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KENTUCKY CENTER
FOR MATHEMATICS

Developing Geometric Thinking

with Bonny Davenport

Welcome!

Your host

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KCM Website

## www.kentuckymathematics.org

Today's Agenda

- Let's Do Math!
- Standards
- Research
  - Van Hiele Model
  - Principles and Standards for School Mathematics
- Manipulatives

Let's Do Some Math!

Which One Doesn't Belong?

Look at this set of four pictures. Decide which one doesn't belong with the other three. Describe your thinking using math words. There are many ways to think about each one!
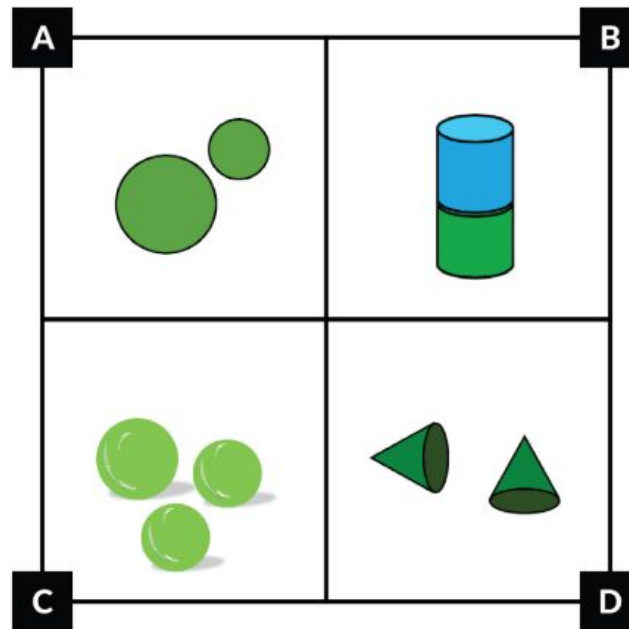
Challenge
See if you can find reasons why *each* of the pictures might not belong with the other three.
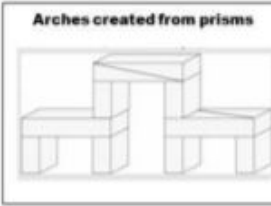
Share
Explain your thinking to someone else. Do they have different reasons why one doesn't belong?

Kindergarten Standards

Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres).

Standards	Clarifications

KY.K.G.1 Name and describe shapes in the environment.
  a. Describe objects in the environment using names of shapes.
  b. Describe the relative positions of these objects using terms *above, below, in front of, behind and next to*.
MP.6
	For objects in student's environment, the student accurately provides a shape name (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres). ("The clock on the wall is a circle." "The desktop is a rectangle.")
	Students use positional language to describe the relationships between objects ("The clock is above the bulletin board." "My desk is next to the computer table.")
	Coherence KY.K.G.1 →KY.K.G.4

KY.K.G.2 Correctly name shapes regardless of orientations or overall size. **MP.7**	Students identify and name shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres) regardless of size, orientation, or positioning. (The classroom window is a rectangle and this paper is a rectangle, too.) Coherence KY.K.G.2→KY.K.G.4
KY.K.G.3 Identify shapes as two-dimensional or three-dimensional. **MP.3, MP.6**	When presented with a shape or object, students determine whether it is two-dimensional (square, circle, triangle, rectangle, or hexagon) or three-dimensional (cube, cone, cylinder, sphere). Students express mathematical reasoning regarding their responses. (The block is three-dimensional because it's thick and not flat like paper.) Coherence KY.K.G.3→KY.1.G.1

Cluster: Analyze, compare, create and compose shapes.

Standards	Clarifications
KY.K.G.4 Describe the similarities, differences and attributes of two and three dimensional shapes using different sizes and orientations. **MP.3, MP.7**	When considering two-dimensional shapes (square, circle, triangle, rectangle, hexagon) or objects and three dimensional shapes (cube, cone, cylinder, sphere) or objects, students describe similarities, differences and attributes. ("The window and paper are both rectangles, but the window sits sideways and my paper is long ways." "My book and my paper both look like rectangles, but my book is three-dimensional because it is thicker.") Coherence KY.K.G.4→KY.1.G.1
KY.K.G.5 Model shapes in the world by building figures from components and drawing shapes. **MP.1, MP.5**	Students construct and draw models of shapes (square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere) in the world around them. Students create shapes with materials that include but are not limited to straws, pipe cleaners, popsicle sticks or clay and describe the shape they create. (Students use sticks and a ball to replicate an ice cream cone.) Coherence KY.K.G.5→KY.1.G.1
KY.K.G.6 Compose simple shapes to form larger shapes. **MP.3, MP.5**	Students explore by using simple shapes to construct a larger shape. (Students arrange paper triangles to form a rectangle. Students arrange triangle pattern blocks to form a hexagon.) Coherence KY.K.G.6→KY.1.G.2

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First Grade Standards

| Cluster: Reason with shapes and their attributes.	
Standards	**Clarifications**
KY.1.G.1 Distinguish between defining attributes versus non-defining attributes; build and draw shapes to possess defining attributes. **MP.7**	Defining attributes include, but are not limited to, number of sides or open/closed shapes. Non-defining attributes include, but are not limited to, color, orientation or overall size. Coherence KY.K.G.4→KY.1.G.1→KY.2.G.1

KY.1.G.2 Compose shapes.
  a. Compose two-dimensional shapes to create rectangles, squares, trapezoids, triangles, half-circles, quarter-circles and composite shapes to compose new shapes from the composite shapes.
  b. Use three-dimensional shapes (cubes, right rectangular prisms, right circular cones and right circular cylinders) to create a composite shape and compose new shapes from the composite shapes.
MP.1, MP.4
	Students do not need to learn formal names such as "right rectangular prisms."
	b.
	Arches created from prisms
	Right rectangular prisms are composed with prisms with right triangle bases. Note the dimensions of the triangular prism on the top arch differ from the dimensions of that on the right.
	
	Coherence KY.K.G.6→KY.1.G.2

KY.1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths* and *quarters*, and use the phrases *half of*, *fourth of* and *quarter of*. Describe the whole as two of or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **MP.3, MP.6**	Students see the relationship of taking the same shape and partitioning it into equal pieces. For example, they compare the size of the pieces when it's half of a shape or a fourth of the shape. Coherence KY.K.G.6→KY.1.G.3→KY.2.G.3

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Second Grade Standards

Cluster: Reason with shapes and their attributes.	
Standards	**Clarifications**
KY.2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or sides. Identify triangles, quadrilaterals, pentagons, hexagons and cubes (identify number of faces). **MP.4, MP.7**	Sizes are compared directly or visually, not compared by measuring. Coherence KY.1.G.1→KY.2.G.1→KY.3.G.1
KY.2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **MP.6, MP.8**	The rectangle should not be divided up into anything larger than 5 rows and 5 columns to correlate with KY.2.OA.4. Coherence KY.2.G.2→KY.3.MD.6
KY.2.G.3 Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words *halves, thirds, half of, a third of,* etc.; and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. **MP.2, MP.3**	Students explore rectangles and circles being partitioned in multiple ways to recognize that equal shares may be different shapes within the same whole. halves thirds fourths Coherence KY.1.G.3→KY.2.G.3→KY.3.NF.1

Van Hiele Model
Levels of Geometric Thinking
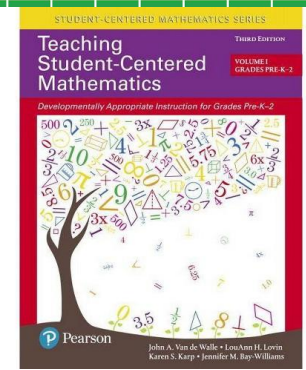
Level 0: Visualization

Level 1: Analysis

Level 2: Informal Deduction

Level 3: Deduction

Level 4: Rigor

- Levels are sequential.
- Not age dependent.
- Geometric experience is key
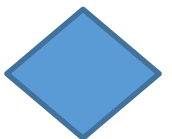- Instruction must match student's level of thought.

Level 0: Visualization

- Figures are judged by appearance.

- Grouping of shapes that seem to be alike.

  ☐ *A circle is a circle because it looks like a loop.*

  ☐ *I grouped these together because they are all pointy.*

  ☐ *A square is a square because it looks like a square.*

  ☐ *A rotated square is not a square to this level of thinker.*

Level 1: Analysis

- Descriptive level

- Properties of shapes

  □ A square is a square because it is a rectangle with four congruent sides.
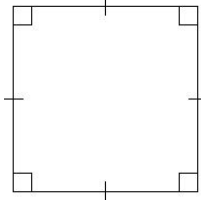
- Relationships among properties not developed

  □ Won't see the relationships between squares, rectangles, and parallelograms.

Implications for Instruction

Principles and Standards for School Mathematics
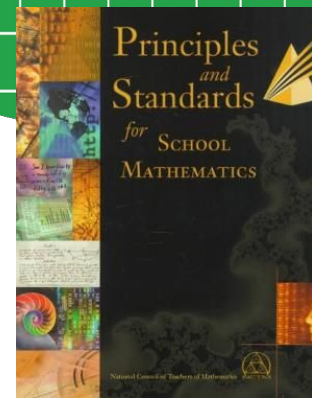
- Analyze characteristics and properties of 2 and 3 dimensional shapes and develop mathematical arguments about geometric relationships.
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems.
- Apply transformations and use symmetry to analyze mathematical situations.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

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Properties of Shapes

K- 2 Expectations:
- Building and drawing shapes
- Comparing shapes by attributes
- Putting together and taking apart shapes
- Identifying shapes in real world
- Examples and non-examples of shapes

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Shapes In Our World

Frank Knight- Spottsville KY
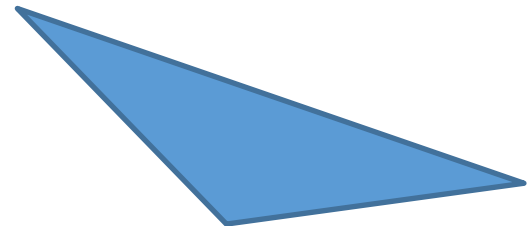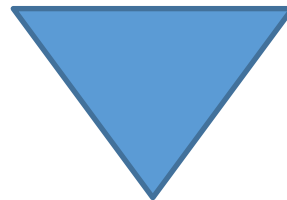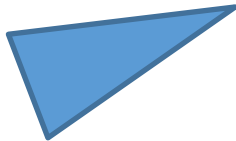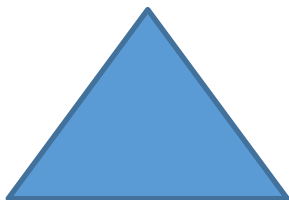
Sort and Classify
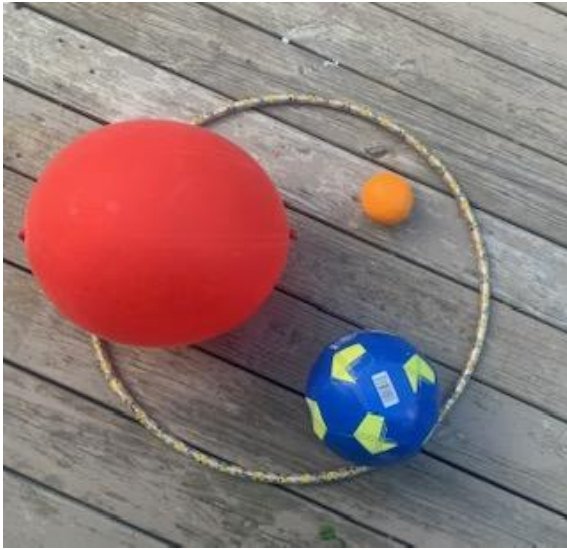
Locations

K- 2 Expectations:

- Learn everyday positional descriptions such as *above, below, beside, behind, in front of* and *next to*.

- Describe landmarks and the space around them adding the concepts of distance and direction.

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Transformations

K- 2 Expectations:

- Naturally use their own physical experiences with shapes to learn about transformations such as slides, flips, and turns.

- Use these movements intuitively when they solve puzzles, turning the pieces, flipping them over and sliding them into new arrangements.
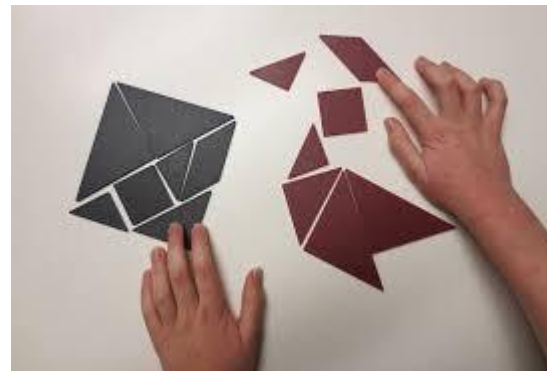
Visualization "Mind's eye"

K- 2 Expectations:
- Create mental images of shapes
- Imagine the shaped turned
- Imagine the shape cut into two pieces
- Predict how a shape would look from a different viewpoint

Composing and Decomposing Shapes

Tangrams

Grandfather Tang's Story
A TALE TOLD WITH TANGRAMS
Ann Tompert
Illustrated by
Robert Andrew Parker

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Developing-Geometry-Understanding-with-Tangrams/

https://toytheater.com/tangram/
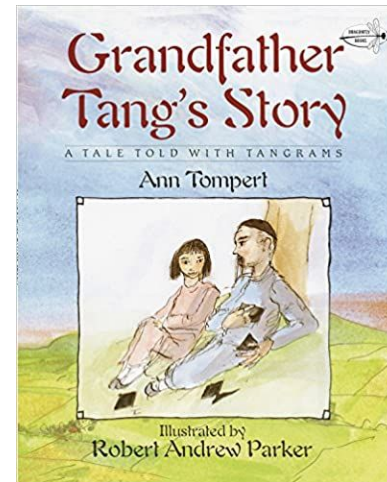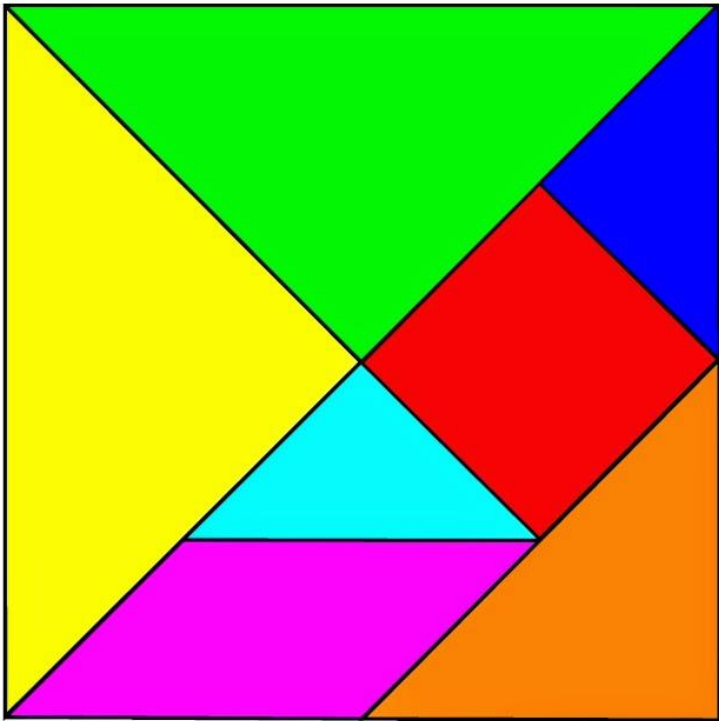
Composing and Decomposing Shapes

Pattern Blocks

Investigations Games

https://apps.mathlearningcenter.org/pattern-shapes/

https://www.coolmath4kids.com/manipulatives/pattern-blocks

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Shape-Tool/

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Patch-Tool/
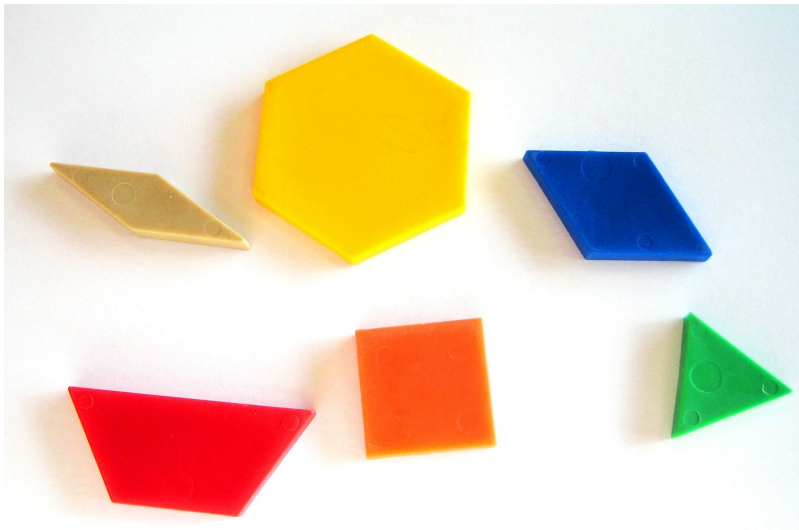
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Composing and Decomposing Shapes

Geoboards

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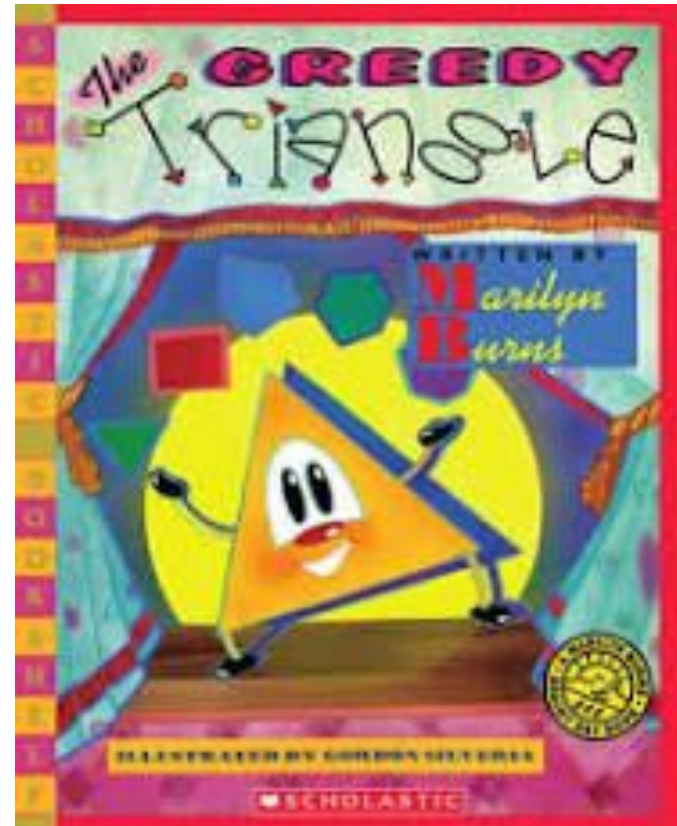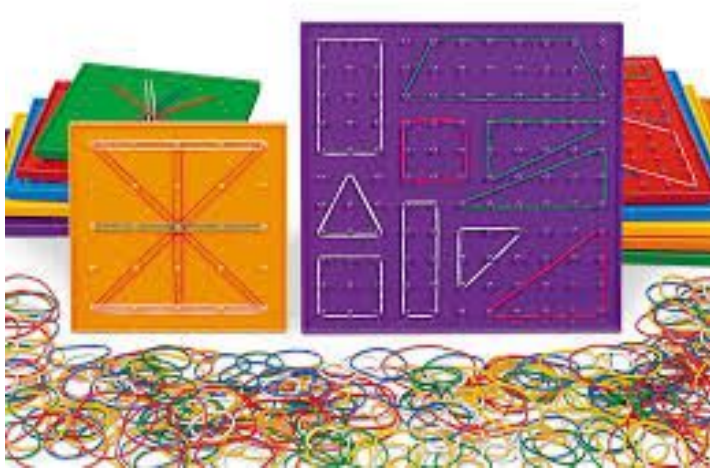
Glow Stick Geometry

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KCM is here to support you!

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Time to Share!

A POINT (or 3!) you would like to make?
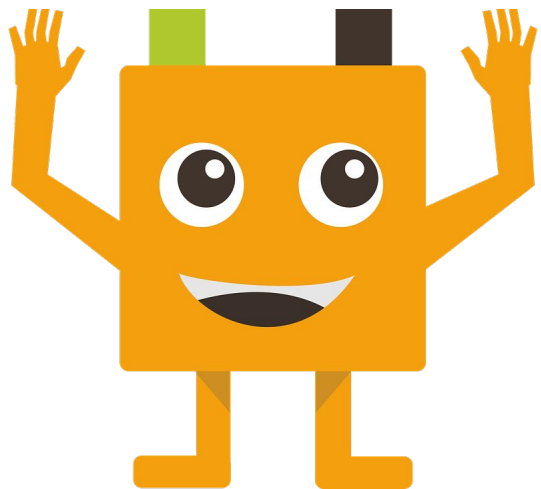
Anything SQUARE with your way of thinking?

Anything still CIRCLING in your mind?