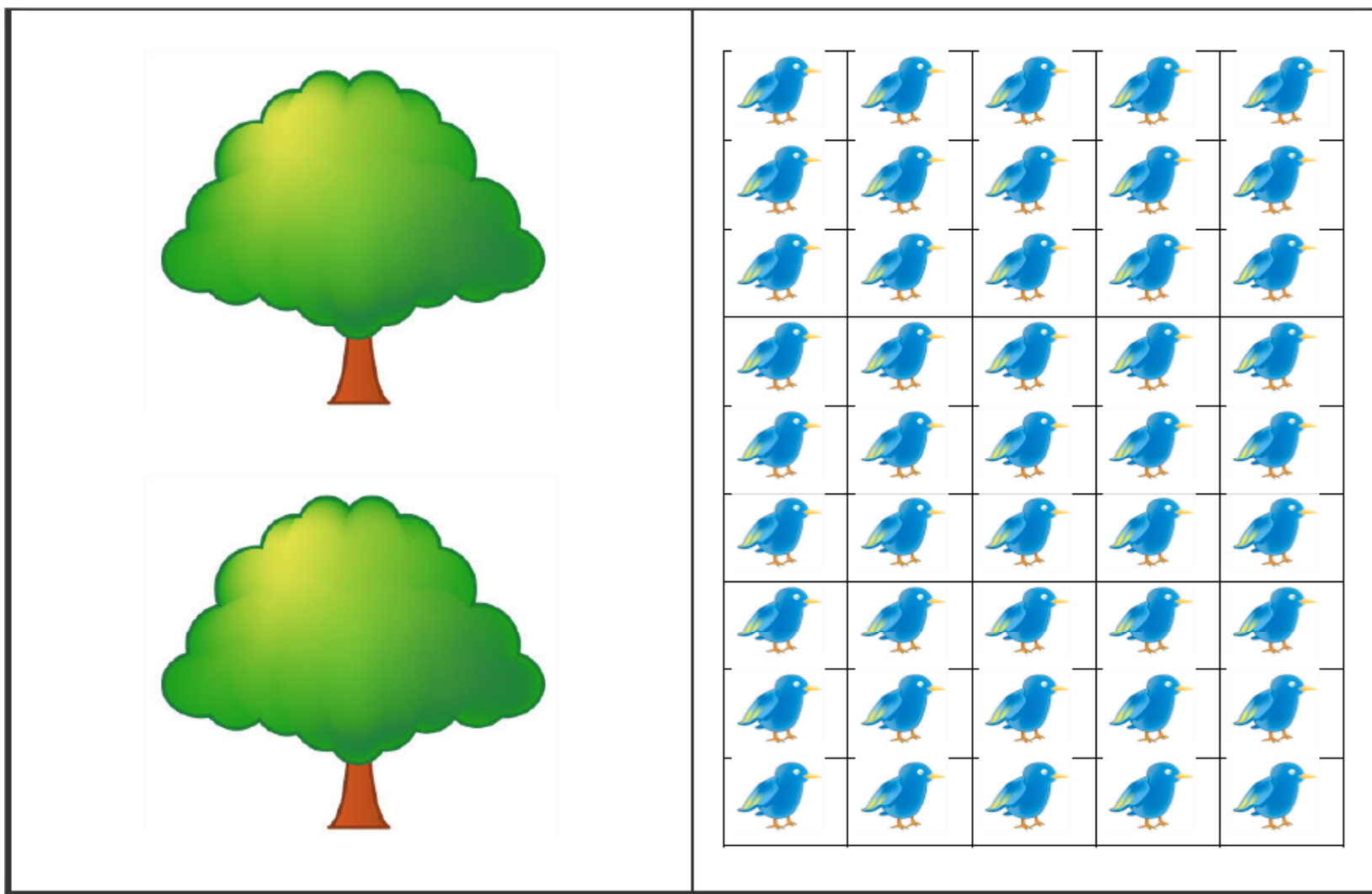# M 435.2

KNP	Kentucky Common Core Academic Standard (KCAS)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Teaching Level	Non-Numeracy Level	Target	"I CAN ……" (*see glossary)
M 435.2	3.OA.1. Interpret products of whole numbers (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects ea., i.e. describe a context in which a total number of objects can be expressed as 5×7	Operations and Algebraic Thinking	Represent & solve problems involving multiplication/division	Multiplicative images (see link), writing surface	Place out a picture showing items in equal groups. Say how many groups, how many items in each group and how many items in all. Record a matching equation.	Multiplication and Division	1 to 2 BLUE		Perceptual counting in multiples	… find the product when all items are visible and write the matching equation.
M	3.OA.1. Interpret	rati and ese	& ve ems	Multiplication	The teacher or another	pric	go	EN nati	e	… find the

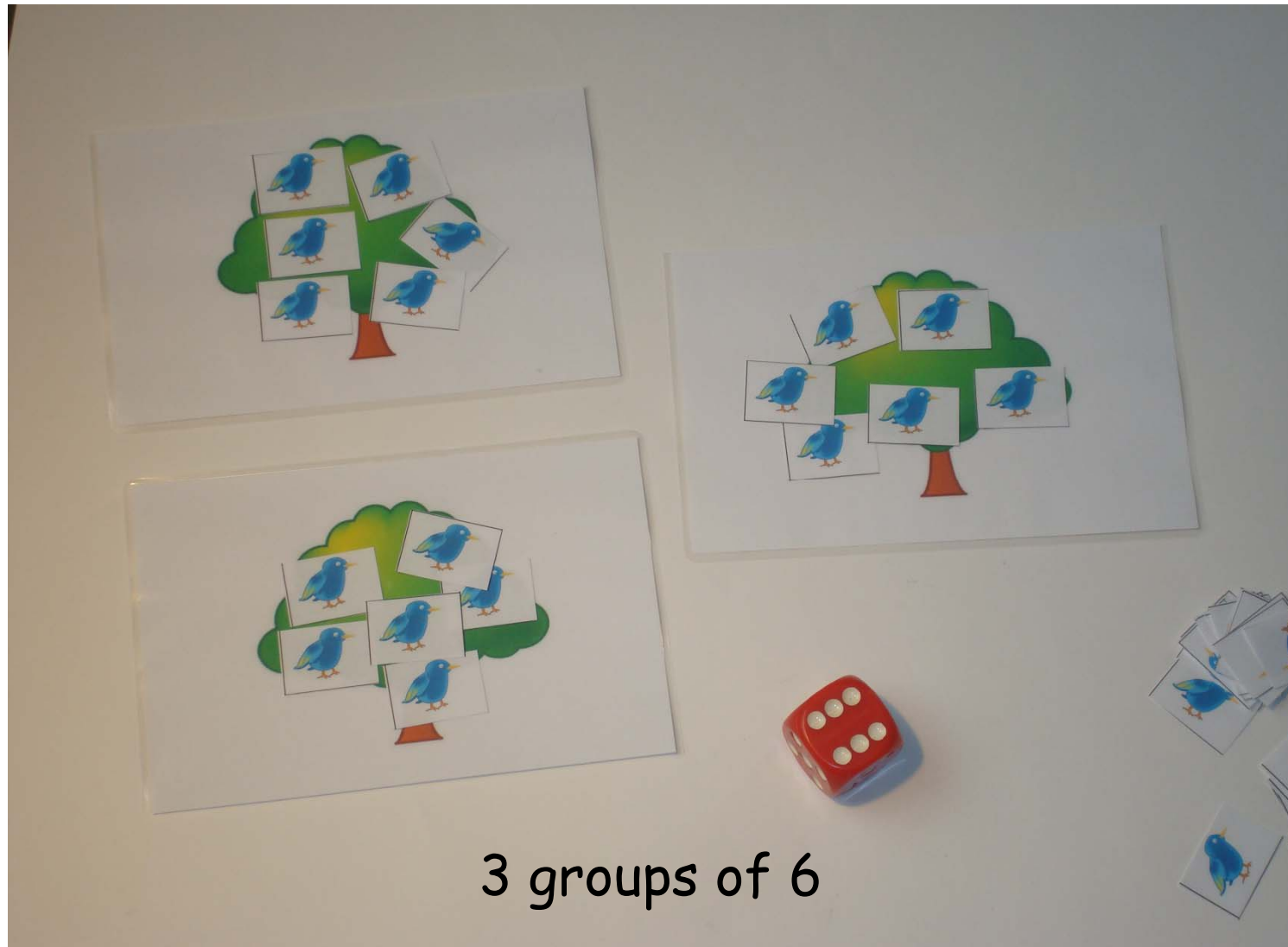# M 435.2

| Setting (Situation & Materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct Level (from AVMR)	Numeracy Target	"I CAN ......" (*see glossary)	Assessment for Learning	Student	Video Link	Print Link	Interactive	Reference	
...ltiplicative ...nages (see ...k), writing ... surface	Place out a picture showing items in equal groups. Say how many groups, how many items in each group and how many items in all. Record a matching equation.	Multiplication and Division	1 to 2 BLUE	Perceptual counting in multiples	... find the product when all items are visible and write the matching multiplication equation.	Place out a picture showing 5 groups of 4. Ask student "What do you see" Ask follow-up questions such as "How many groups? How many items in each group?" and "how many items in all?" Prompt student to write a matching addition and a matching multiplication equation.	various		http://kymath.org/intervention/doc/NumeracyP			Note... bein... take... the... (i.e. ... thro... and... im... co... being... usin... will a... this...

M 435.2 – print link sample
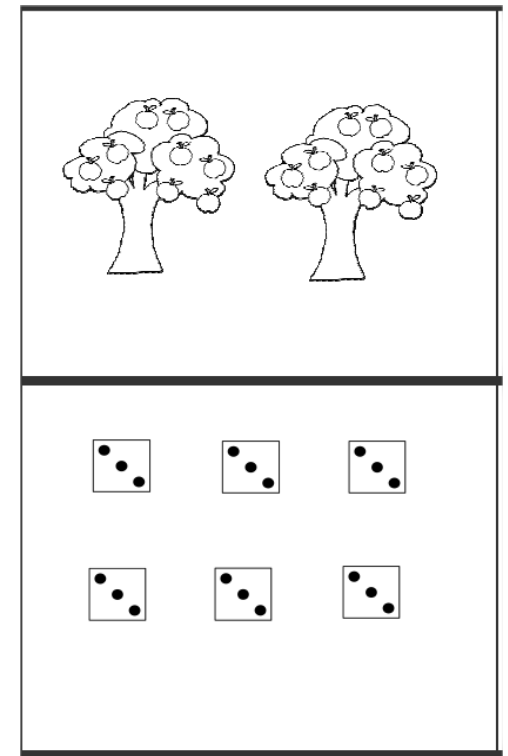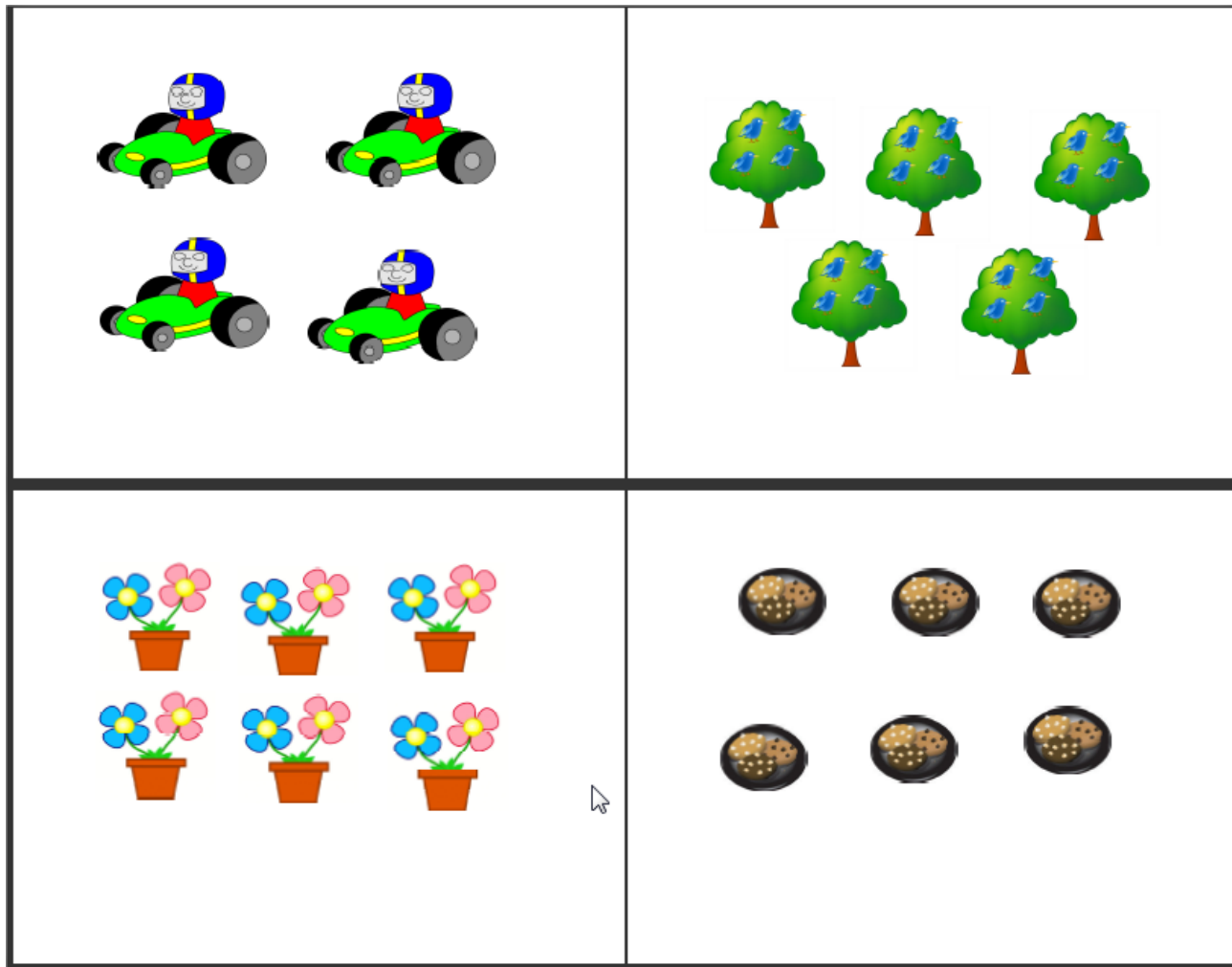
M 435.2
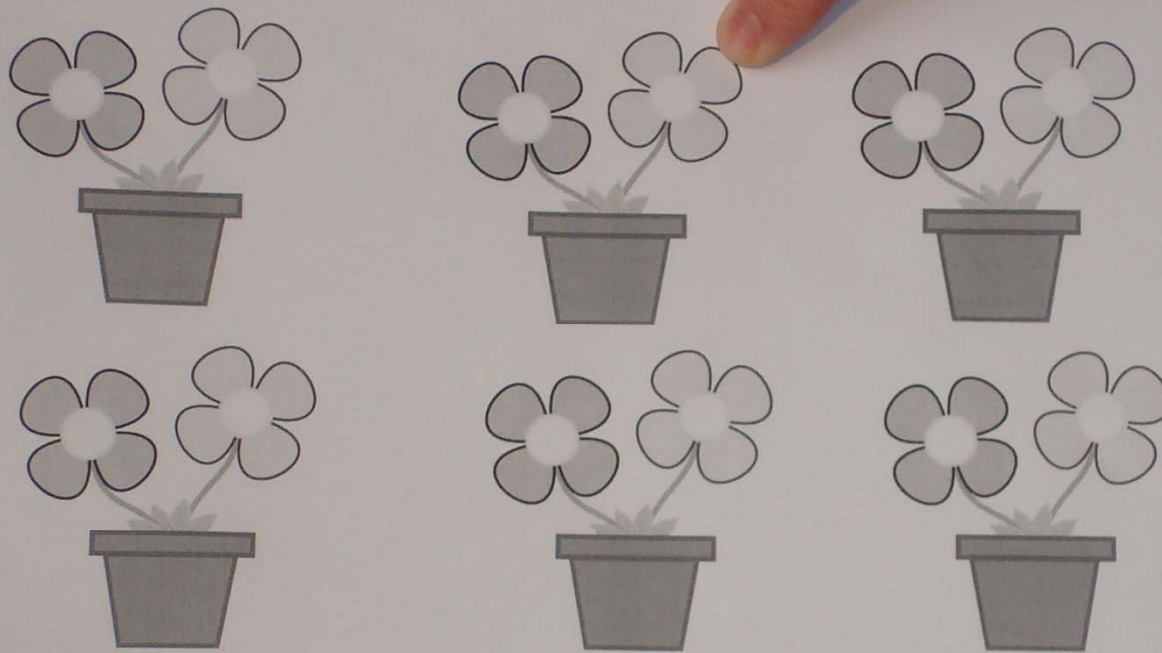
Items remain visible

| t for Learning	Student	Video Link	Print Link	Interactive	Reference	Teacher Notes	S
picture showing 4. Ask student see" Ask follow- s such as "How ps? How many ch group?" and items in all?" dent to write a ddition and a multiplication ation.	various		ttp://www.kentuckymathematics.org/inter			Note carefully how the total number of items are being counted. If the student's counting does not take advantage of the organization of the groups, the student may need more time building groups (i.e. activity 435.1). Additional images are available through the powerpoints used with activities 435.3 and 435.4. Initially students will likely need a printed image that can be touched. However, students should progress to being able to working with a projected image. When using the powerpoint at this level, keep the image will all items visible. When students are able to do this easily, students are ready to move to the next level.	C

Powerpoint for M 435.3
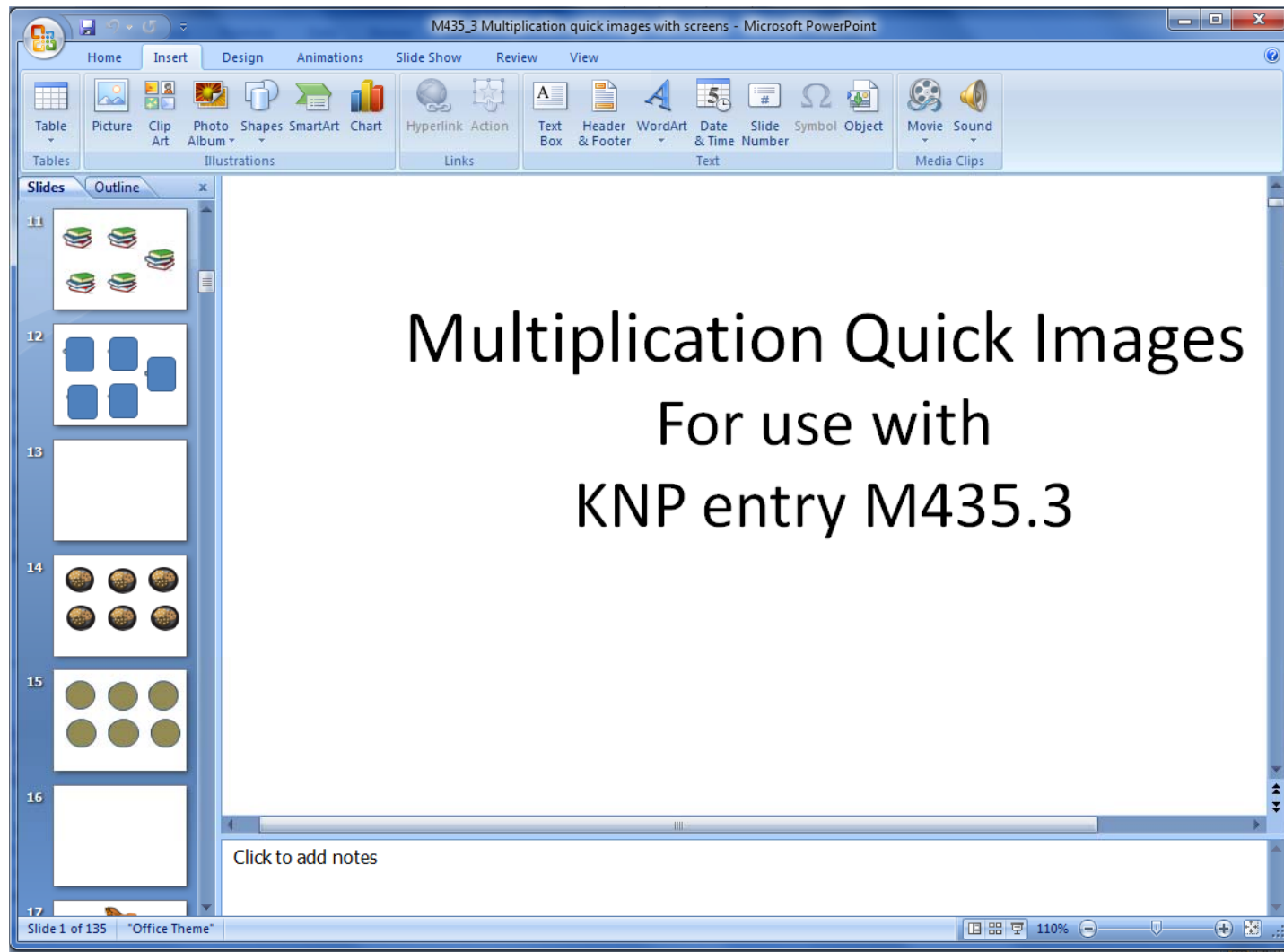
M 435.2
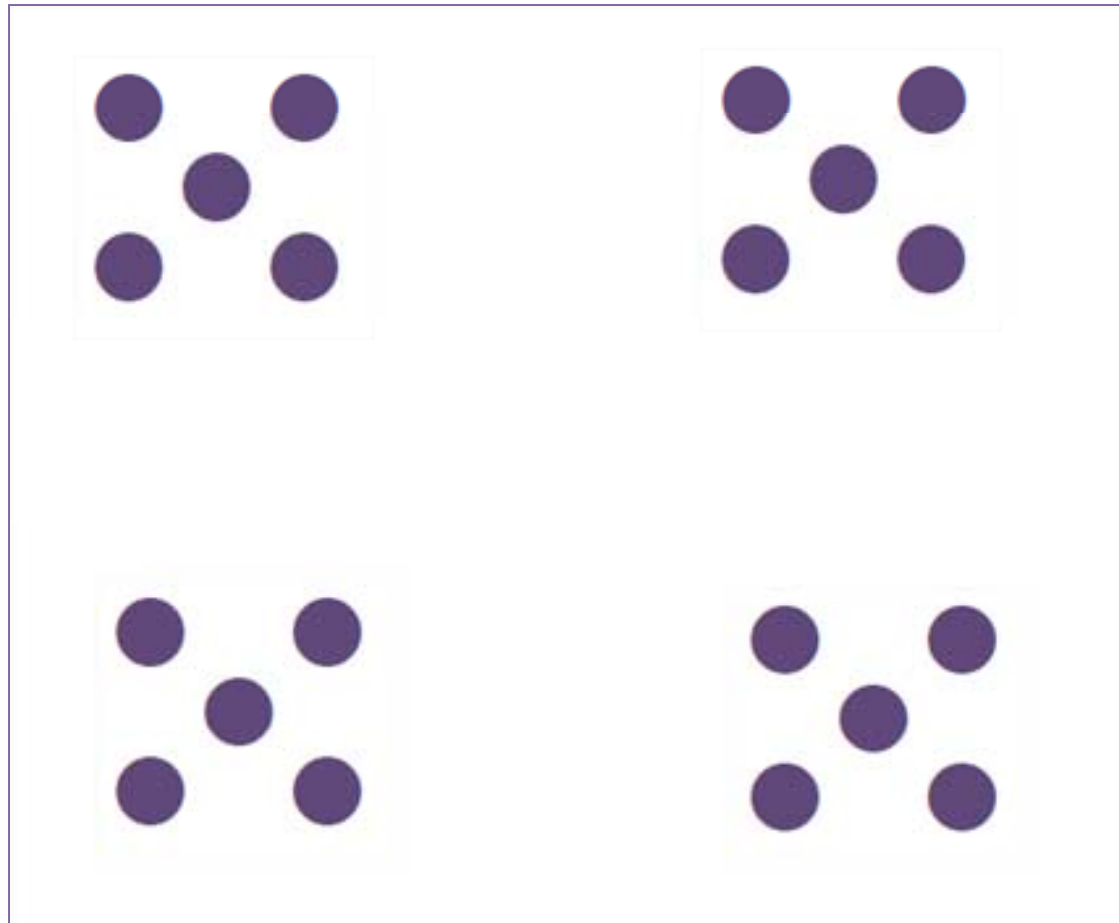
PowerPoint image remains visible

M 435.2
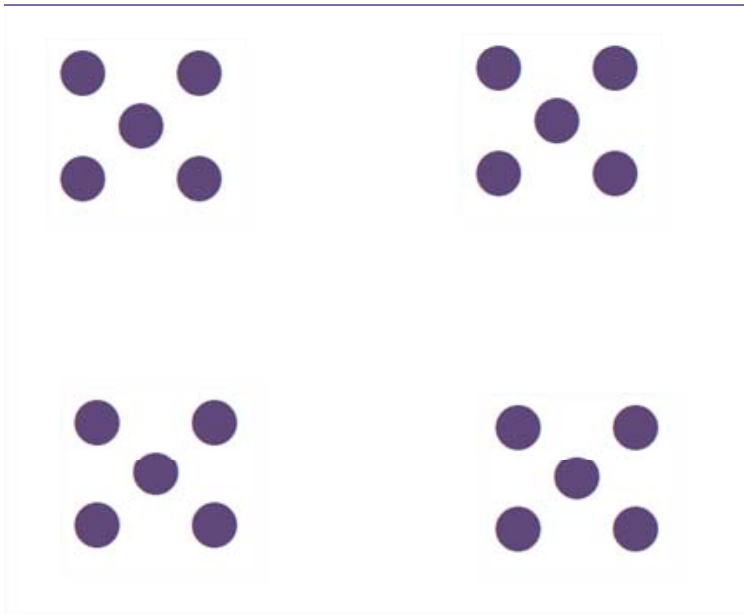
Allow students "think time"

Students may use a "thumbs up" to indicate readiness to answer.

Students may be asked to record their thinking on paper.

M 435.3

KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Structure Level (from AVMR) Numeracy Target	"I CAN ......" (*see glossary)
M 435.3	3.OA.1. Interpret products of whole numbers (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects ea., i.e. describe a context in which a total number of objects can be expressed as 5×7	Operations and Algebraic Thinking	Represent & solve problems involving multiplication/division	Multiplication images with screens PowerPoint (see link) - may be printed or shown directly, writing surface	The teacher or another student will flash the PowerPoint quick image for approximately 3 seconds, then display the image with the groupings screened. Students will determine how many dots or items in all. If using printed images, briefly show the image and then place the screened version on top of the unscreened version. Students should record a matching multiplication equation.	Multiplication and Division	2 to 3 GREEN Figurative composite grouping	... find the product with only group markers visible and write the matching multiplication equation.

Use the powerpoint available through the print link.

Let's do an example...

Are you ready?

M 435.3

Flash image briefly – approximately 3 seconds

M 435.3
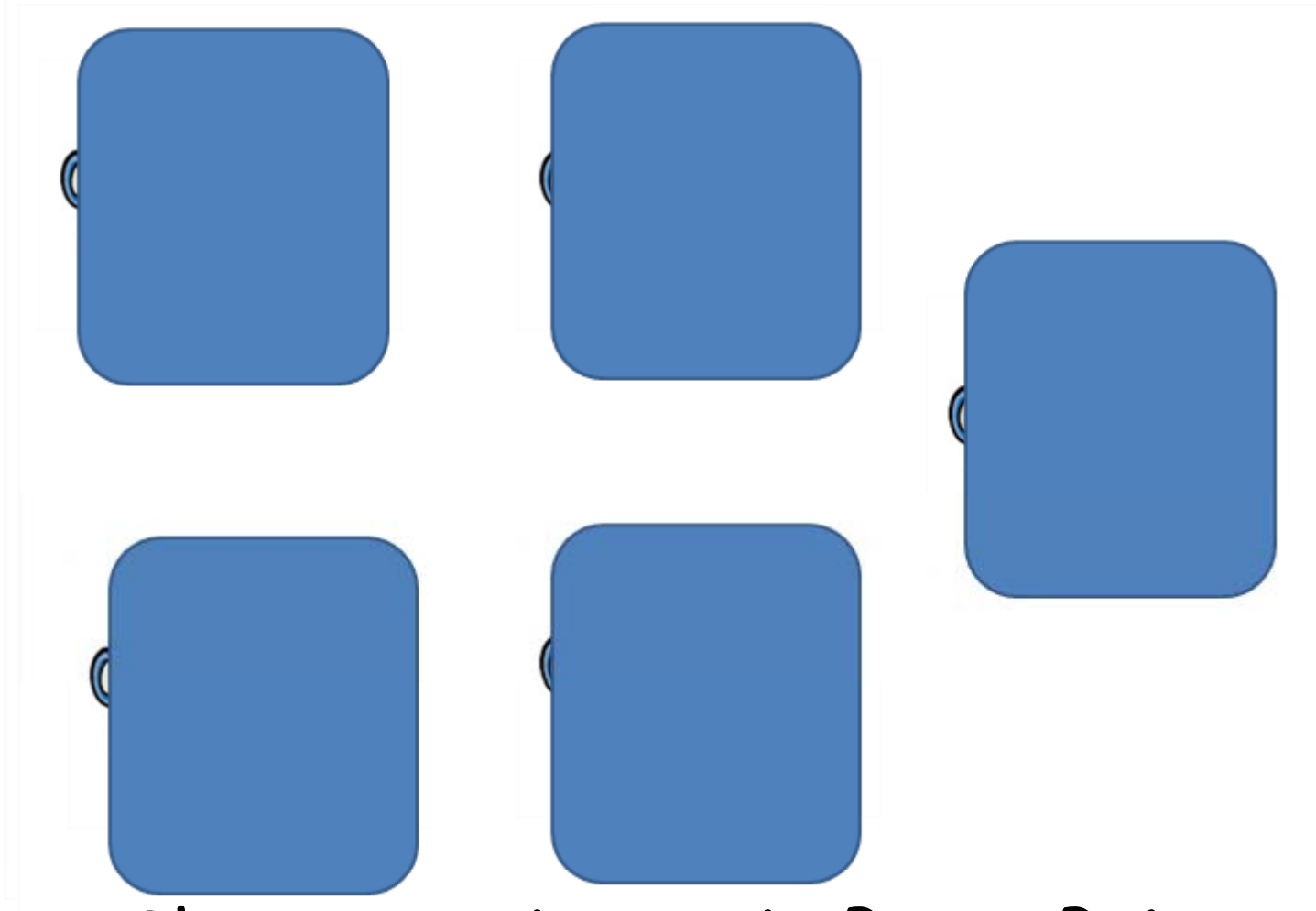
Show next image in PowerPoint

M 435.3
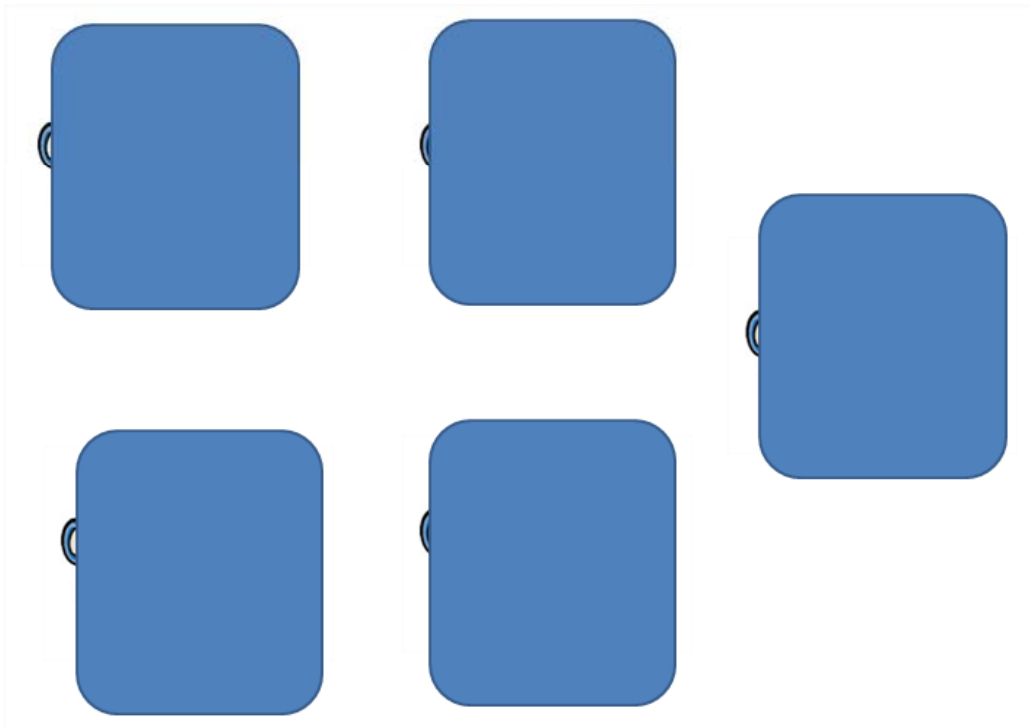
Group Markers

The images of the covered books can be used by the students to keep track of the number of groups.

M 435.3
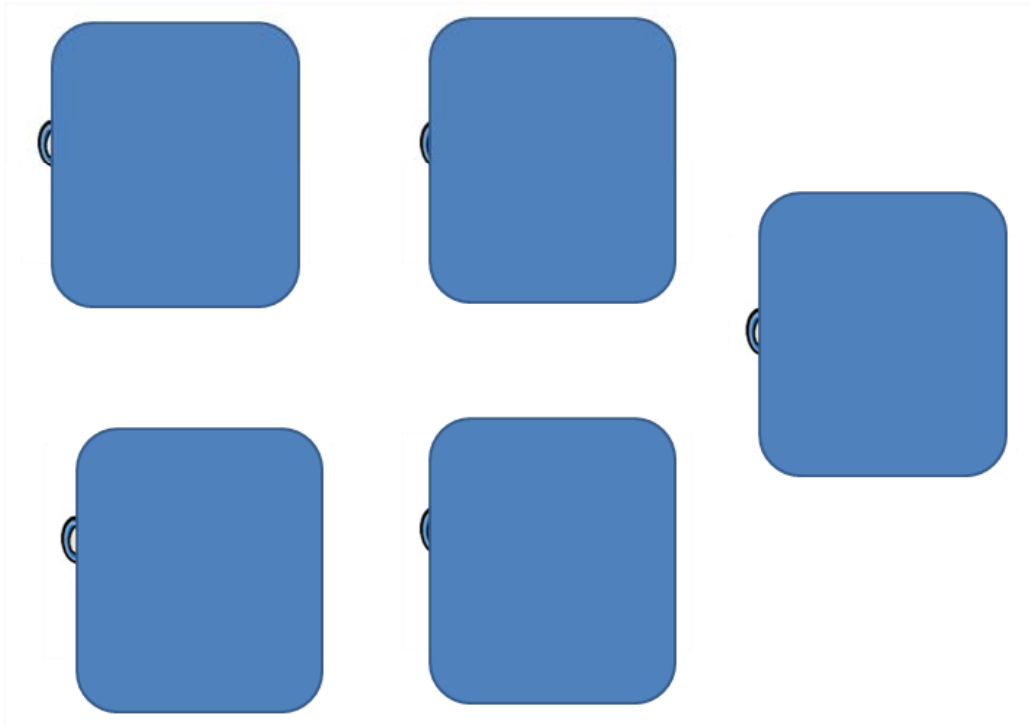
Questions:
- What do/did you see?

- How many books in each stack?

- How many stacks?

- How many books?

- How did you figure that out?

M 435.3
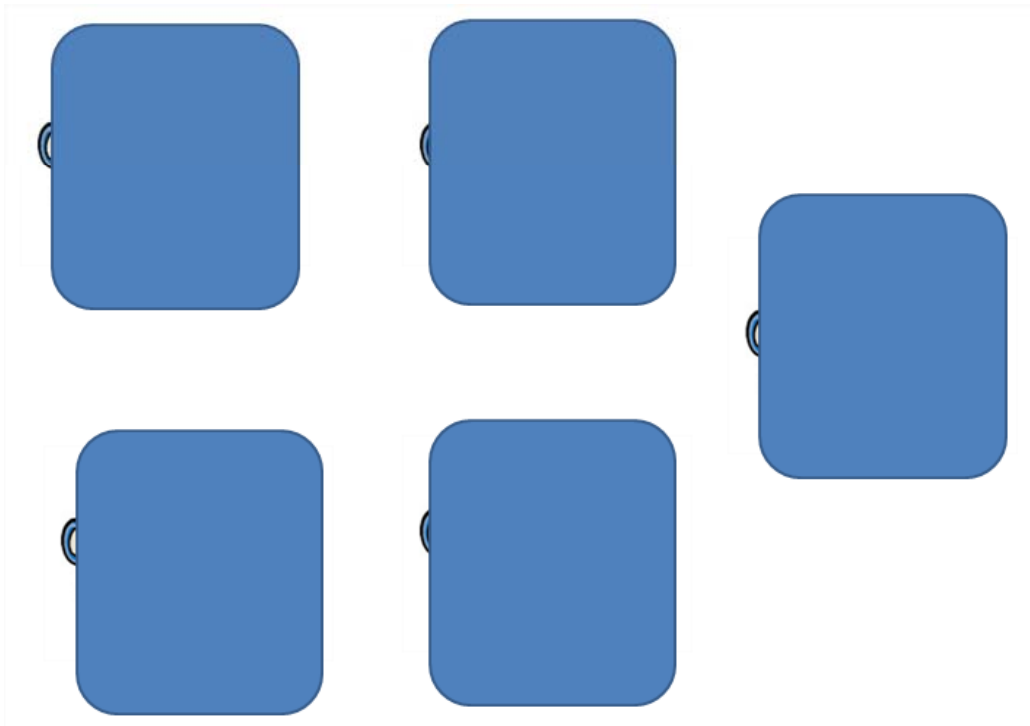
Possible equations:

3+3+3+3+3 = 15

5 x 3 = 15

Bring out connection between multiplication and repeated addition

Using the powerpoint

-May be presented small or whole group

-10 to 15 minutes per lesson

-Present only a few slides per lesson

-Can be used for daily computation practice

-Student driven discussion
(see Webinar 6 for more information about fostering student thinking through discussion)

M 435.3 - printout
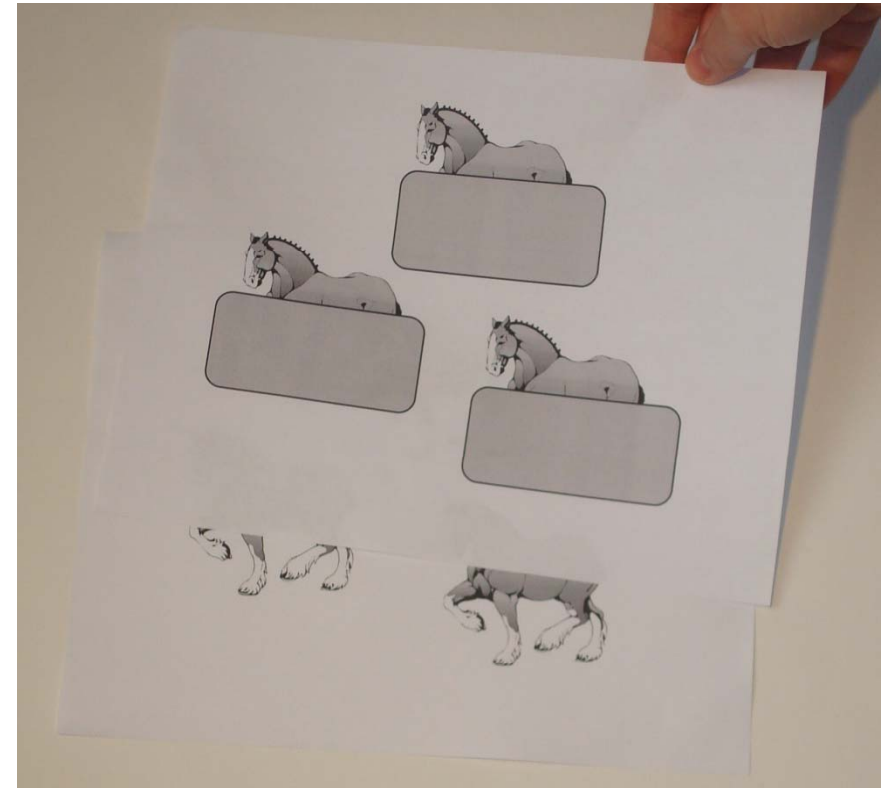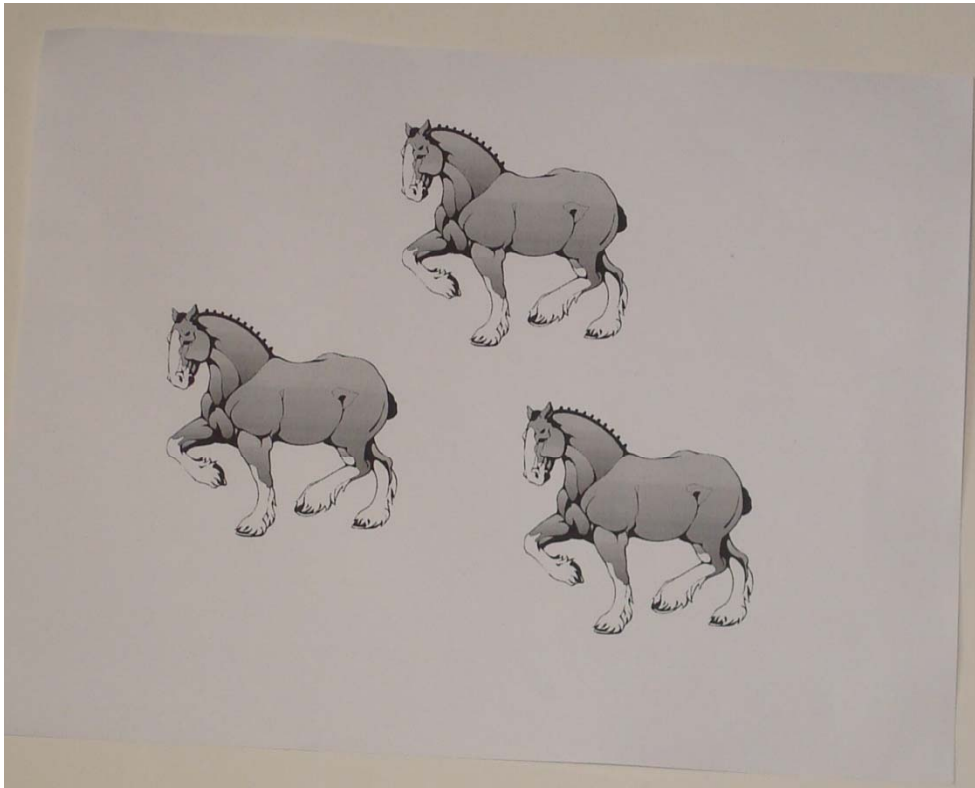
Show image for a few seconds, then screen items

M 435.4

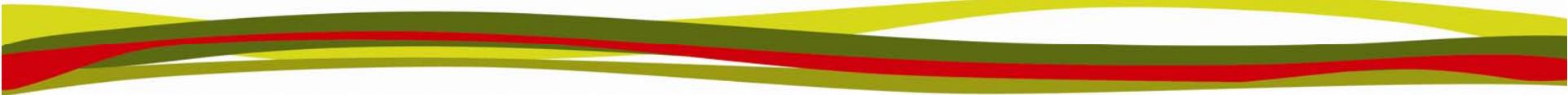| KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct Level (from AVMR)	Numeracy Target	"I CAN ......" (*see glossary)	
M 435.4	3.OA.1. Interpret products of whole numbers (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects ea., i.e. describe a context in which a total number of objects can be expressed as 5×7	Operations and Algebraic Thinking	Represent & solve problems involving multiplication/division	Multiplicative quick images PowerPoint (see link) - may be printed or shown directly, writing surface	The teacher or another student will flash the PowerPoint quick image for approximately 3 seconds, then show only a white screen. Students will determine how many dots or items in all. If using printed images, briefly show the image and then screen image. Students should record a matching multiplication equation.	Multiplication and Division	3 to 4 PURPLE	Repeated abstract composite grouping	... find the product when all items are screened and write the matching multiplication equation.	s
	3.OA.4	d	p	ns	g	One student or the teacher			m	find the	c

Another powerpoint is available using the print link.

Let's do an example...

Are you ready?
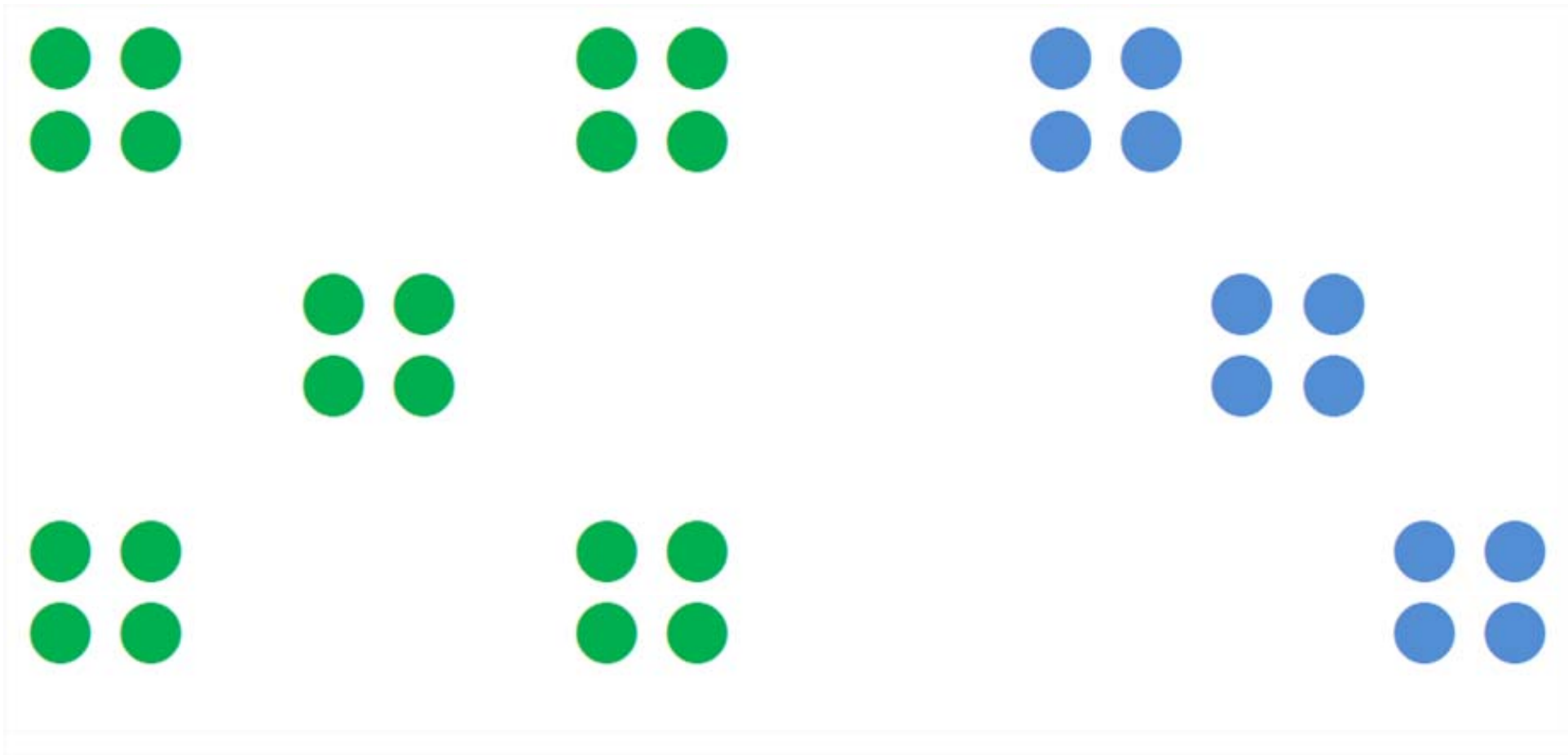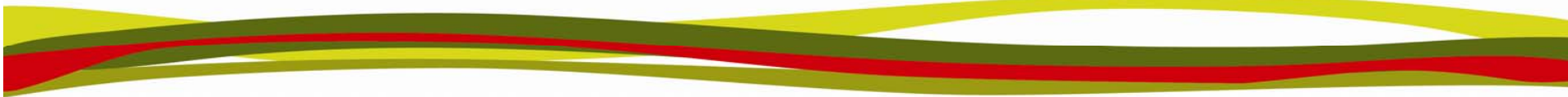
Go to a white screen

Give wait time

Questions:

•Tell me something about what you saw. Tell me something else.
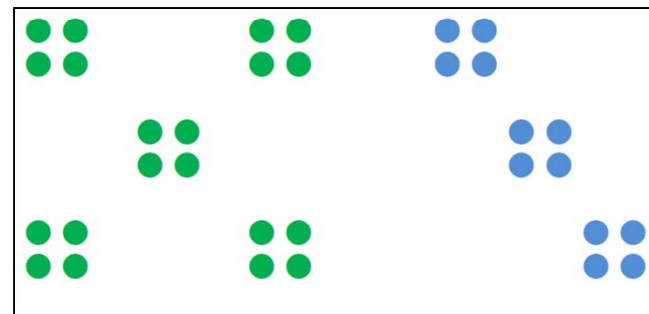
•What groups did you see?

•How many dots? How did you figure that out?

•How many green dots? How many blue?

•Write an equation that matches what you saw. Write another.

Reflash image or display image during discussion

Let's do another example...

Are you ready?

M 435.4
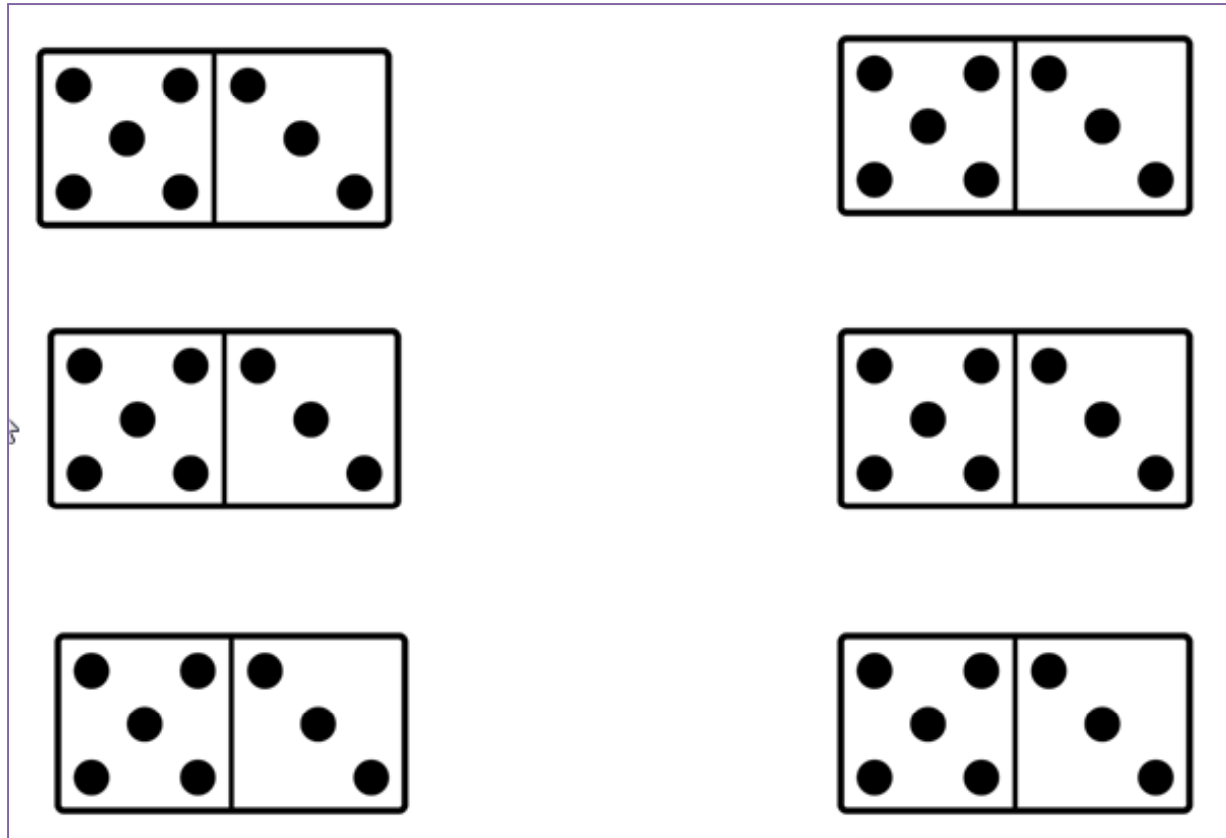
Flash image for approximately 3-5 seconds...

M 435.4

...then fully screen

M 435.4

Questions:
- Tell me something about this picture. Tell me something else.

- What groups did you see?

- How many dots?

...ask open questions

M 435.4

Questions:

•How did you work that out?

•Does anyone have a
    different way?

•What is a matching equation? What is another
equation?
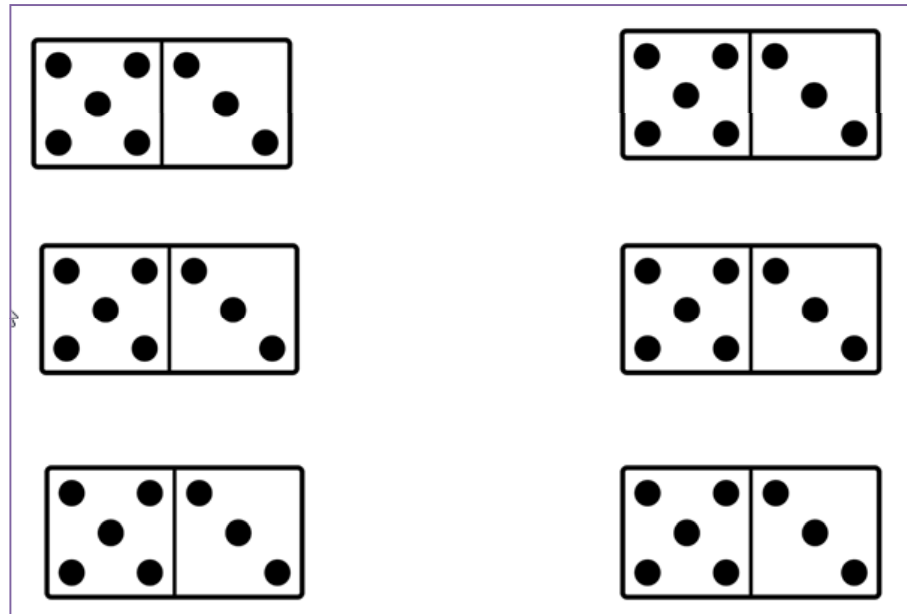
M 435.4

Possible student equations

6 groups of 8
6 x 8 = 48
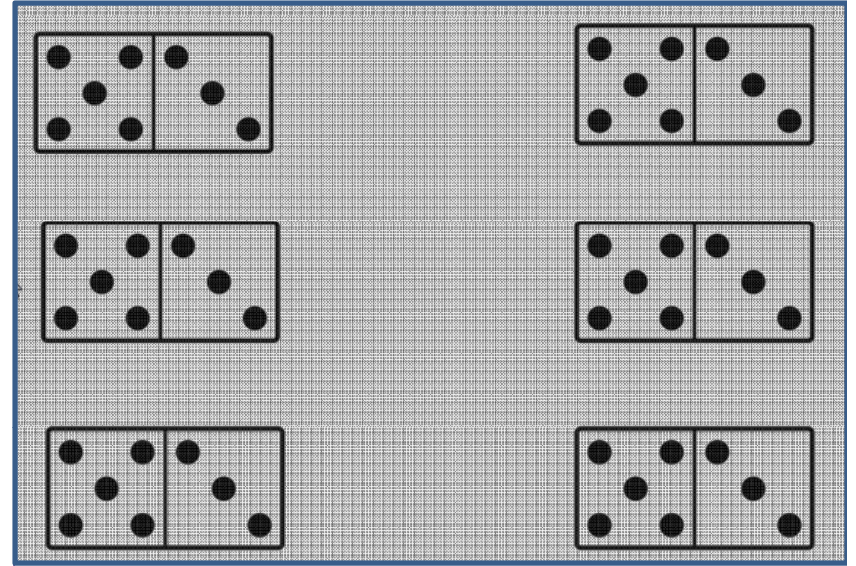
6 groups of 5 and 6 groups of 3
6x5 + 6x3 = 30 + 18 = 48

Develop idea of distributive property
6 x (5+3) = (6 x 5) + (6 x 3)

M 435.4
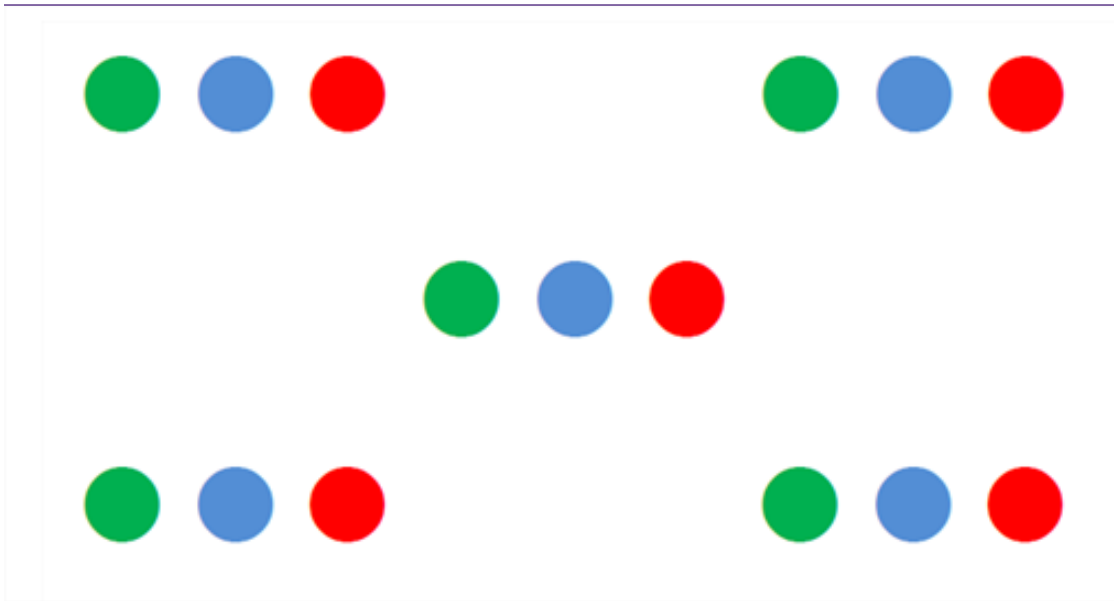
Flash for 3 seconds, then screen image

Questions:
- What did you see?

- What groups did you see?

- Tell me about the groups?

- How many dots?

- How many red dots?

Possible student equations

5 groups of 3
$5 \times 3 = 15$
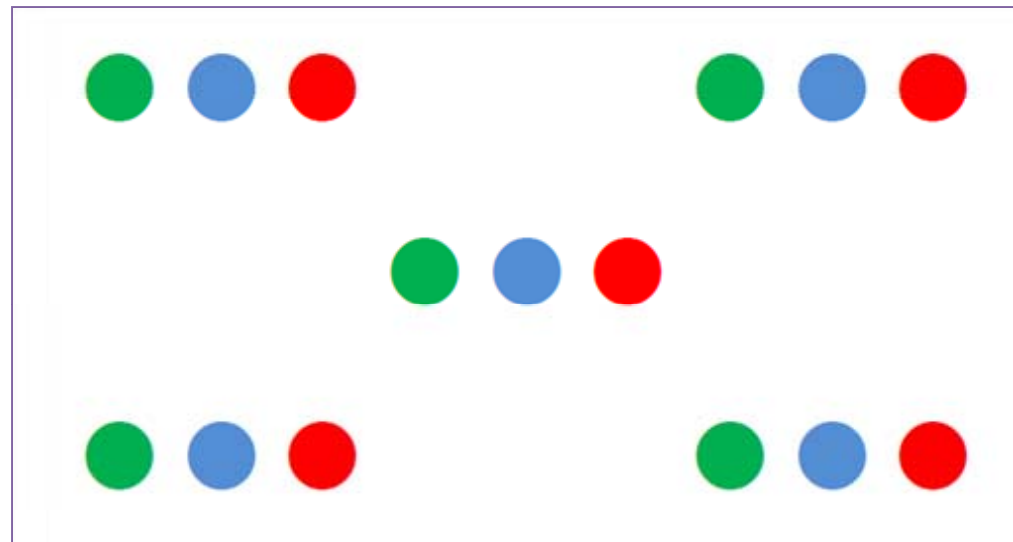
3 groups of 5
$3 \times 5 = 15$

Develop understanding of commutative property
$5 \times 3 = 3 \times 5$

Discuss relationship of multiplication & division
$15 \div 3 = 5$ or $15 \div 5 = 3$

M 435.5

KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct Level (from AVMR) Numeracy Target	"I CAN ......" (*see glossary)
M 435.5	3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in	Operations and Algebraic Thinking	Represent and solve problems involving multiplication and division.	Multiplicative quick images, writing surface	One student or the teacher will describe a quick image in words and then ask a question. The other student(s) will answer the question. For example, suppose the picture shows 5 stacks of 3 books. The student might say "There are 5 stacks of 3 books - how many books in all?" or "There are 5 equal stacks and 15 books in all - how many books are in each stack?" The	Multiplication and Division	4 to 5 PINK Multiplication and division as operations	... find the product or quotient

M 435.5 print link
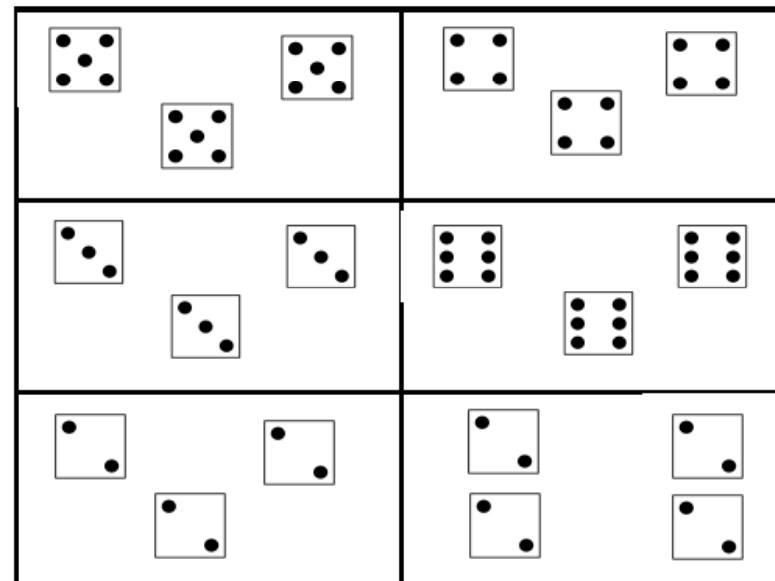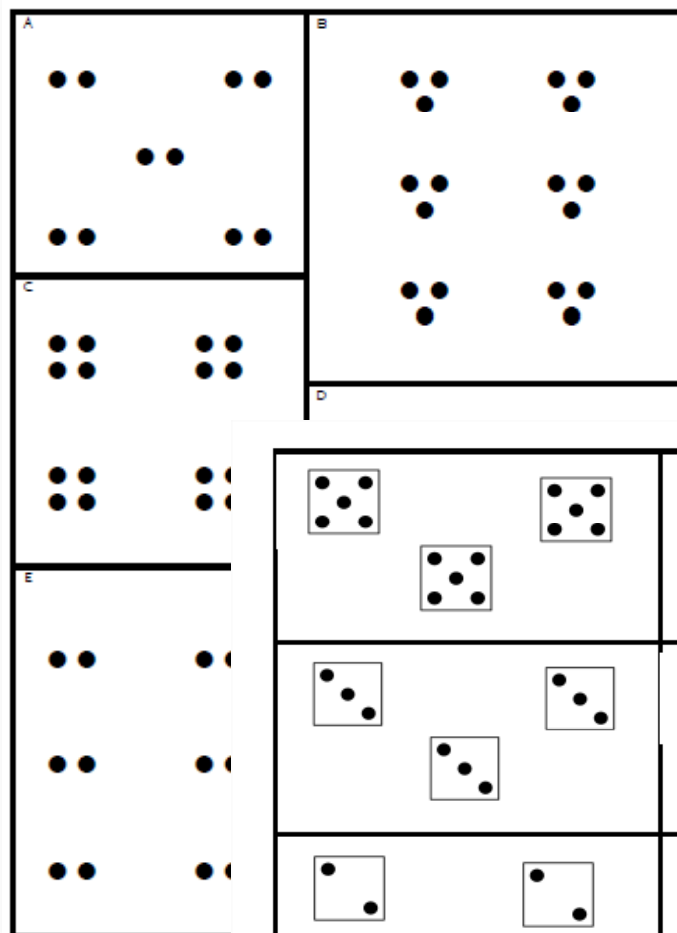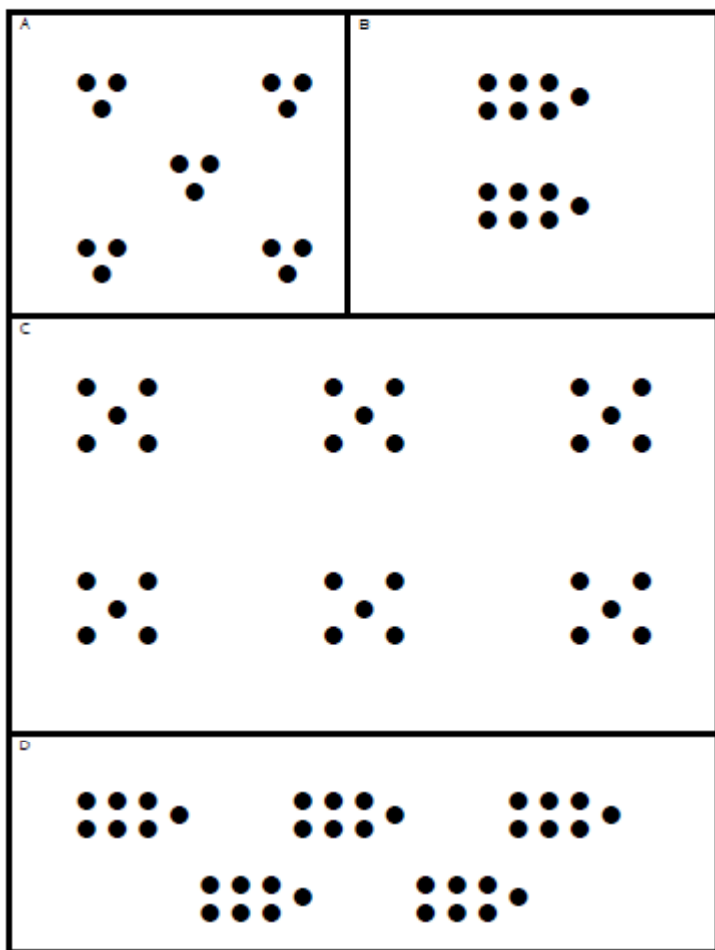
M 435.5 Example

Student A looks at picture

Student A might say:
"My picture has 6 dice all showing the same amount. There are 18 dots showing. How many dots are on each die?"
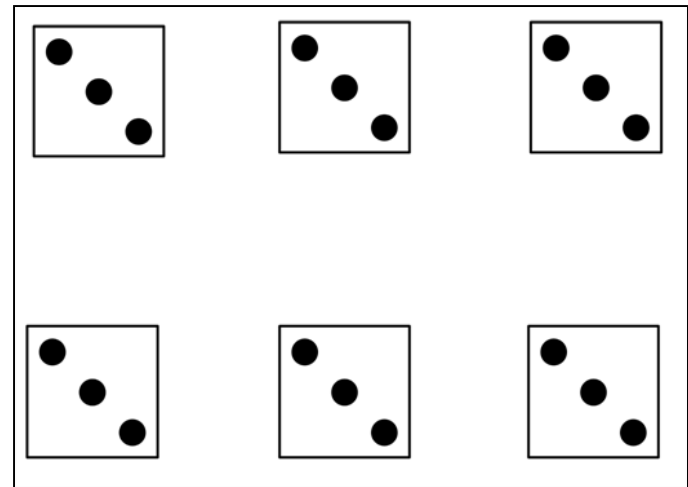
Student B
"Since 6 times 3 is 18, each die is showing 3"

M 435.5 Example

Record work
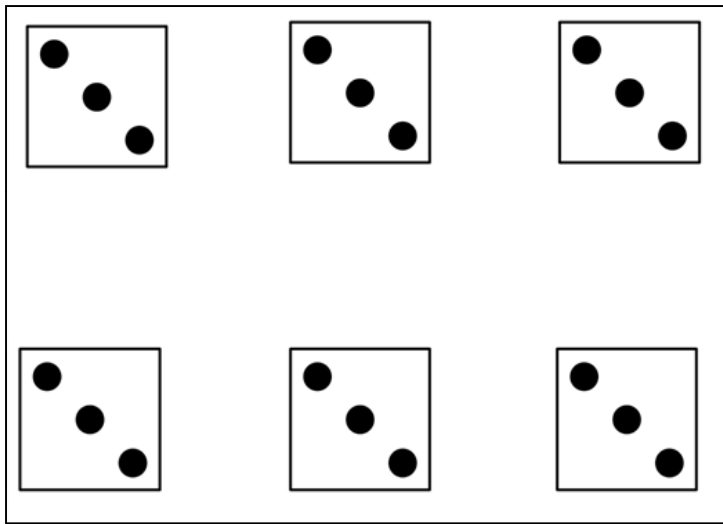
6 x n = 18

n = 3
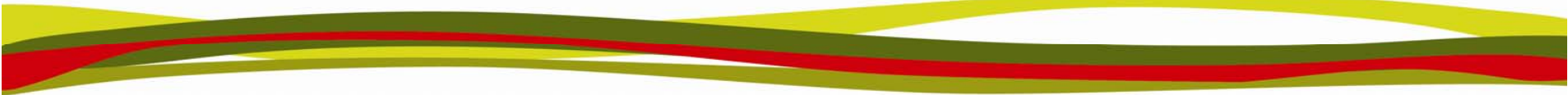
OR

18 ÷ 6 = n

n = 3

M 435.6

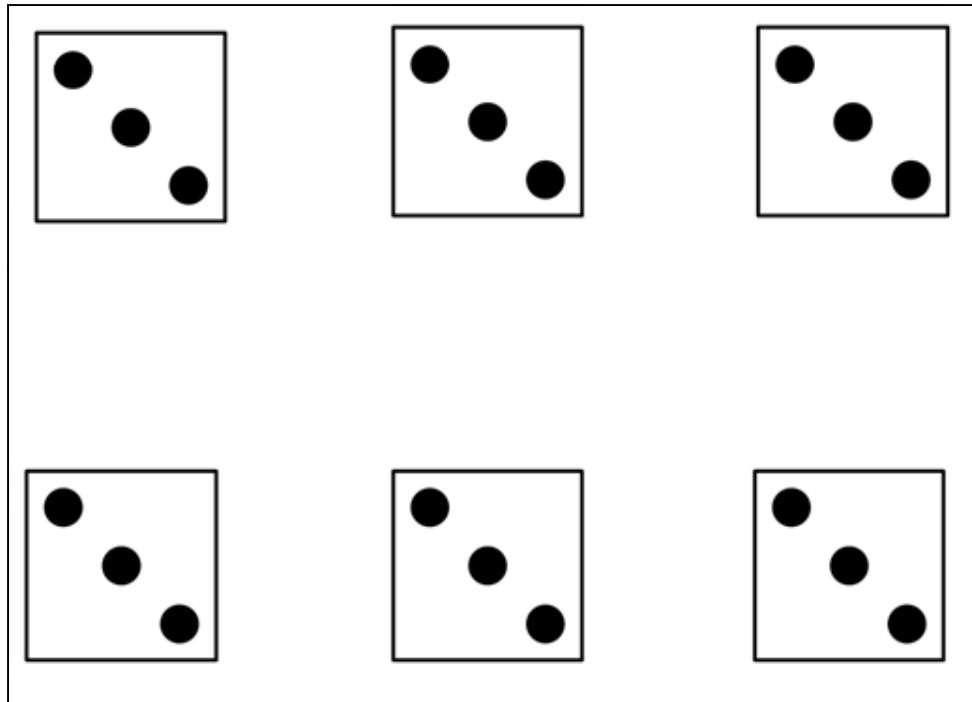| KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct/Level (from AVMR)	Numeracy Target	"I CAN ......" (*see glossary)
M 435.6	3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?	Operations and Algebraic Thinking	Represent and solve problems involving multiplication and division.	Multiplication /Division Strings	The teacher will present a series of related problems, one at a time. The student(s) will answer and justify the answer to each problem in succession. The teacher will invite discussion of each problem and relationships between the problem to explore mathematical ideas.	Multiplication and Division	5+ ORANGE	Multiplication and division as operations	... find the product and/or quotient to a string of related problems.

Problem Strings

- Present a series of related computation problems

- Use questioning and student discussion to explore mathematical ideas and increase computational fluency

See print link and view Webinar 6 for more information

M 435.6 – print link

MENTAL MATH PROBLEM STRINGS
(Multiplication and Division)

What is a _Mental Math Problem String_?
A problem string is a _sequence of related arithmetic tasks_ (typically 5-10 problems presented horizontally) that are designed to call attention to a particular mathematical feature. They can be posed with mat... such as showing arrays, or posed with just numerals. The tasks are presented one at a time. Often, t... previous tasks and their solutions remain visible as each successive task is presented.

Problem strings create opportunities for students to discover and strengthen composite arithmetical strategies and foster number sense. The types of tasks and the way successive tasks build on preced... will influence which strategies students will discover and use. Teacher questioning, student think tim... student discussions are critical to the process.

Some examples of multiplication and division problem strings are:

Doubling	Inverse relationship of multiplication and division	Distributive property
2 x 7 =		5 x 8 =
4 x 7 =	6 x 4 =	2 x 8 =
8 x 7 =	24 ÷ 6 =	7 x 8 =
3 x 8 =	7 x 4 =	5 x 9 =
6 x 8 =	28 ÷ 4 =	6 x 9 =
	28 ÷ __ = 4	7 x 6 =

Fostering student thinking
Use open-ended questions to prompt student thinking, such as
- How did you work that out?
- Why does that work?
- Does anyone have another way to work that out?
- What did you know that helped you work that out?
- Do you see a pattern? What is it?
- What is a new problem that is similar to these?
- Can you use the previous problem(s) to help with this problem?

Allow think time before students respond. A "thumbs up" or other hand single can be used by stude... indicate their readiness to answer. Foster an expectation that students listen to each other and may ... questions about each other's strategies. Students might show agreement through a waggling "hang t... gesture or other agreed upon signal. At times, ask one student to paraphrase another student's expl... or use another student's method. Invite multiple strategies for a single task.

Modeling Student Strategies
When leading activities involving mental problem strings, the teacher can support student thinking and student discussion by acting out and/or recording the students' explanations. For example, the teacher might draw a set model or array model to match an explanation.

Written methods like those shown below can be used to record student thinking. Match the recording model to the strategy used. As students become more familiar with investigating and discussing strings, transition from teacher modeling to the students recording their own thinking and sharing those recordings with the class. (A class document camera is very helpful for these kinds of discussions!)

The teacher's role is to keep the mathematical discussion focused, help student express and record their own thinking and support the students in listening to and understanding each other. Be careful not to show a "teacher" way of solving or thinking - this will discourage students from pursuing their own ideas and listening to peers. Be open - students may come up with ideas you did not anticipate but contain wonderful mathematics!

Equations with pictures   7 x 8 = 56
"I split the 7 apart into 5 and 2. 5 sets of 8 is 40, 2 sets of 8 is 16, 40 + 16 = 56."

2 x 8 = 16

5 x 8 = 40

7 x 8 = (5 x 8) + (2 x 8)
= 40 + 16
= 56

JUMP MODEL   7 x 8 = 56
(Empty Number Line)
"I knew that 5 x 8 = 40 and then counted by 8 two more times... 40, 48, 56"

40	48	56
5 x 8	6 x 8	7 x 8

Teacher Resources

The following two teacher resources use a series of problem strings over the course of 1-3 years to develop multiplication and division ideas. Copy the URL below the title for sample information. Copy the final link for ordering information.

Minilessons for Early Multiplication and Division; A Yearlong Resource, Fosnot, C. T. & Uittenbogaard, W. Portsmouth, (2007) NH: Heinemann.
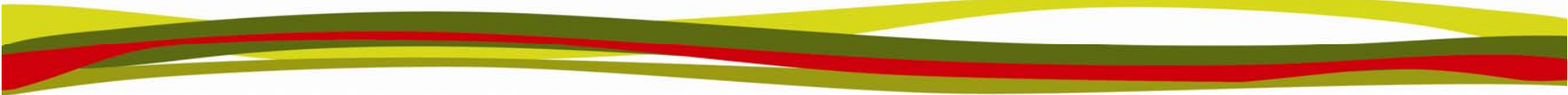http://www.contextsforlearning.com/samples/RG35EarlyMultDivConSum.pdf

Minilessons for Extending Multiplication and Division; A Yearlong Resource, Fosnot, C. T. & Uittenbogaard, W. Portsmouth, (2007) NH: Heinemann.
http://www.contextsforlearning.com/samples/RG35ExtMultDivConSum.pdf

Ordering Information:
http://www.heinemann.com/search/searchResults.aspx?s=all&q=minilessons%20multiplication%20and%20division

Additional Examples of Mental Math Problem Strings

Inverse relationship of multiplication and division and Doubles	Doubling and Halving	Using related facts
2 x 7 =	2 x 7 =	5 x 4 =
2 x 3 =	14 ÷ ____ = 7	6 x ____ = 24
4 x 3 =	4 x 7 =	24 ÷ ____ = 4
12 ÷ 2 =	8 x 7 =	4 x 7 =
2 x 4 =	28 ÷ 4 =	28 ÷ 4 =
4 x 4 =	28 ÷ 2 =	___ x 8 = 32

Doubling and Halving	Commutative property of multiplication and inverse relationship of mult. & div.	Doubling
4 x 3 =	20 ÷ 10 =	2 x 6 =
__ x 3 = 24	20 ÷ 5 =	4 x 6 =
8 x 6 =	20 ÷ 4 =	6 x 6 =
4 x 6 =	16 ÷ 2 =	4 x 8 =
2 x 6 =	16 ÷ 8 =	8 x 8 =

Distancing the Setting

Concrete   Abstract/Symbolic

Distancing the Setting in M435

-Objects that can be moved and touched. (435.1)

-Images that can be touched. (435.2)

-Images in the distance. (435.2)

-Images that are flashed and then partially screened. Only group markers are visible. (435.3)

-Images that are flashed and then fully screened. (435.4)

-Images that are described but not seen. (435.5)

-Bare number problem strings. (435.6)

M436.1

KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand	Construct/Level
M 436.1	3.OA.1. Interpret products of whole numbers (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects ea., i.e. describe a context in which a total number of objects can be expressed as 5×7	Operations and Algebraic Thinking	Represent & solve problems involving multiplication/division	beans or other counters; small plastic portion cups	Distribute to each student 5 cups and a handful of beans. Give instructions for each student to take a given number of beans and share them equally into a given number of cups.	Multiplication and Division	0 to 1 RED

M436.1

Exemplary Experiences (summary)	Numeracy Strand	Construct/Level	Numeracy Target (from AVMR)	"I CAN ......" (*see glossary)	Assessment for Learning	Student Grouping	Video Link	Interactive Website	Reference	Teacher Notes
student 5 of beans. for each ven number re them number of	Multiplication and Division	0 to 1 RED	Initial grouping	...create equal groups by sharing.	Ask a student to take 14 beans and make 2 equal groups; then to take 16 beans and make 4 equal groups; etc.	Whole class				

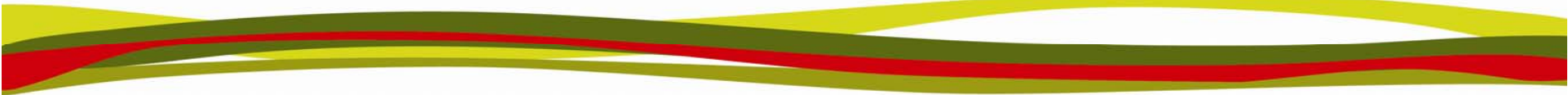# Materials for M436.1
(could also use counters, bears, etc.)

"There are 16 beans. Can you put the beans in 4 equal groups?"

"What if there were only 2 groups – could you put 16 beans into 2 equal groups?"

"How many beans are in each group?"

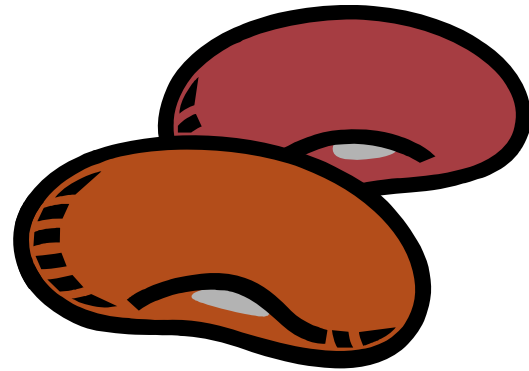"Get 8 beans and make 4 equal groups. How many beans are in each group?"

"Pull 12 beans from the bag. Can you put the beans into equal groups? How many equal groups do you have? Could you group them equally in a different way?"

"You have 5 cups. How many beans will you have altogether if you put 3 beans in each cup?"

"Take 13 beans and make 2 equal groups. How many beans are in each group?"

M436.2

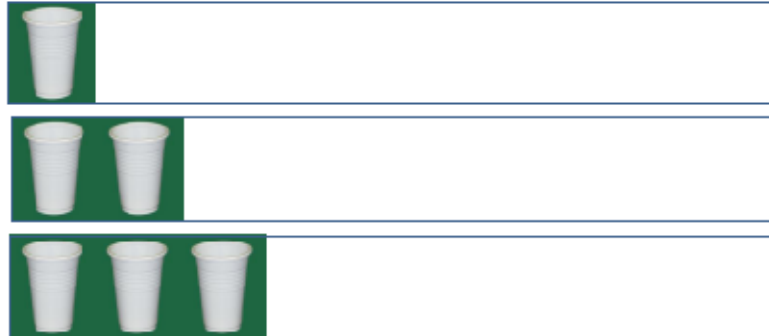| KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activ...
M 436.2	3.OA.1. Interpret products of whole numbers (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects ea., i.e. describe a context in which a total number of objects can be expressed as 5×7	Operations and Algebraic Thinking	Represent & solve problems involving multiplication/division	For each pair of students: 2 sets of Cups Game Cards with 1 to 5 cups (see print link); dot die with 1 to 6 pips; beans; writing space	Print on car... Game Card... with 1 to 5 ... in random ... Card and ro... (actual or ... related mu... process. ... turns. Playe... five tur...

M436.2

Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand (from AVMR)	Construct/Level (from AVMR)
For each pair of students: 2 sets of Cups Game Cards with 1 to 5 cups (see print link); dot die with 1 to 6 pips; beans; writing space	Print on card stock, cut out, and laminate two sets of Cups Game Cards for each pair of students (use only the cards with 1 to 5 cups for this version). Turn all cards face down in random order. The first player will take a Cups Game Card and roll the die. The player will put that many beans (actual or drawn) repeatedly on each cup and write the related multiplication equation. Player B will repeat the process. The game ends when both players have had 5 turns. Players should find the sum of the products from all five turns. The player with the highest sum wins.	Multiplication and Division	1 to 2 BLUE

M436.2

Construct/Level (from AVMR)	Numeracy Target (from AVMR)	"I CAN ......" (*see glossary)	Assessment for Learning	Student Grouping	Video Link	Print Link	Interactive Website	Reference	
1 to 2 BLUE	Perceptual counting in multiples	...make equal groups of visible objects, find the product, and write the multiplication equation.	Ask a student to show four groups of three, to find the product and to write the multiplication sentence. Repeat for three groups of four and ask the student to think about and explain why the products are the same.	Partners		cups game cards			Students may wish t actual beans on the p cup pictures indicat actual portion cups (i range 1 to 5 or by a students' counting str count by ones, others skip counting or em Awareness of the grou count group markers

Cups Game Cards

M436.2

...arning	Student Grouping	Video Link	Print Link	Interactive Website	Reference	Teacher Notes
...ow four ...find the ...te the ...ence. ...ups of ...dent to ...ain why ...e same.	Partners		cups game cards			Students may wish to draw the beans on the cups instead of placing actual beans on the pictures of the cups. Instead of using the printed cup pictures indicating the number of groups, you may wish to use actual portion cups (indicated by a random numeral card chosen in the range 1 to 5 or by a spinner or different color number cube). Note students' counting strategies. Whereas some students will continue to count by ones, others will indicate awareness of the group structure by skip counting or emphasizing the final number word in each group. Awareness of the group structure reveals that students may be ready to count group markers while visualing the hidden items in each group.

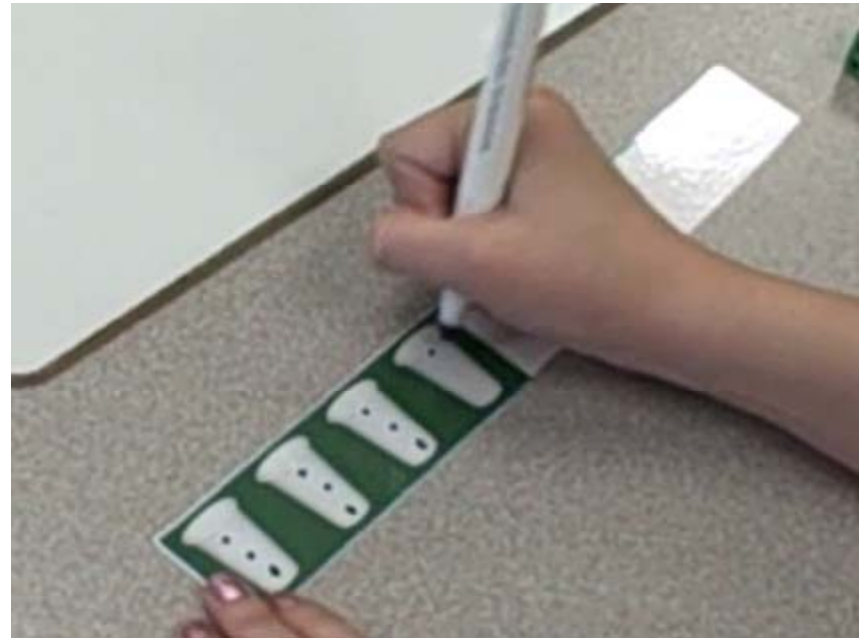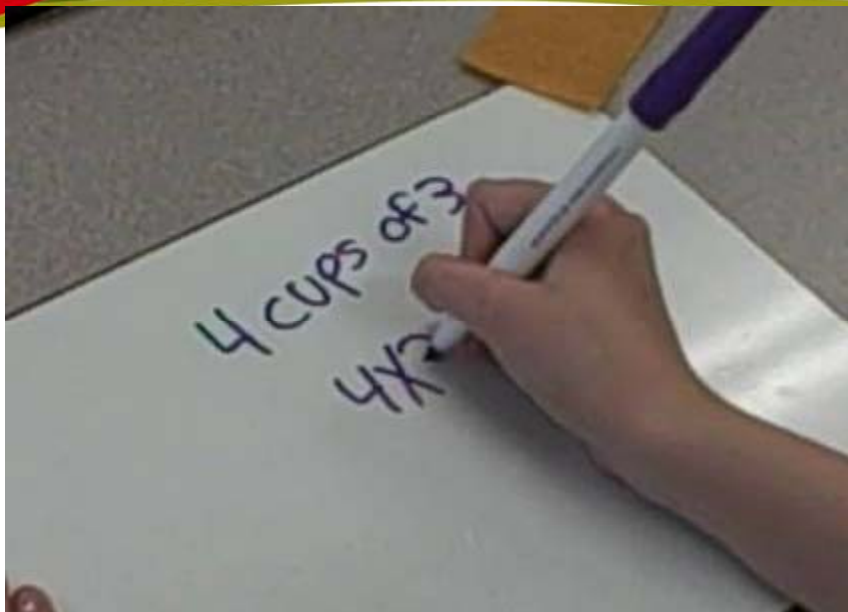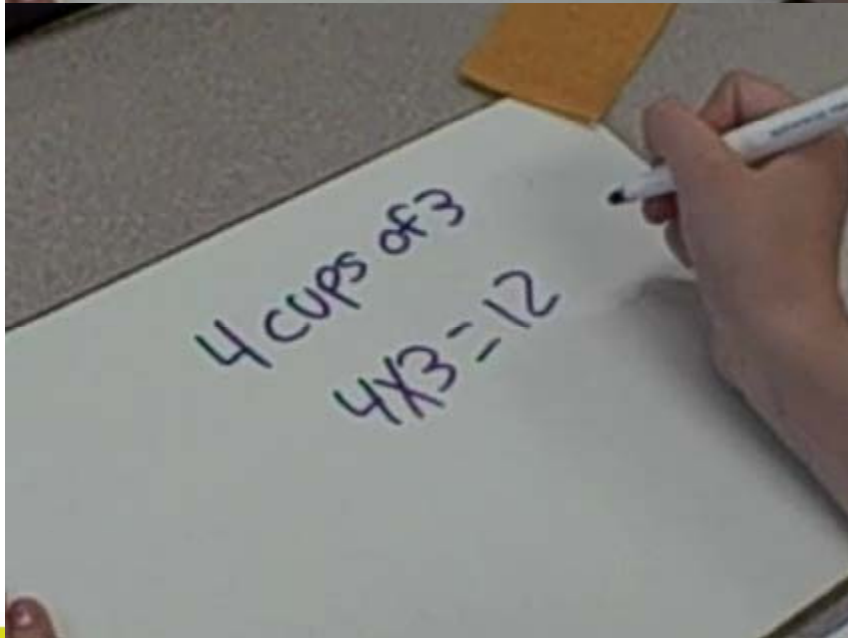# Materials for M436.2
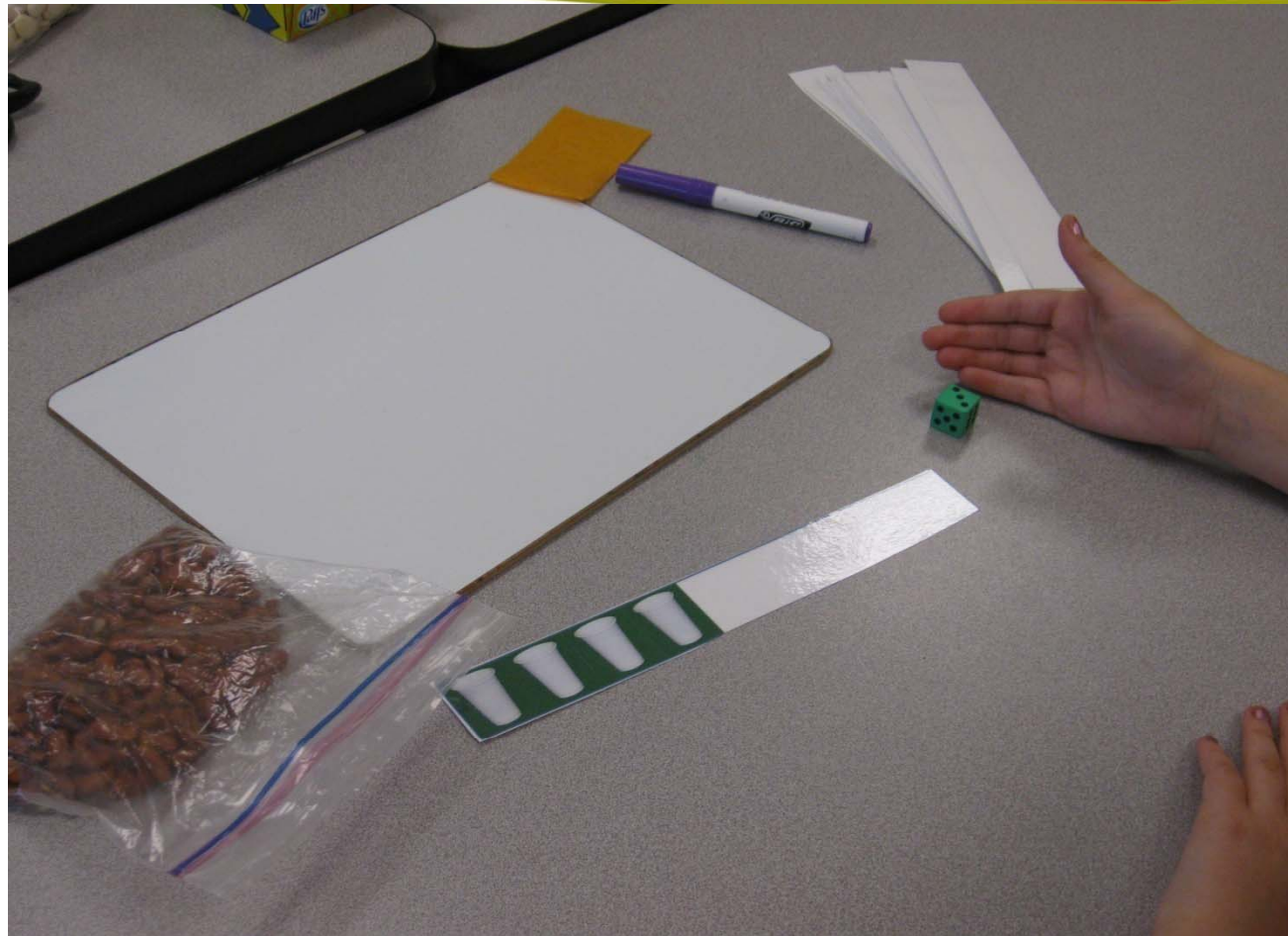
The student chooses a cups card, then rolls the number cube to determine how many counters (or marks) to put on each cup.

3 beans "in" each cup   OR   3 dots on each cup

Record the related multiplication fact

M436.3

KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand (from AVMR)	Construct/Level (from AVMR)
Operations and Algebraic Thinking	Represent & solve problems involving multiplication/division	For each pair of students: 2 sets of Cups Game Cards with 1 to 9 cups (see print link); numeral die with numerals 2 to 7; dry erase marker; writing space	Print on card stock, cut out, and laminate two sets of Cups Game Cards for each pair of students. Turn all cards face down in random order. The first player will take a Cups Game Card and roll the die. The player will put the numeral repeatedly on each cup and write the related multiplication equation. Player B will repeat the process. The game ends when both players have had 5 turns. Players should find the sum of the products from all five turns. The player with the highest sum wins.	Multiplication and Division	2 to 3 GREEN

M436.3

Experiences	Numeracy Strand (from AVMR)	Construct/Level (from AVMR)	Numeracy Target (from AVMR)	"I CAN ......" (*see glossary)	Assessment for Learning	Student Grouping	Video Link	Print Link	Interactive Website	Reference
te two sets of Cups Turn all cards face r will take a Cups will put the numeral lated multiplication ess. The game ends yers should find the The player with the	Multiplication and Division	2 to 3 GREEN	Figurative composite grouping	...find the product when only the group markers are visible and the objects in each group are hidden.	Show a student a card with 6 cups and the numeral 4 written on each cup. Ask the student to tell how many in all and to write the multiplication sentence. Repeat with 4 cups, each labeled with the numeral 6. Ask the student to write the multiplication sentence and to explain how it relates to the first equation.	Partners		cups game cards		

M436.3

|	Assessment for Learning	Student Grouping	Video Link	Print Link	Interactive Website	Reference	Teacher Notes
duct he are he ch den.	Show a student a card with 6 cups and the numeral 4 written on each cup. Ask the student to tell how many in all and to write the multiplication sentence. Repeat with 4 cups, each labeled with the numeral 6. Ask the student to write the multiplication sentence and to explain how it relates to the first equation.	Partners		cups game cards			When students become proficient at efficiently counting the product without reliance on the group markers, they may have sufficiently internalized the quantitty and may be ready to multiply abstractly using numerals only.

Materials for M436.3

Students choose a cups game card, then roll the cube and write that number on each cup
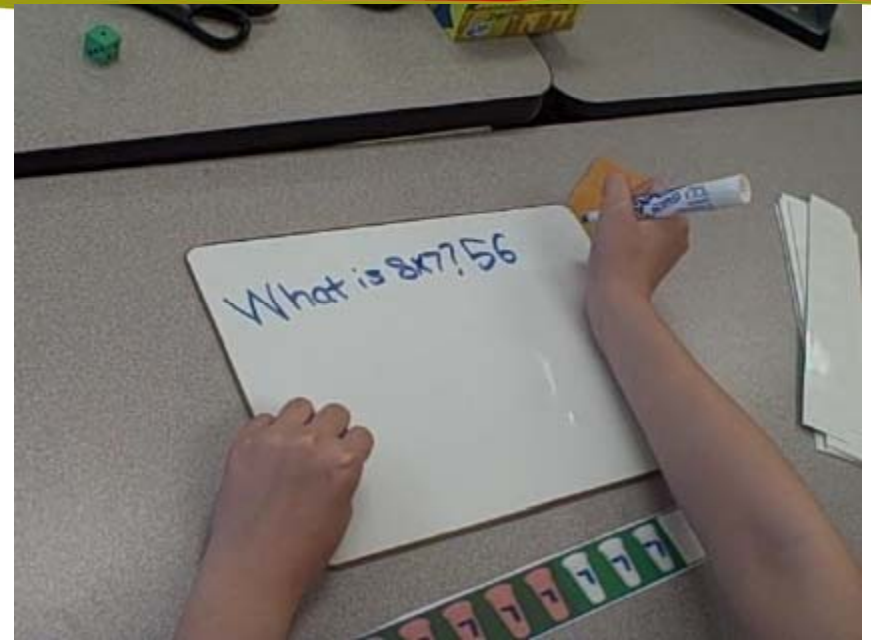
Students write the related multiplication fact

M436.4

KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exe
M 436.4	3.OA.3. Use multiplication & division w/in 100 to solve word problems in situations involving = groups, arrays, & measurement quantities (use drawings & equations w/ a symbol for the unknown number representing the problem	Operations and Algebraic Thinking	Represent & solve problems involving multiplication & division	numeral cards 1 to 12; ten-sided numeral die with numerals 0 to 9; writing space	Player A chooses a n... of cups. Player A t... number of beans in product and writes... continues, alternating... 5 turns. The winner i...

M436.4

| KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand (from AVMR)	Construct/Level (from AVMR)
Operations and Algebraic Thinking	Represent & solve problems involving multiplication & division	numeral cards 1 to 12; ten-sided numeral die with numerals 0 to 9; writing space	Player A chooses a numeral card to represent the number of cups. Player A then rolls the numeral die to find the number of beans in each cup. Player A then finds the product and writes the multiplication sentence. Play continues, alternating between players until each have had 5 turns. The winner is the player with the higher sum of all five products.	Multiplication and Division	3 to 4 PURPLE

M436.4

"I CAN ......" (*see glossary)	Assessment for Learning	Student Grouping	Video Link	Print Link	Interactive Website	Reference	Teacher Note
...find the product when numerals are used to represent the number of groups and the number in each group.	Strategically choose a numeral card (1 to 12) and say, "This is the number of cups". Roll the numeral die and say, "This is the number of beans in each cup." "Find the total number of beans and write the multiplication sentence."	Partners					Be cautious about using numerals explicit quantitative representa children may not have sufficient u sense of the abstract o

M436.4

| Assessment for Learning	Student Grouping	Video Link	Print Link	Interactive Website	Reference	Teacher Notes
Strategically choose a numeral card (1 to 12) and say, "This is the number of cups". Roll the numeral die and say, "This is the number beans in each cup." "Find the total number of beans and write the multiplication sentence."	Partners					Be cautious about using numerals too early without the explicit quantitative representation, because some children may not have sufficient understanding to make sense of the abstract operations.

Materials for M436.4
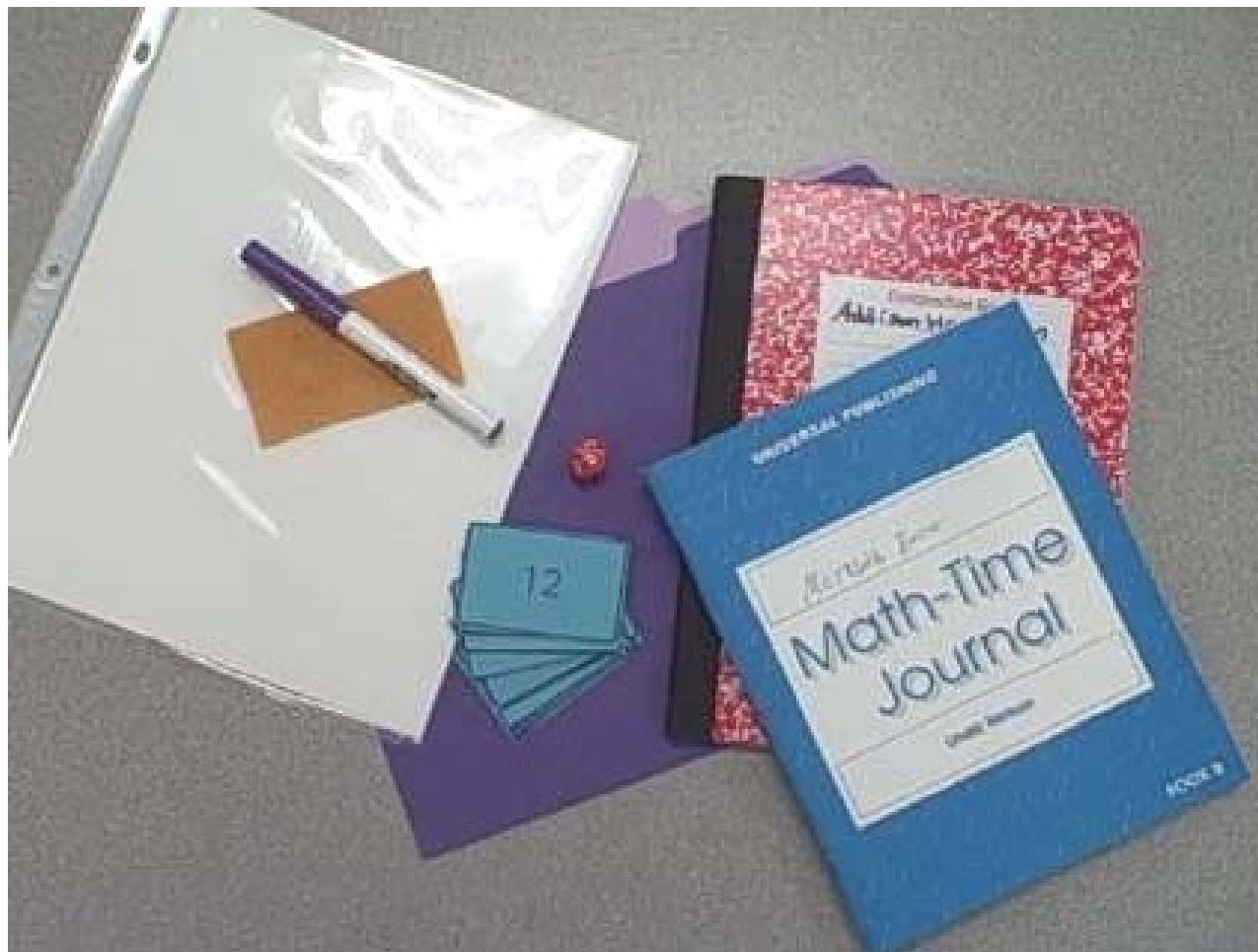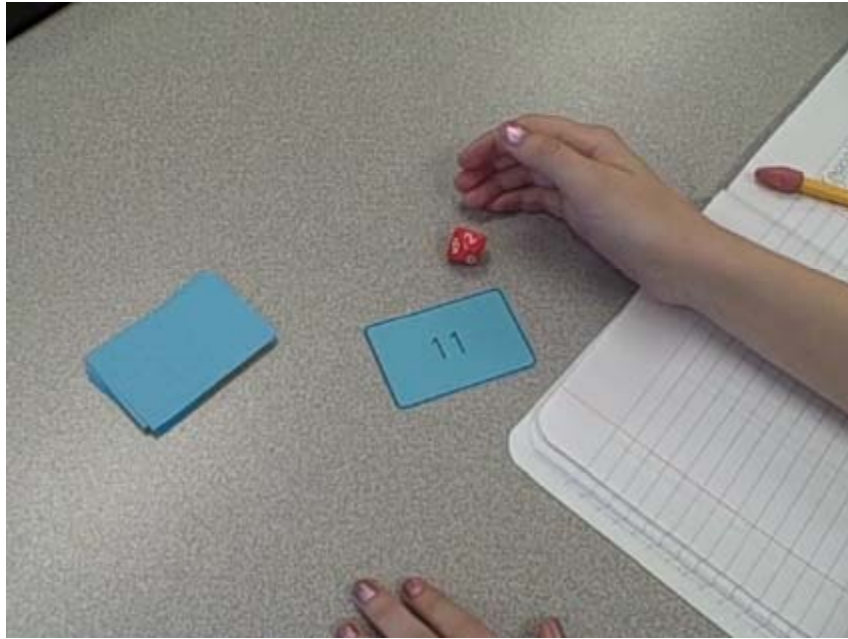
Student draws a numeral card and rolls the die to generate the "number of cups and beans"

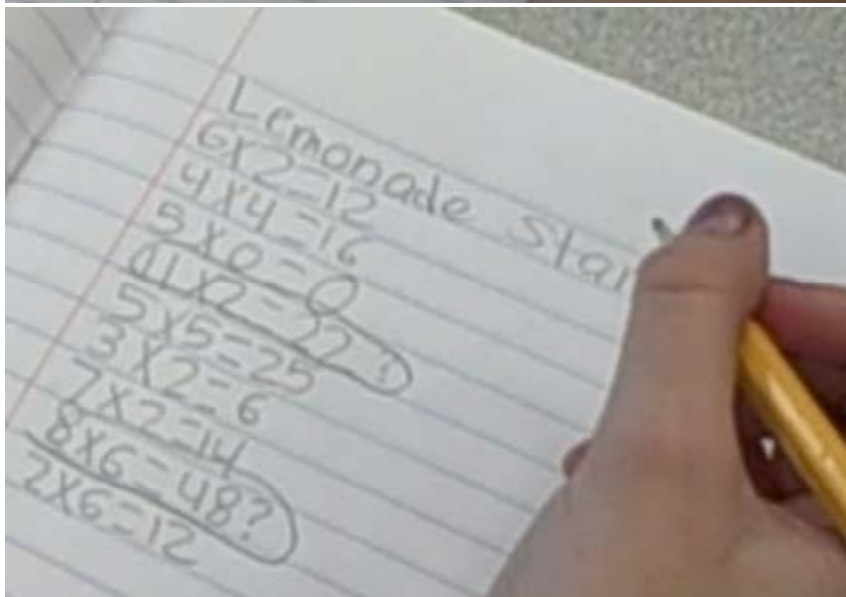**Student tells the product and records the corresponding multiplication sentence.**
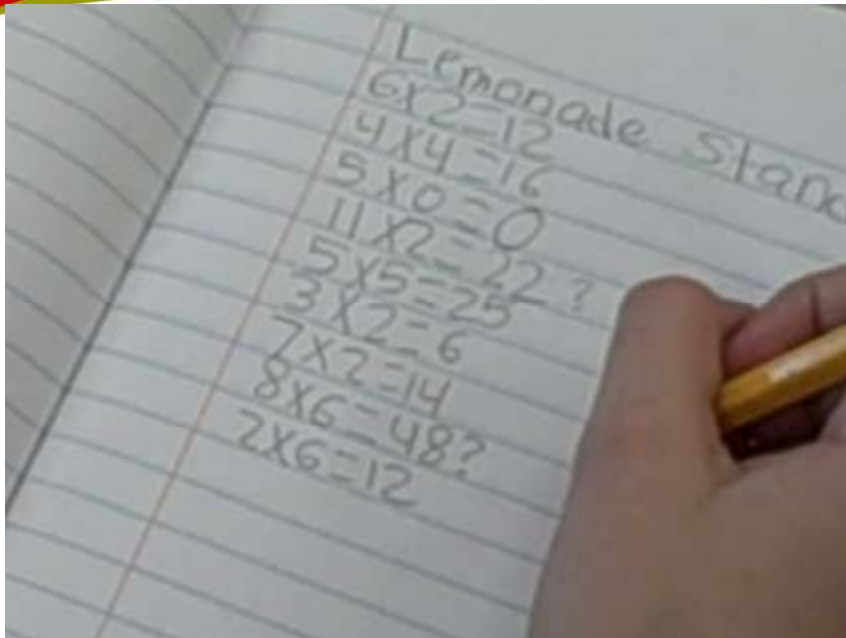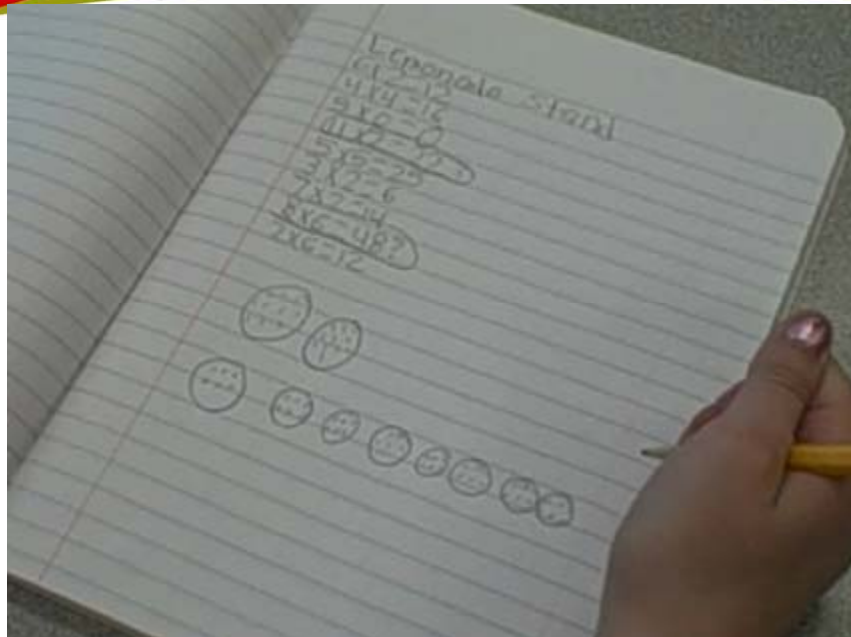
"I think 11 x 2 is 21."

"I don't know my 8's!"

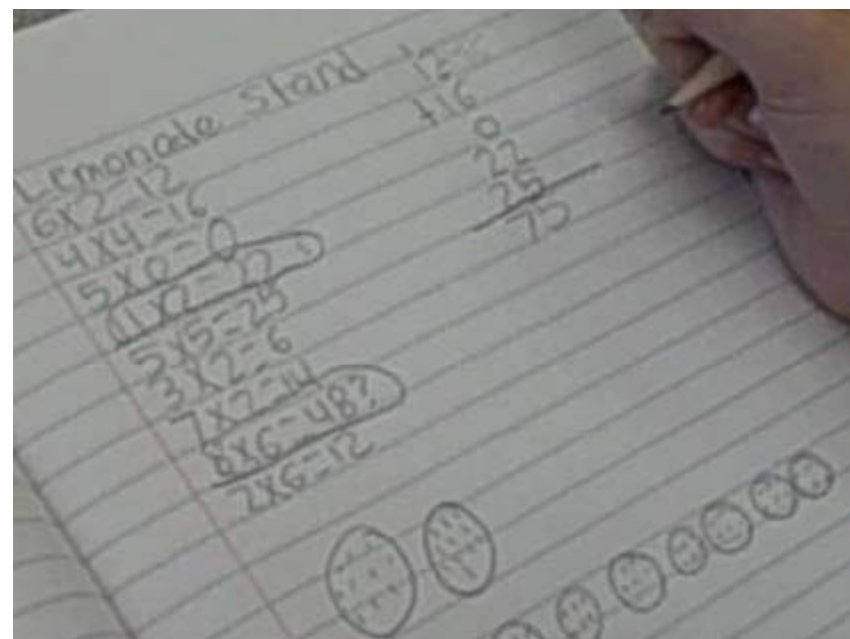"I'm not sure about this one."

"Is this right?"

"What does 11 x 2 look like?"

"How can we show 8 x 6 in another way?"

Find the sum of all the products to see which player has the highest score (and to reinforce addition!)

M436.5

| KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities: Exe
M 436.5	3.OA.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Operations and Algebraic Thinking	Solve problems involving the 4 operations, identify & explain patterns in arithmetic	numeral cards 1 to 12; ten-sided numeral die with numerals 0 to 9; writing space	Player A will take a Player A will then roll player B. Player A wil either the number of Player A will write t secret number. If pla s/he will write n = 4 (will take four turns ar who has the highest

M436.5

| Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand (from AVMR)	Construct/Level (from AVMR)	Numeracy Target (from AVMR)
numeral cards 1 to 12; ten-sided numeral die with numerals 0 to 9; writing space	Player A will take a card without showing it to player B. Player A will then roll the numeral die without showing it to player B. Player A will then tell the product, but keep secret either the number of groups or the number in each group. Player A will write the equations with "n" in place of the secret number. If player B is able to tell the missing factor, s/he will write n = 4 (or whatever is accurate). Each player will take four turns and the winner of the game is the player who has the highest sum of the value of all 5 "n" values.	Multiplication and Division	4 to 5 PINK	Multiplication and division as operations

M436.5

| "I CAN ......" (*see glossary)	Assessment for Learning	Student Grouping	Video Link	Print Link	Interactive Website	Reference	Te
...find the missing factor, given the other factor and the product.	Write this equation and ask the student to find the missing factor: 4 X n = 8. Present additional equations, such as: n X 8 = 16 5 X n = 15 7 X n = 21 n X 6 = 18	Partners					

Materials for M436.5
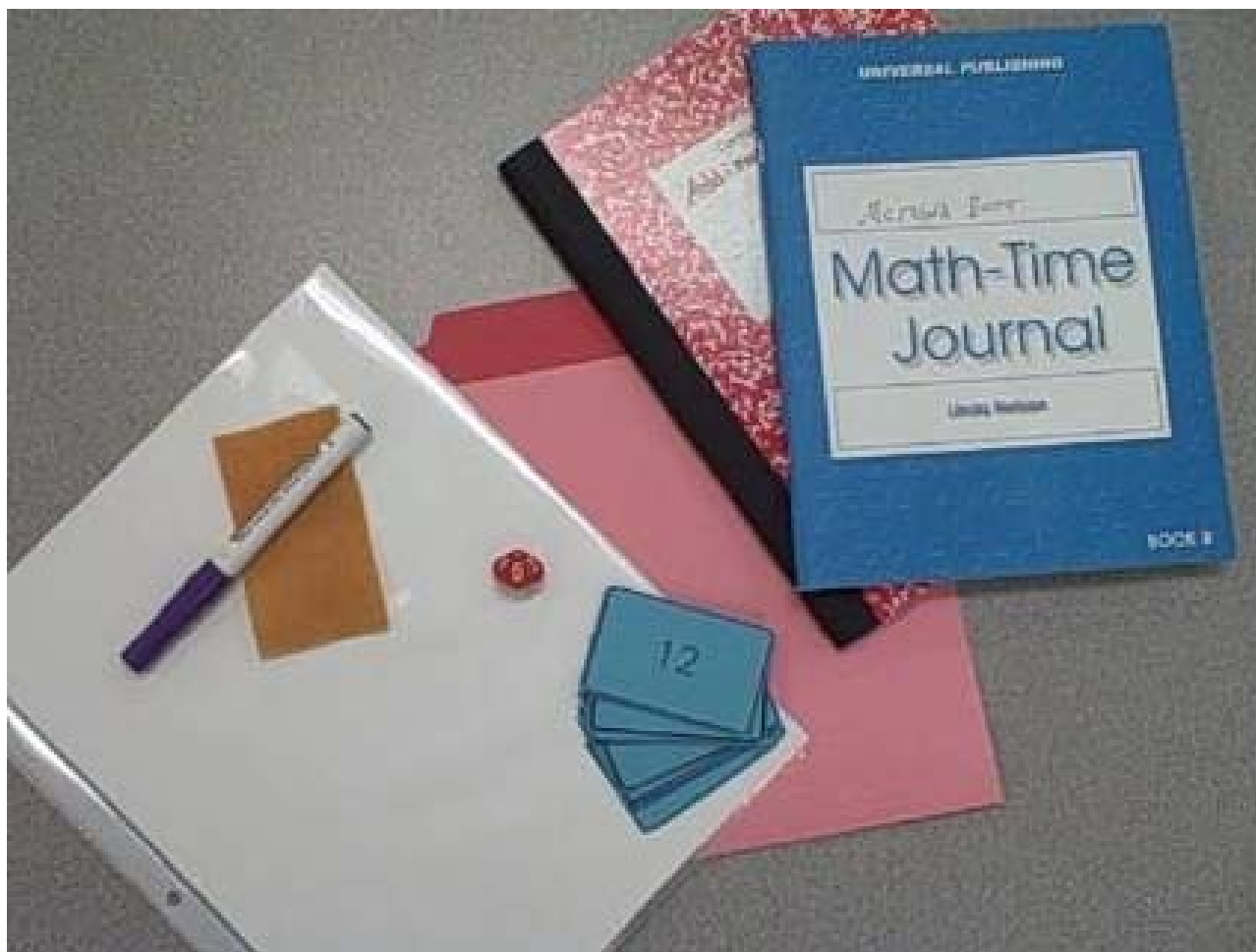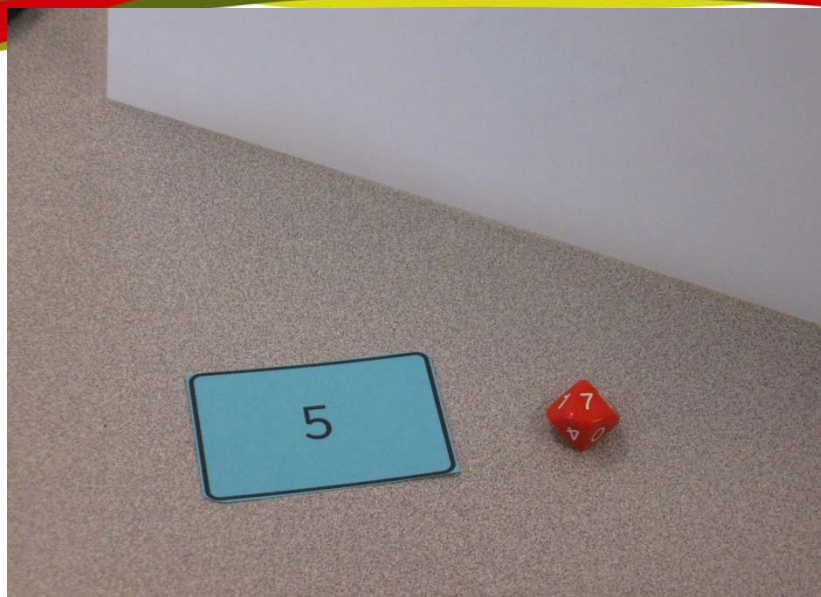
Player 1 draws a numeral card and rolls the die to generate a multiplication fact, tells her partner the product, but keeps the factors hidden from Player 2
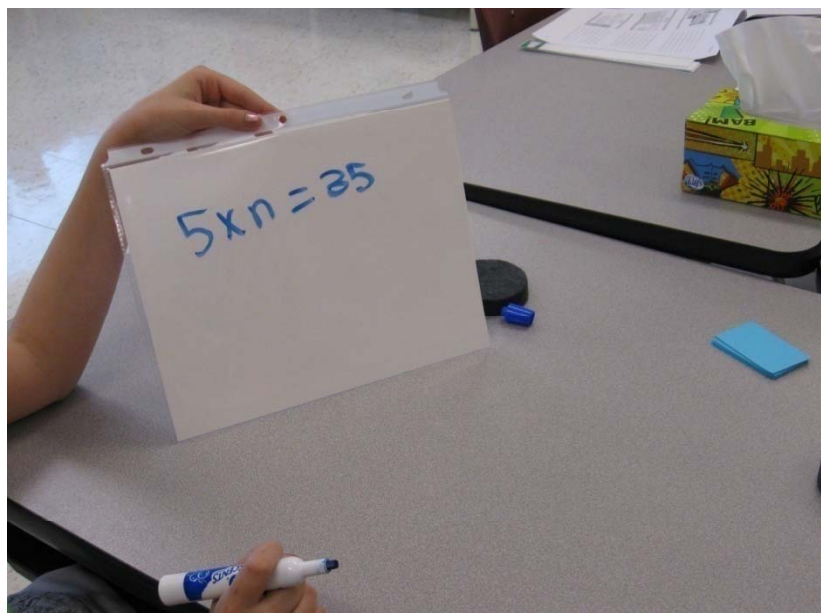
Player 1 writes an equation using the numbers from the card and die, but uses a variable to represent one of the factors
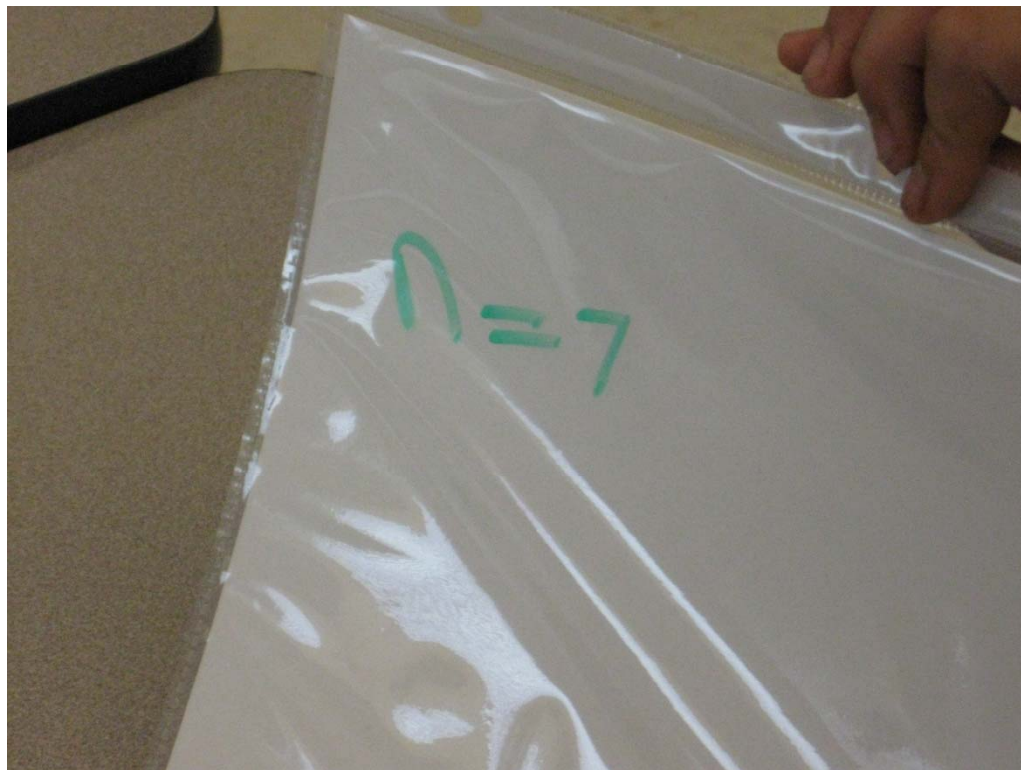
Player 2 writes her answer in terms of the unknown factor

M436.6

| KNP Entry	Kentucky Common Core Academic Standard (KCAS) (*see glossary)	KCAS Domain	KCAS Cluster	Setting (situation & materials)	Activities:
M 436.6	3.OA.5. Apply property of operations as strategy to multiply & divide (ex. 1: If 6×4=24 is known, 4×6=24 is also known. (Commutative property of multiplication) 3×5×2 can be found by 3×5=15 & 15×2=30, or by 5×2=10 & 3×10=30 (Associative property of multiplication) ex. 2: Knowing that 8×5=40 & 8×2=16, find 8×7 as 8×(5+2)=(8×5)+(8×2)=40+16=56 (Distributive property)	Operations and Algebraic Thinking	Understand properties of multiplication & relationship of multiplication / division	writing space	Partners will pla with three fact decide if it's true making a check will be indicated sign. Each time the thinking) a p pa

M436.6

Setting (situation & materials)	Activities: Exemplary Learning Experiences (*see glossary)	Numeracy Strand (from AVMR)	Construct/Level (from AVMR)	Numeracy Target (from AVMR)
writing space	Partners will play "Equal or Not?" by writing an equation with three factors on each side and asking the other to decide if it's true. A true statement will be indicated by making a check mark underneath and a false statement will be indicated by putting a slash mark through the equal sign. Each time a player answers accurately (and explains the thinking) a point is earned. Play continues until each partner has taken five turns.	Multiplication and Division	5+ ORANGE	Multiplication and division as operations

M436.6

| "I CAN ......" (*see glossary)	Assessment for Learning	Student Grouping	Video Link	Print Link	Interactive Website	Reference	Te
...compare multiplication expressions.	Give the student an equation with several factors on each side and ask if the statement is true. For example: 4 X 5 X 2 = 5 X 4 X 2 3 X 8 X 2 = 12 X 2 X 2 4 X 10 X 2 = 4 X 5 X 2	Partners					

Materials for M436.6
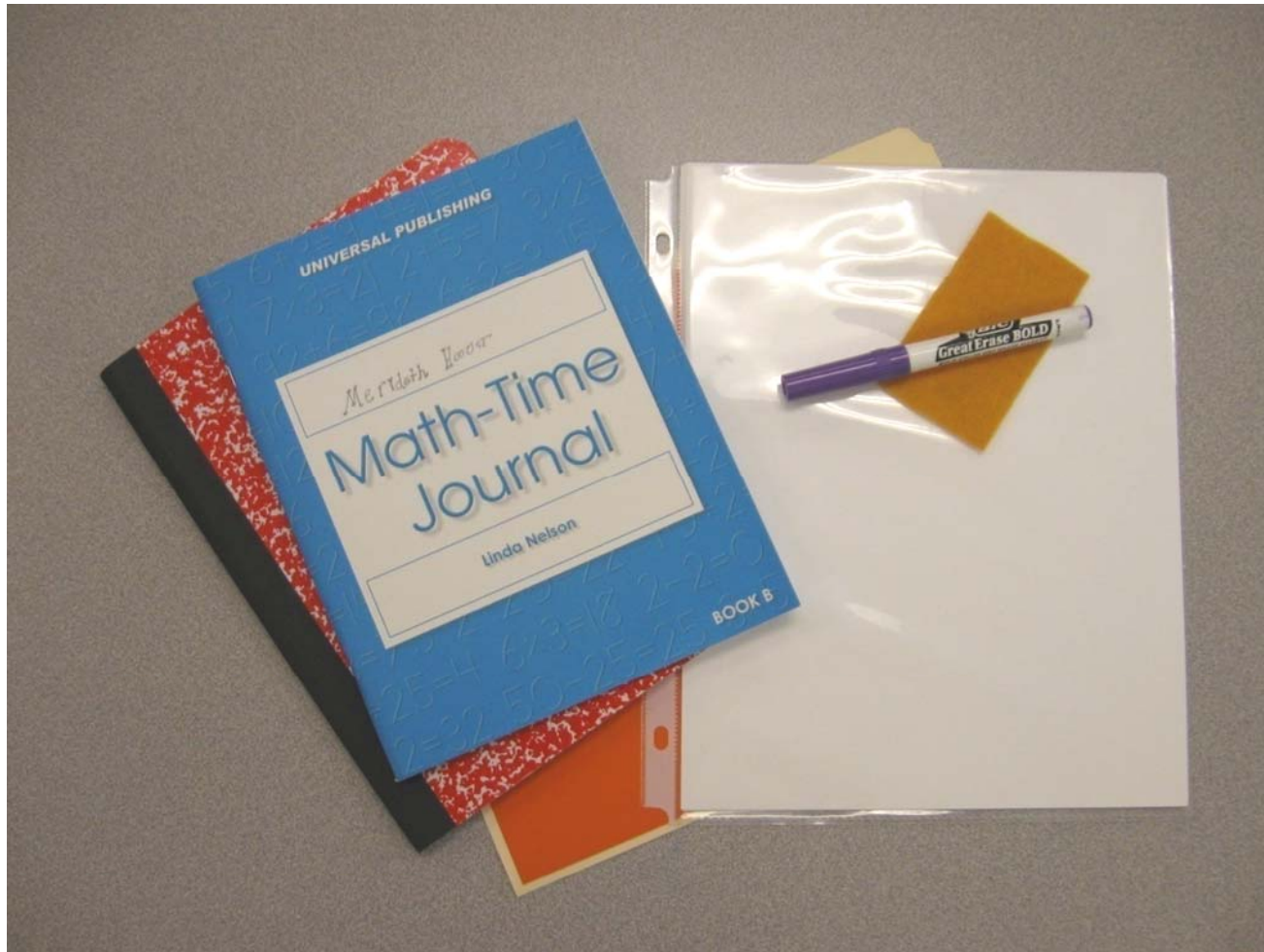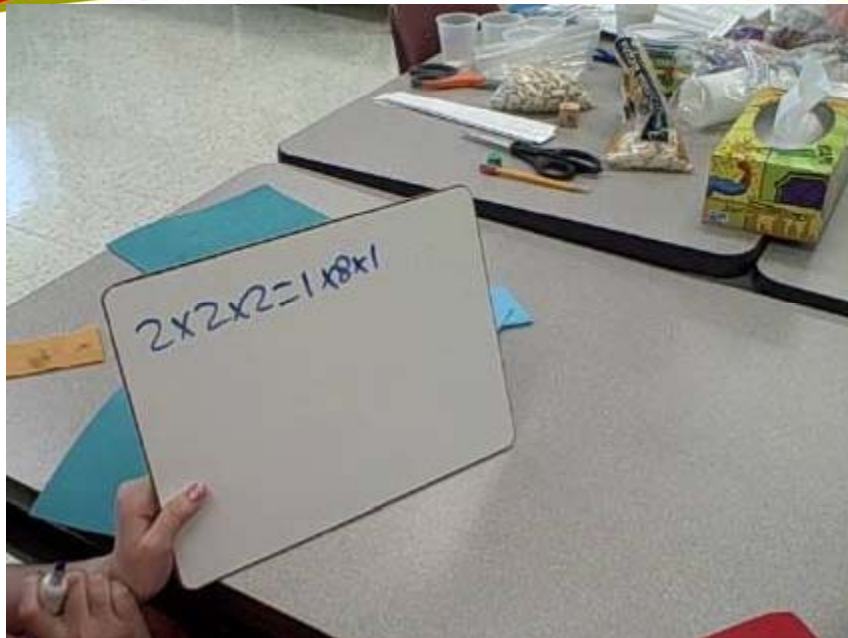
Partner 1 writes a multiplication equation using 3 factors on each side

Partner 2 decides if the equation is true and records
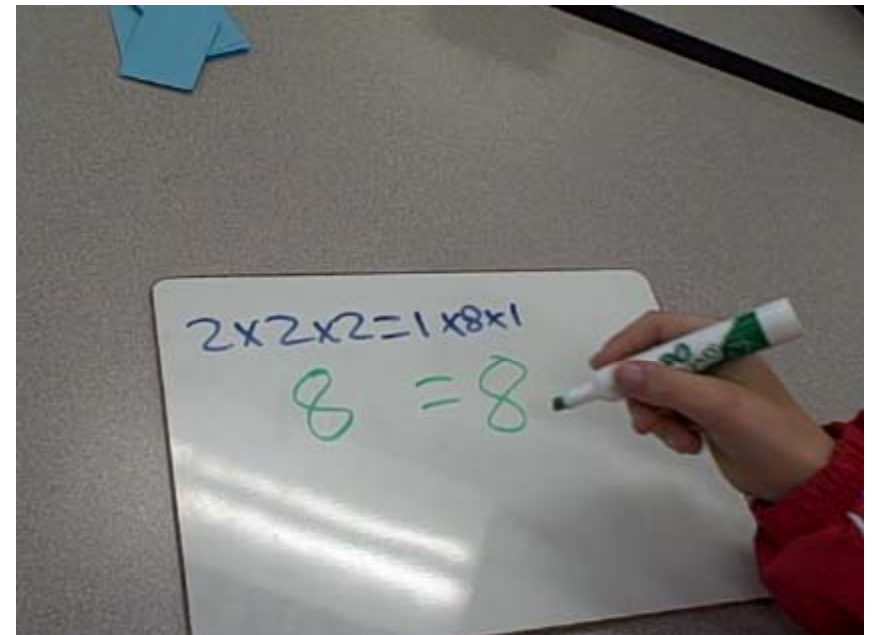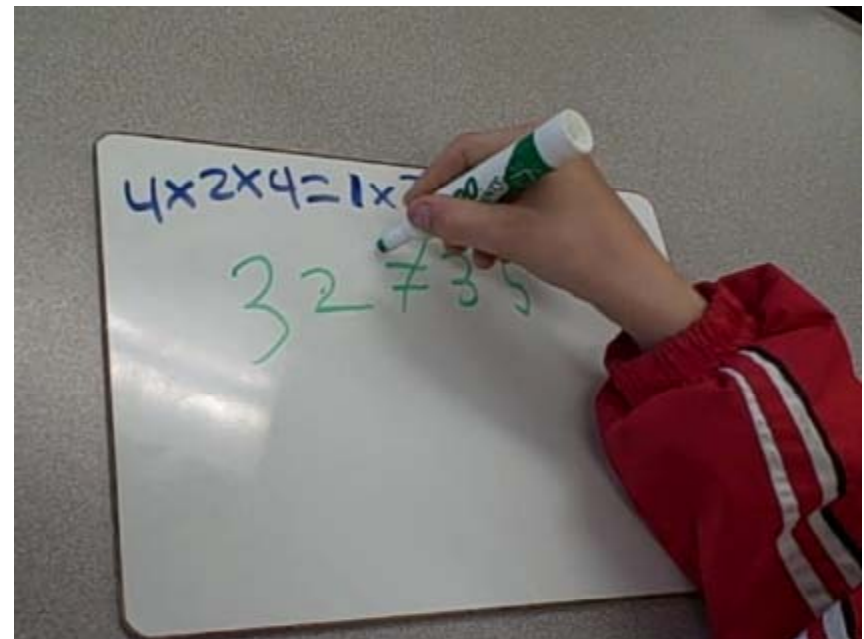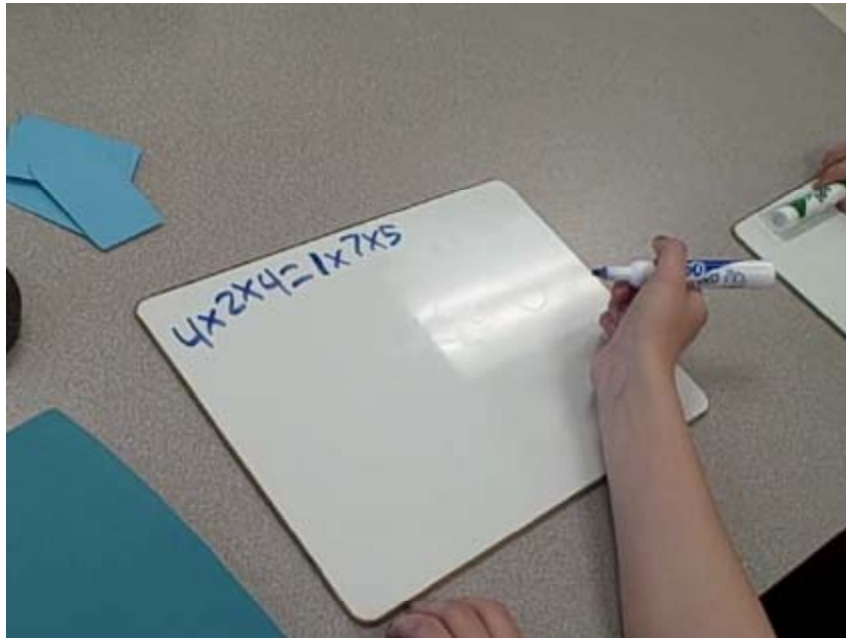
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INTRODUCTION

The Kentucky Center for Mathematics - Intervention Growth Opportunities - Windows Internet Explorer

http://kymath.org/intervention/iTraining.asp

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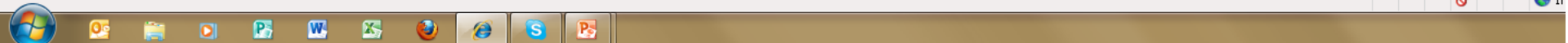
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Growth Opportunities

- **Kentucky Numeracy Project Intensive Course: Flyer, Expectations and Registration**

- **MIT Community Membership: View the Flyer, Registration**
  Posted April 22, 2011

- **Add+Vantage MR Course 2: View the Flyer, Registration**
  Posted April 14, 2011

- **Kentucky Numeracy Project Webinars - *Archive Access:* Registration**
  Kentucky educators may register to view the KNP archives
  Cost: Free
  For more information: View the ET Flyer or View the CT Flyer

See the KCM Calendar for additional event site details such as specific locations and recomme

Online Registrations Contact: Alice Gabbard, gabbardal@nku.edu

Last live KNP Webinar
(in this series)
3:30 to 4:30 p.m. ET

JUN 2 –

Base Ten Arithmetical Strategies

http://kymath.org/intervention/iKNP.asp

Reflection Questions

- How might you determine if your students are connecting the idea of quantity when performing the operations of multiplication and division?

- What are the algebraic properties that are apparent in fluent, advanced mental computation for multiplication and division?

- Why is it important to support students' intuitive sense of number and operation by guiding them to develop increasingly sophisticated, efficient mental computation strategies, according to their readiness?

- What does it mean to strategically remove manipulatives (de-scaffold) to aid students in quantitative reasoning, development of advanced, efficient mental computation strategies and sense-making for efficient multiplication and division with understanding?

KENTUCKY CENTER FOR MATHEMATICS