1

Everyone has the right to learn.

Heinemann
Dedicated to Teachers™

2

We respect the ideas of others.

3

Challenging problems help our brains grow stronger.

4

Mistakes are great!

Heinemann
Dedicated to Teachers™
Thinking Together: 9 Beliefs for Building a Mathematical Community

5

Good mathematicians are brave and try new things.

Heinemann
Dedicated to Teachers™

6

There are different strategies for solving a problem.

Heinemann
DEDICATED TO TEACHERS™

7

It's not just about the answer.

Heinemann
DEDICATED TO TEACHERS™

8

Good learners ask questions.

9

Questions from the teacher help us learn and grow.

Heinemann
DEDICATED TO TEACHERS™

1

Everyone has the right to learn.

Heinemann
Dedicated to Teachers™

2

We respect the ideas of others.

Heinemann

Dedicated to Teachers™

3

Challenging problems help our brains grow stronger.

Heinemann
Dedicated to Teachers™

4

Mistakes are great!

Heinemann
Dedicated to Teachers™

5

Good mathematicians are brave and try new things.

Heinemann
Dedicated to Teachers™

There are different strategies for solving a problem.

6

It's not just about the answer.

8

Good learners ask questions.

Heinemann
Dedicated to Teachers™

9

Questions from the teacher help us learn and grow.

Heinemann
DEDICATED TO TEACHERS™

Math Attitude Survey

When I don't know what to do in math, I keep trying.	Yes	Sometimes	No
I share my thinking with others in math.	Yes	Sometimes	No
Math can be done in different ways.	Yes	Sometimes	No
I use math only at school.	Yes	Sometimes	No
I like math.	Yes	Sometimes	No
I can be good at math.	Yes	Sometimes	No
I talk to others in class about my ideas to solve math problems.	Yes	Sometimes	No

Thinking Together

9 Beliefs for Building a Mathematical Community

Rozlynn Dance and **Tessa Kaplan**

HEINEMANN
Portsmouth, NH

Heinemann
361 Hanover Street
Portsmouth, NH 03801–3912
www.heinemann.com

Offices and agents throughout the world

The authors and publisher wish to thank those who have generously given permission to reprint borrowed material:

Eight principles of effective instruction from *Principles to Actions: Ensuring Mathematical Success for All* by National Council of Teachers of Mathematics (NCTM). Copyright © 2014 by NCTM. Published by NCTM, Reston, VA. Reprinted by permission of the publisher, conveyed through Copyright Clearance Center, Inc.

Library of Congress Cataloging-in-Publication Data
Names: Dance, Rozlynn, author. | Kaplan, Tessa, author.
Title: Thinking together : 9 beliefs for building a mathematical community /
Rozlynn Dance and Tessa Kaplan.
Description: Portsmouth, NH : Heinemann, [2018] | Includes bibliographical references.
Identifiers: LCCN 2017045594 | ISBN 9780325098180
Subjects: LCSH: Mathematics teachers—Training of. | Mathematics—Study and
teaching (Elementary). | Elementary school teachers—Training of. | Effective teaching.
Classification: LCC QA10.5 .K37 2018 | DDC 372.7/044—dc23
LC record available at https://lccn.loc.gov/2017045594

Editor: Katherine Bryant
Production: Victoria Merecki
Cover design: Suzanne Heiser
Text design: Sue Godel
Typesetter: Kim Arney
Cover photo: © Getty Images/FatCamera
Author photos: Tami Baumgartner Photography
Manufacturing: Steve Bernier

Printed in the United States of America on acid-free paper
22 21 20 19 18 RWP 1 2 3 4 5

Contents

To download full-size versions of the Appendices, please visit **http://hein.pub/ThinkingTogether** and click on the *Companion Resources* tab.

Foreword

The members of my family have varied relationships with math. My wife Bridget loves math and even today enjoys figuring out challenging math problems when helping our high school-age children with math. I'm not as confident with math, so I'm happy to let her handle math homework (not that I'd be much help), but I absolutely loved teaching math as a first-grade teacher. Our son Harrison is an actuary, and his whole work life is high-level math. Our daughters Meredith and Molly have both positive and negative feelings toward math that haven't shifted as much over time. But it is our daughter Natalie who has the most tenuous relationship with math. Some years she loves math and feels confident. Other years she dislikes math, and you can see her confidence wane.

Natalie's a fairly introspective fourteen-year-old, so I asked her to talk about what affected her feelings toward math from year to year. She quickly rattled off the influences on her mathematical confidence.

- She talked about the positive impact of *low-pressure* opportunities to solve mathematical problems with *guided support* from the teacher.
- She said it was easier to *form questions* when you are actually solving a problem, and much harder to form questions when listening to a lesson.
- She mentioned the importance of frequent *feedback* at school, rather than trying to figure it out on your own at home (and stressing about whether it was right).
- She said her favorite years were those where the focus was on *understanding* mathematical ideas (process) as opposed to getting answers correct (product).
- She said that *exploring and doing* was a better way of transferring understanding than telling and explaining.

The years where teachers made classroom decisions that supported these ideas, Natalie flourished.

Simply put, the decisions teachers make matter and have lasting effects. Throughout this book, Rozlynn Dance and Tessa Kaplan provide specific strategies that will help *you* make decisions that benefit all of the learners in your class.

You're about to read a math book that isn't only about math. It's really a book about the conditions that maximize learning for any child in any content area. Just to prove this, I read a whole chapter and each time I came to the word *math*, I skipped it. The meaning of the sentence didn't change. Good teaching, no matter the subject, is rooted in fundamentally similar key practices, which form the foundation for this book. Much of the professional work I do is in writing, and I was continually struck by the parallels to writing instruction as I read this book. For example, in writing we think about the importance of composition, the ability to create meaning on a page, in addition to conventions (spelling, punctuation, etc.). The authors talk about the important role of sense making in math (Chapter 4) and how understanding is the goal, in addition to being able to solve problems. In math as in writing we want children to have a sense of ownership and control, whether it's deciding on a strategy to solve a math problem or deciding which strategy to use to convey feelings in a piece of writing.

But, it's important that the focus of this book is on math, because so many children (and teachers) lack confidence in math. Students and adults often feel that their disposition toward math is set, thinking that they are either good at math or not good at math. Rozlynn and Tessa show us that isn't true. They show us strategies teachers can use that will help improve children's disposition toward math and solving problems (and other content areas as well). And those dispositions are critical. Lilian Katz defines a disposition as "a pattern of behavior exhibited frequently and in the absence of coercion and constituting a habit of mind under some conscious and voluntary control, and that it is intentional and oriented towards broad goals" (Katz 1993). Although it is possible to learn despite a negative disposition toward what you are learning, learning is more efficient, lasting, and enjoyable when the learner has a positive disposition toward the learning, especially math.

By focusing on dispositions and key principles about learning math, Rozlynn and Tessa provide teachers and students with ideas and skills that will last long beyond the school year. Just to be sure, I ran some of these ideas by my son Harrison. As an actuary, he spends his days doing high-level math related to probability. To become a fully certified actuary you have to take a series of exams over the course of many years. So, I asked him, to pass these exams, isn't it all about getting the right answer and knowing how to solve the algorithms? He told me that although he certainly has to know formulas and algorithms, that wasn't enough. He said that to solve the problems he encounters, he has to be able to *understand the process and theory* and decide how to solve it. Even at a high level, there are *multiple strategies* to get to an answer, but to consider and use multiple strategies you have to truly understand the process. The ideas of multiple pathways to solve a problem, mistakes as opportunities for learning, and the importance of understanding permeate this book. Both first graders and actuaries need to be able to do more than solve an algorithm. They have to be able to play with ideas, consider multiple possibilities, and apply what they know to new situations.

Throughout the book, Rozlynn and Tessa are respectful of children and childhood. They embed their decisions in what they know is right for children based on who they are right now. That sounds obvious, but actually isn't easy in the "pushdown" world of education, where, in a race to always go further faster, depth and understanding are pushed aside. Rozlynn and Tessa make the case for pushing from the ground up and providing children with the skills, strategies, and dispositions that they need now *and* going forward.

The authors also support positive dispositions toward math for teachers, by thinking about adult learning in the same way as they do student learning, and with the same respect for a community of learners. Just as they value mistakes as opportunities for learning, they anticipate that teachers will encounter challenges along the way. In each chapter, you will find a section called "When Things Don't Seem to Be Working." These sections provide valuable, practical tips for what to do when the inevitable happens and our teaching doesn't go as planned. But, more importantly, these sections reflect the belief of the authors that things won't always go smoothly, whether as a teacher or student, and that's OK. They anticipate and celebrate struggles as opportunities for learning.

All children deserve to be in a classroom that nurtures their identities as mathematicians, writers, and readers. To have a positive mathematical identity, all children have to willingly and enthusiastically engage in mathematical thinking each day. Teachers, and the decisions they make, significantly impact children's disposition and identity whether they realize it or not. This book supports your ability to create classroom communities that nurture all children as thinkers.

Chapter One Creating a Community of Mathematicians

As elementary school teachers, we have watched math instruction change and grow drastically during our careers. When we first started teaching math to young children, we taught the way we had been taught as children ourselves. The teacher introduced a new topic or idea, directly showed the students how to do it, and then the students practiced that idea by repeating problem after problem on a worksheet or from a textbook.

In our early years of teaching, however, we started to notice something. After teaching a topic and having our students complete practice problems correctly, we assessed them and most could score well on a test. Yet two to three weeks down the road, when we reviewed or touched on that topic again, our students seemed to have forgotten everything! The problem was this: most of our children could memorize and follow procedures long enough to pass a test, but the minute the knowledge was no longer at the front of their minds, it slipped away. Students weren't deeply understanding and internalizing math concepts; they were just getting by. They were never asked to grapple with complex topics and find solutions to problems on their own. They were never asked to understand why a procedure works; they simply carried out the procedure to get to an answer.

We became unsatisfied with our math instruction and sought to change it. We wanted the students, not the teachers, to be at the center of math learning. We wanted students to learn how to be problem solvers, trying different strategies and persevering until they came to a conclusion. We wanted our students to construct their own knowledge instead of acting as receptacles as we dumped knowledge upon them.

Through collaboration with colleagues and work with an instructional coach who was integral to our growth as teachers, we slowly changed our beliefs about mathematics instruction. The more we read, the more we realized that research supported our changing beliefs, too. In *Principles to Actions,* the authors identify several characteristics of math instruction that promote deep mathematical learning for children. Students must engage with challenging tasks, make connections to what they already know, develop both conceptual and procedural knowledge, construct knowledge through discourse, receive productive feedback, and develop awareness of themselves as learners (NCTM 2014).

Qualities of Effective Mathematics Instruction

Over the years, we learned that in order for students to truly learn and understand mathematical concepts, they must live in classrooms that support cooperative learning and mathematical discourse. They must learn to persevere and focus on the process of math, not the product. And most importantly, they need to construct their own knowledge of mathematics from the ground up in an atmosphere where they feel safe to learn, take risks, make mistakes, and grow.

A SAFE SPACE

In Ms. Kaplan's first-grade classroom, she has just introduced the day's math problem: "Gabriella was helping her mom plant her garden over the weekend. They planted 35 pea seeds. They also planted 54 pumpkin seeds. How many seeds did they plant altogether?" The students have discussed the problem to ensure they understand what they are being asked to do and are released to begin problem-solving.

As soon as the children receive their papers, which are blank except for the day's problem written at the very top, they head back to their seats, pulling out their pencils. Some stop at the math center on their way to their tables, pulling out base ten blocks, connecting cubes, or ten frame mats. When the students are settled with their supplies, a quiet buzz falls over the room as they get to work.

Leo is drawing tally marks. Ashlyn is pulling out tens blocks, quietly asking herself, "How many tens are there?" Some students are drawing ten frames and filling them with circles on their papers. Others are drawing tens and ones mats, making sticks and dots for tens and ones. Some are writing equations and sentences that explain their thinking. A few haven't touched their papers yet and are busy counting connecting cubes or base ten blocks. The students are engaged in solving a problem they haven't been taught how to solve, using tools and strategies that make sense to them.

Students in Ms. Kaplan's class feel safe making their own choices. They try out strategies even when they aren't sure if they'll work, and when they don't work, they try to solve the problem again or give it a go with a different strategy. Beyond that, they trust that their teacher won't be upset if they get the answer wrong or don't finish right away. They trust that their classmates won't make fun of them when they make mistakes but will celebrate them for those mistakes that create new learning. Creating respect between and among the students as well as the teacher in a classroom is the first major building block in creating a classroom community where students can deeply learn math.

CONSTRUCTING KNOWLEDGE

The goal of mathematics instruction in our classrooms is to teach for understanding. In order for students to deeply understand math, they must construct their own knowledge, connecting new ideas with their prior knowledge. Truly developing mathematical concepts means intricately weaving what students know with what they are learning, then looping back repeatedly to deepen and strengthen those connections. According to John A. Van de Walle et al. (2014, 4), "At the heart of constructivism is the notion that learners are not blank slates but rather creators (constructors) of their own learning. All people, all the time, construct or give meaning to things they think about or perceive." When we tell students what to think or how to do math, we take from them the chance to construct their own understanding.

Listen in as Ms. Kaplan talks with Katie, who is struggling to start the problem about Gabriella and her garden. Katie is using base ten blocks to model the first number in the problem, 35.

> Ms. Kaplan: *How many do you have here?*
>
> Katie: *10, 20, 30, 40 . . . oh wait.* [slowing down, pointing to each ten stick] 10, 20, 30. [picking up one ten stick] *But the 20 is in here.*
>
> Ms. Kaplan: *The 20 is in here? . . . 10, 20, 30? What number are we trying to make here?*
>
> Katie: [looks at the problem on her paper and points] *35.*
>
> Ms. Kaplan: *35, okay. If we have 30 here, what do we need now?*
>
> Katie: *One more?*
>
> Ms. Kaplan: *Let's try that and see if it works.*

Katie pulls out a single one block. She and Ms. Kaplan count together and find that she now has 31. Through some more trial and error, Katie eventually concludes that she needs 5 more.

In this vignette, Katie is still learning about basic place value concepts. In order to help Katie develop a model of the number 35, Ms. Kaplan acknowledges where she is in her understanding and guides her through questioning toward a deeper understanding. She encourages her to try out ideas and test them to see if they work. Katie is not following the steps in a procedure but persevering through this task as she tries to figure out how to accurately model her thinking.

We can help our students construct their own understanding by teaching them not to rely on the teacher for the answer. We want our students to look within themselves for connections that might help them construct understanding or to talk with their classmates to formulate ideas. We want them to confirm and justify their own thinking. As teachers, we act as facilitators and learning coaches. The teacher is not the source of knowledge but merely a guide on the path toward knowledge construction.

PERSEVERANCE

We also believe that one of the most important factors in successful student-centered instruction and learning is perseverance. For students to be successful mathematicians, they must be willing to struggle through mistakes, take risks in their learning, and try again when things don't quite work out. In his 2010 TED Talk, Dan Meyer discusses the importance of helping our students learn to be patient problem solvers. Children of the twenty-first century are used to quick problems with easy solutions, but real mathematical problems don't work out that way. We need to teach our children to tackle complex problems worth solving and persevere in solving them (Meyer 2010).

According to Jo Boaler (2016, 13), "Studies of successful and unsuccessful business people show . . . what separates the more successful people from the less successful people is not the number of their successes but the number of mistakes they make, with the more successful people making *more* mistakes." When we teach students that mistakes not only are okay but are steps on the path to success and innovation, we help build perseverance. As mathematicians, we want our students to be able to try different paths to a solution, to experiment with new strategies, and to keep trying until they find an idea that works.

We believe that if we let our students give up after the first try or get overly frustrated when they make a mistake, we are doing them a great disservice. The same goes for when we dive in to save them, correcting their thinking or giving them the answer. We are not doing them any favors by bailing them out when they are struggling. The structures we set up in our classroom and the community we build can support our students as they persevere through productive struggle, encouraging them to continue to persist when things get tough.

COOPERATIVE LEARNING AND MATHEMATICAL DISCOURSE

To understand the importance of mathematical discourse, let's step back into Ms. Kaplan's room. The students have finished working on their problem and are gathered on the carpet listening to Harper share how she solved it.

> Harper: *My sentence was "I knew that 5 + 4 was the same as 9 and I knew that 30 + 50 was the same as 80. So I put them together and it made 89."*
>
> Ms. Kaplan: *Okay, turn to your neighbor and tell them what Harper's sentences mean.*

As Ms. Kaplan listens in on the partner conversations, she can see some confusion about Harper's thinking. Some students are unclear where her numbers came from. Once students turn back to the whole group, Ms. Kaplan begins facilitating a discussion of these ideas. The students work out together, with guiding questions from the teacher, that the 5 and the 4 are the ones in the problem and the 30 and the 50 are the tens. At the end of the discussion, Elliot comments, "I was confused at first, and then when I started talking to Amity . . . um . . . when she was going to talk, I . . . I knew what she meant."

By asking students first to turn and talk about Harper's thinking and then to discuss her thinking as a whole group, Ms. Kaplan was able to guide students toward understanding Harper's mathematical ideas. Students made connections between their own thinking and Harper's thinking, deepening their understanding. Elliot was even able to articulate how helpful it was to be able to hear another student explain Harper's thinking in a different way.

In order to construct knowledge and deepen understanding, students must be able to talk about their learning and understand the ideas of others. The Common Core Standards for Mathematical Practice (CCSSO 2010) specify that students should be able to "construct viable arguments and critique the reasoning of others." When students can justify their own thinking and explain their ideas to their classmates, they are deepening their own understanding of those concepts. When they can critique the reasoning of others, students show that they are thinking deeply about mathematical ideas, making connections between their own thinking and the ideas of others.

On a more complex level, students must not only be able to talk to and listen to one another but also be able to work cooperatively in partners and groups. They must find and build upon the strengths of others, which in turn strengthens their own skills and understanding. Students need the skills to be able to work with one another as they think creatively and problem-solve together. It is our job as teachers to ensure equity of voice among all our students, regardless of skill level or ability. We know that when we encourage cooperative learning, there is a tendency for some students to dominate the discussion. Therefore, to be effective teachers, we must have a variety of techniques on hand to encourage all students to contribute to a discussion (Danielson 2014). Developing those skills requires deliberate and clear structures to be in place in the classroom.

LETTING STUDENTS DO THE THINKING

The study of mathematics is not just about finding answers but about figuring out ways to solve complex problems successfully. Simply put, the process is more important than the product. Because of this, it is important that students take ownership of their learning process as the teacher steps back from being the center of knowledge. During Ms. Kaplan's lesson, three students share their thinking, justify their answer, and answer questions from their classmates about how they found their answer. Not one student ever asks Ms. Kaplan if 89 is actually the correct answer. By talking about and analyzing student strategies for solving the problem, the children come to trust that 89 is the answer without needing confirmation from the teacher.

When students learn to think metacognitively about their own processes by comparing, critiquing, and justifying them, they begin to trust themselves as learners. They don't need an answer key in the back of the book because they have chosen and used strategies that make sense to them. They have checked their answers on their own using different strategies and critiqued their answers to be sure they are reasonable in the context of the problem. When students are more focused on the process of math, the correct answers come more naturally and fluidly in the course of time.

Key Beliefs That Support Effective Mathematics Instruction

In our first few years of teaching, we spent a lot of time thinking about what mathematics instruction *shouldn't* be. The more difficult part, though, was figuring out what it was that mathematics instruction *should* be. The National Council of Teachers of Mathematics identifies eight key principles of effective mathematics instruction:

1. *Establish mathematics goals to focus learning.*
2. *Implement tasks that promote reasoning and problem-solving.*
3. *Use and connect mathematical representations.*
4. *Facilitate meaningful mathematical discourse.*
5. *Pose purposeful questions.*
6. *Build procedural fluency from conceptual understanding.*
7. *Support productive struggle in learning mathematics.*
8. *Elicit and use evidence of student thinking. (NCTM 2014, 10)*

These eight brief sentences seem simple yet are incredibly complex and daunting. Each of the eight principles, to be adequately implemented in elementary classrooms, must be thought about and planned strategically. Daily, we must ask ourselves:

- Is my task challenging enough, and does it meet my mathematical goal?
- How will I facilitate the connection between mathematical ideas and representations?
- What questions will I ask or encourage my students to ask?
- How will I support students in building their conceptual understanding as they struggle through a task?
- How will my students justify and share their thinking?

After years of teaching through problem-solving-based, student-centered methods, we have realized that there is one common thread that underlies success in all these tasks. As teachers, we must cultivate the structures and beliefs in a classroom community that lay the foundation for the mathematical growth of our students. We must create a kind, caring, trusting community of learners who feel comfortable tackling the unknown, taking risks, and making mistakes. Our foundation is built on a set of nine key beliefs:

1. *Everyone has the right to learn.*
2. *We respect the ideas of others.*
3. *Challenging problems help our brains grow stronger.*
4. *Mistakes are great!*
5. *Good mathematicians are brave and try new things.*
6. *There are different strategies for solving a problem.*
7. *It's not just about the answer.*
8. *Good learners ask questions.*
9. *Questions from the teacher help us learn and grow.*

In this book, we hope to share with you some of the strategies we've learned and developed over the years to help reinforce these beliefs and build the foundation for successful mathematical learning.

In Chapter 2, we will discuss beliefs 1 and 2, sharing strategies for creating a respectful community of learners who believe that everyone has the right to learn. We will also share some strategies to help students understand how to respect the ideas of others as they learn to listen to and critique their classmates' thinking. We'll discuss the importance of honoring mistakes (which we'll return to in Chapter 3), respectfully disagreeing, and giving everyone the time they need to think and learn.

Beliefs 3, 4, and 5 will be the subject of Chapter 3, as we share strategies for building up our students' mathematical confidence. We'll share some ways to build perseverance, set up a structure for sharing and celebrating mistakes, and encourage students to take risks and try new things.

In Chapter 4, we will discuss the importance of honoring different mathematical strategies and ways of thinking as we examine beliefs 6 and 7. We'll share some important ways to help students understand that there are multiple strategies for solving a problem, and we'll explain how students can share these strategies with the class. We'll also provide some concrete suggestions for helping students justify their thinking.

In Chapter 5, we'll touch on the importance of questioning, both from the teacher and the students, as we consider beliefs 8 and 9. We'll clarify the difference between questions that clarify and probe; questions that guide, challenge, and extend; and questions that assess for understanding. We'll also provide some concrete strategies for teaching students how to question one another.

Finally, in Chapter 6, we'll provide some tips and ideas to help you keep up the momentum as you work to change your math instruction so it aligns with these beliefs.

Our goal is to provide you with concrete lessons, ideas, and strategies that help students feel safe in their classroom, believe in themselves as mathematicians, and think critically about their own and others' mathematical thinking. By implementing the structures and norms we suggest at the beginning of the year and reinforcing them throughout the year, you can help give your students a strong foundation upon which to build their mathematical understanding.