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FOR MATHEMATICS

More Conceptual Understanding of Geometric Thinking

with Dee Crescitelli

Welcome!

Your host

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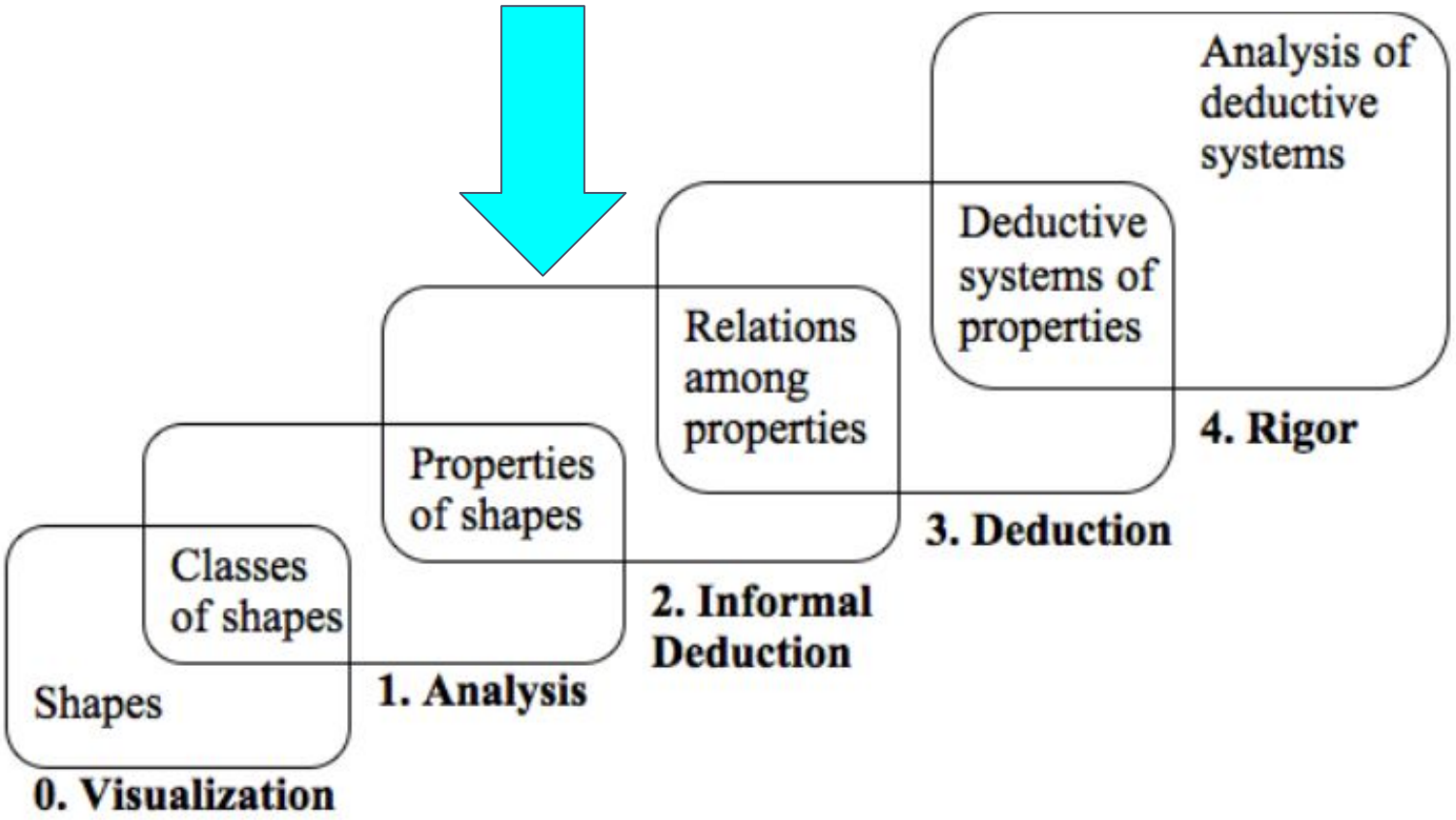
The van Hiele Levels of Geometric Thinking



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Critical Areas

7th and 8th Grade Geometry Clusters

Geometry (G)	Geometry (G)
• Draw, construct and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure, area, surface area and volume.	• Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean Theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

14 Standards!

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Can You Build It?

Can You Build It?

A) A shape with just one square corner and four sides

B) A shape with three square corners

C) A shape with two pairs of parallel lines and NO right angles

D) A triangle with three congruent sides and an obtuse angle

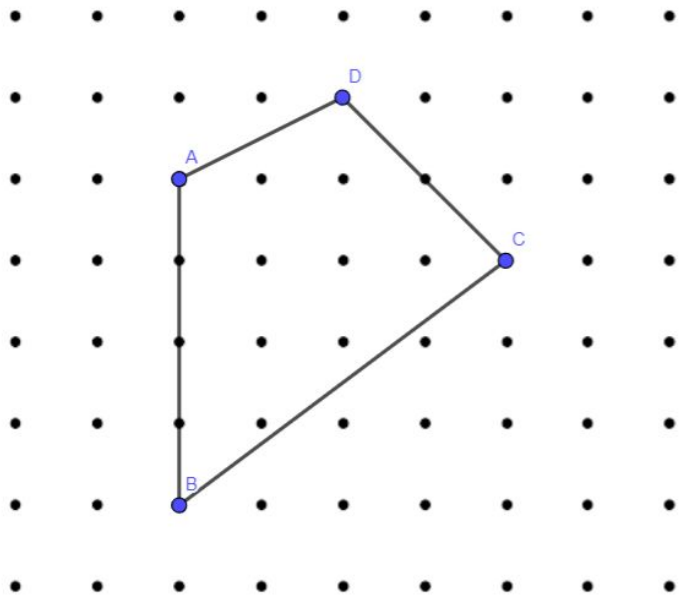
E) A trapezoid with two congruent sides

F) An equilateral triangle with a square corner

G) A triangle with three acute angles and three congruent sides

H) A triangle with no congruent sides, no congruent angles and 1 obtuse angle

Use the geoboard at the link below to build your shapes:
https://www.geogebra.org/m/Hm4GU3gp#material/vpE8GWTY



What is special about D?

Standard Connection

KY.7.G.2 Draw (freehand, with ruler and protractor and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **MP.6, MP.7**	Emphasis is on taking given conditions and converting them to geometric shapes, constructing triangles with given angle measures and side lengths and determining when the given conditions do not meet the conditions of a triangle. Coherence KY.7.G.2→KY.8.G.1

KY.7.G.2

Draw (freehand, with ruler and protractor and with technology) geometric shapes with given conditions.

Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

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Developing Geometric Thinking through Activities That

Begin with Play

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Mosaics

MOSAICS

Invitation to Learn

Provide each student with a *Seven Piece Mosaic Puzzle*. Begin by asking, "What can we do with these pieces?" As students begin to explore, have them share and talk about what they have built. They may use all the pieces or only a few. *"Children need ample time to explore and share their creations. Such play gives teachers a chance to observe how children use the pieces and to assess informally how they think and talk about pieces." (Van Hiele, p. 312)*

Lesson

1. Have students explore all the possible ways to make the *Parallelogram* using their puzzle pieces.
   The students may slide, flip, turn (rotate) their pieces.
   What two-piece combinations are possible?
   Ask the students which pieces were not used?

2. Have students choose any two pieces, set the others to one side, and see how many different shapes can be made by joining them at the sides that match.
   Try pieces five and six.

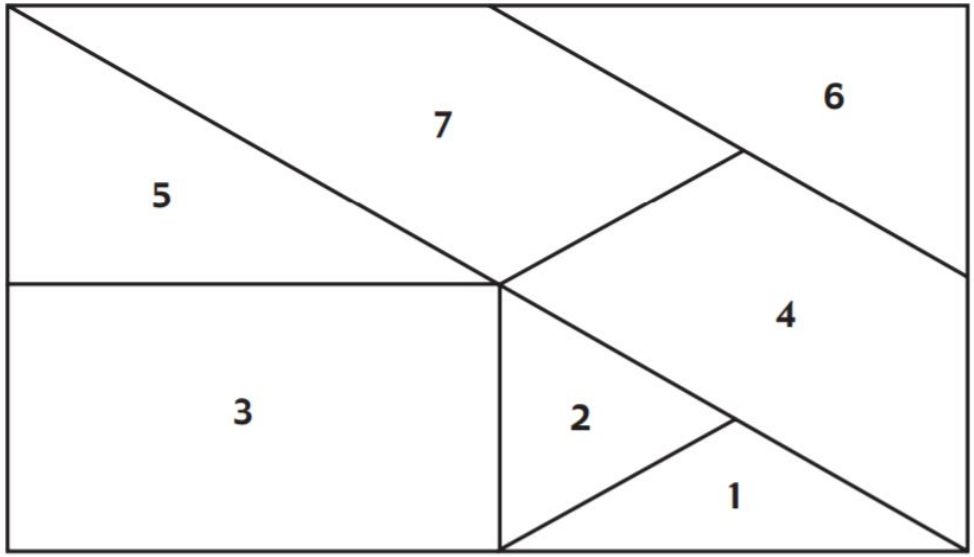
3. Ask what pieces can be made from two others? Which ones cannot?
   Challenge: Find the one piece that can be made from three others.
   Solutions can be recorded by tracing around the larger pieces and then draw how the larger shape was made with the other pieces.

4. Can students make the Parallelogram with three pieces?

5. Have students make a short house and trace around it. Can they make the shape with two other pieces? Three pieces? Four pieces? Can they create a tall house with two pieces? Three pieces? (Remember: Touching edges have to be the same length.)

6. Have each student create their own puzzle using two, three, or four pieces; trace around the shape. Can students make this shape with other pieces? These may be done on cardstock and used as puzzles for other students to build with their puzzle pieces.

Seven Piece Mosaic Puzzle



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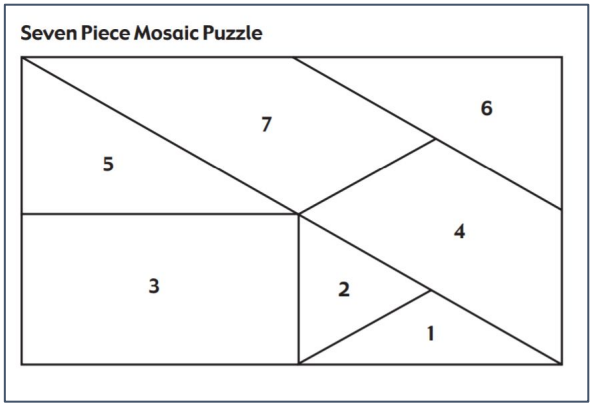
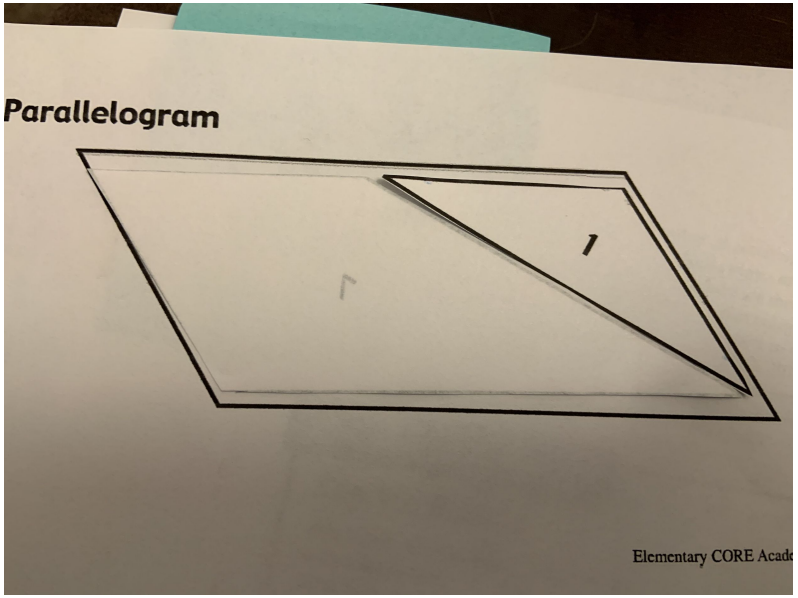
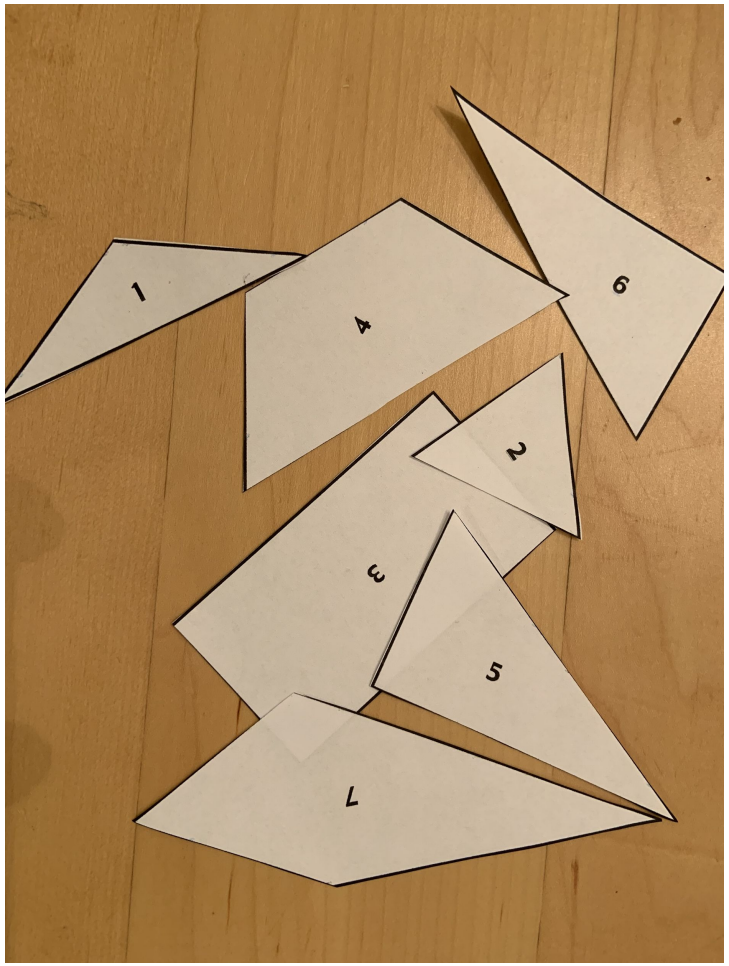
Mosaics







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Standard Connection

KY.7.G.6 Solve problems involving area of two-dimensional objects and surface area and volume of three-dimensional objects. a. Solve real-world and mathematical problems involving area of two-dimensional objects composed of triangles, quadrilaterals and other polygons.	a. Emphasis is on find … es composed of convex polygon… b. Students underst… wo different quantities used … nal figure. Building upon their und… dents use nets of three dimensional obj… rface area.

Manipulating the shapes helps students see composite area...

and prepares them for:

KY.7.G.6a

Solve problems involving area of tw… sional objects and surface area and volume of three-dimensional objects.

a. Solve real-world and mathematical problems involving area of two-dimensional objects composed of triangles, quadrilaterals and other polygons.

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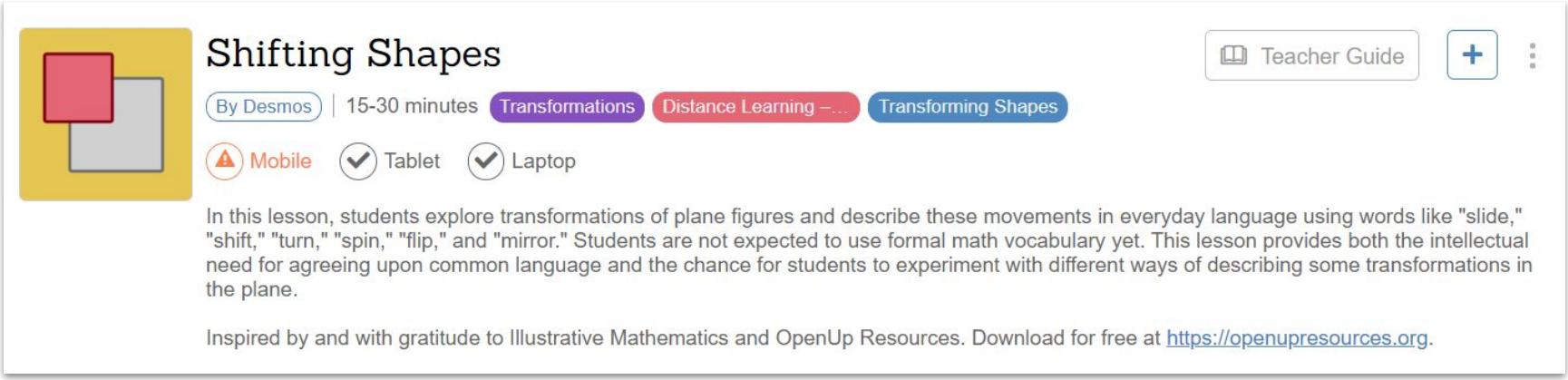
Shifting Shapes



Hey, students!

Go to student.desmos.com
and type in:

JY2 XC7

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Standard Connection

KY.8.G.1 and KY.8.G.2

KY.8.G.1 Verify experimentally the properties of rotations, reflections and translations: • Lines are congruent to lines. • Line segments are congruent to line segments of the same length. • Angles are congruent to angles of the same measure. • Parallel lines are congruent to parallel lines. **MP.5, MP.6**	Emphasis is congruence transformations preserve corresponding congruent lines, segments and angles. KY.HS.G.2 Coherence KY.8.G.1→ KY.HS.G.3(+)
KY.8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations. Given two congruent figures, describe a sequence that exhibits the congruence between them. **MP.2, MP.7**	Students understand a figure, called a pre-image, is congruent to another figure, called the image, if the second figure can be obtained by a sequence of congruence transformations performed on the first figure. Students describe the sequence of congruence transformations necessary to transform one figure to a congruent second figure. KY.HS.G.4 Coherence KY.8.G.2→ KY.HS.G.5

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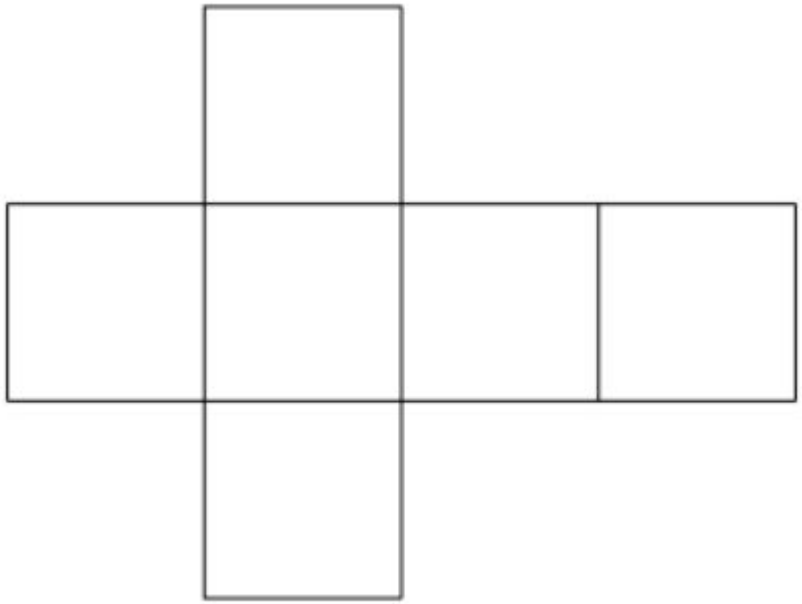
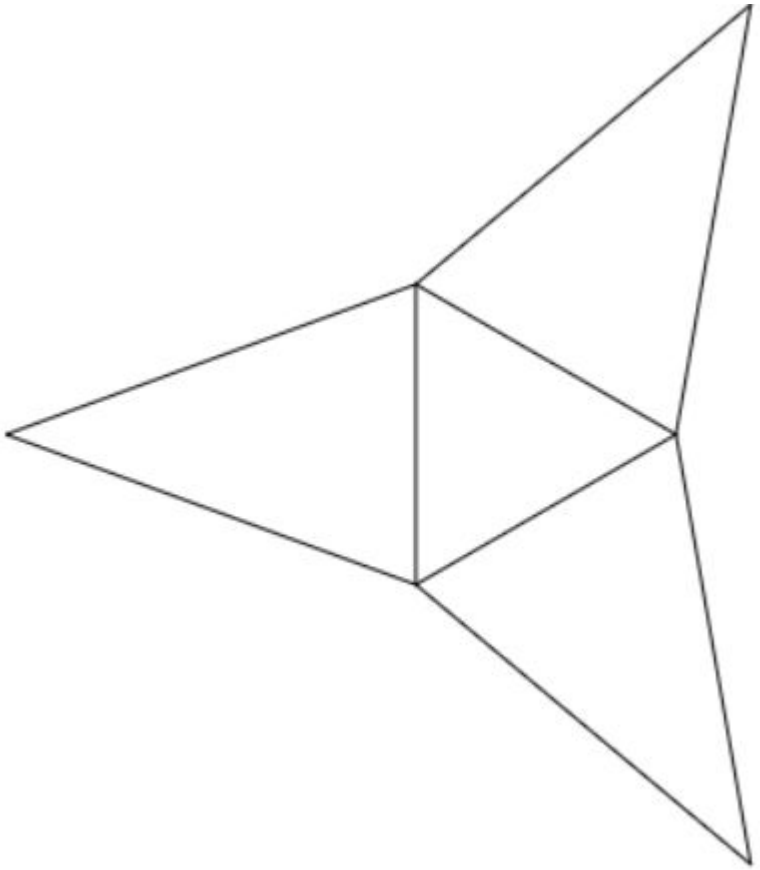
Surface Area

Nets!





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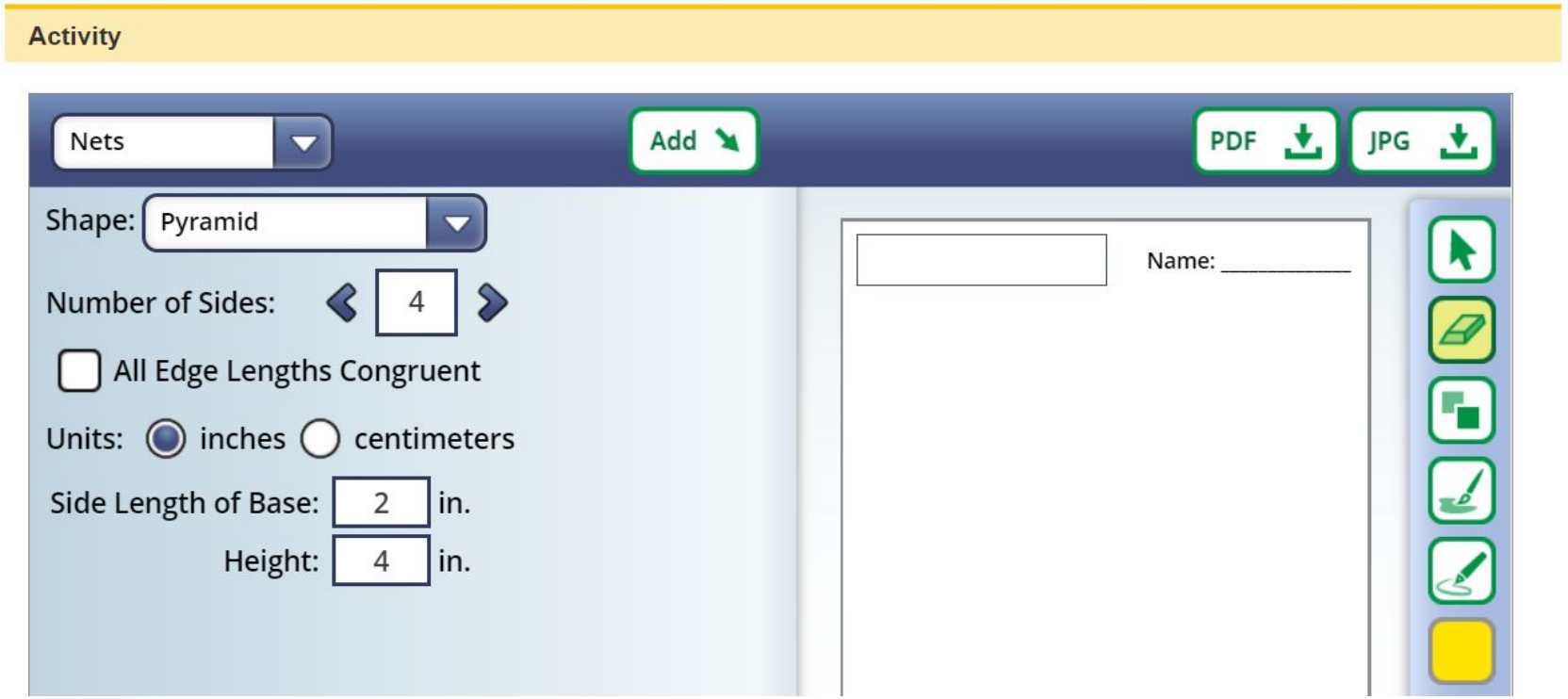
Surface Area

Dynamic Paper - NCTM Illuminations



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Surface & Area

No one needs to memorize all these formulas!!!

Give students experiences WITH the figures & they can build the connections

Shape	Surface Area Formula	Volume Formula
Cube	$SA = 6s^2$ where s = length of the side	$V = s^3$ where s = length of the side
Cuboid	$SA = 2(lw + lh + wh)$ where l = length, w = width, h = height	$V = lwh$ where l = length, w = width, h = height
Prism	$SA = 2B + ph$ where B = area of base, p = perimeter of base, h = height	$V = Bh$ where B = area of base, h = height
Cylinder	$SA = 2\pi r^2 + 2\pi rh$ where r = radius, h = height	$V = \pi r^2 h$ where r = radius, h = height
Hollow Cylinder	$SA = 2\pi rh + 2\pi Rh + 2\left(\pi R^2 - \pi r^2\right)$ where R = radius of the outer surface, r = radius of the inner surface	$V = \pi R^2 h - \pi r^2 h$ where R = radius of the outer surface, r = radius of the inner surface
Cone	$SA = \pi r^2 + \pi rs$ where r = radius, s = slant height	$V = \frac{1}{3}\pi r^2 h$ where r = radius, h = height
Pyramid	SA = area of base + area of each of the lateral faces Regular pyramid = area of base + $\frac{1}{2}ps$ where p = perimeter of the base, s = slant height Square pyramid = $b^2 + 2bs$ where b = length of the base, s = slant height	$V = \frac{1}{3}Bh$ where B = area of the base, h = height
Sphere	$SA = 4\pi r^2$ where r = radius	$V = \frac{4}{3}\pi r^3$ where r = radius
Hemisphere	$SA = 3\pi r^2$ where r = radius	$V = \frac{2}{3}\pi r^3$ where r = radius

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Standard Connection

KY.7.G.6b

Nets as a STRATEGY for finding surface area and volume.

Standards	Clarifications
b. Solve real-world and mathematical problems involving volume and surface area, using nets as needed, of three-dimensional objects including cubes, pyramids and right prisms. **MP.3, MP.4, MP.5**	Students calculate with appropriate units, using nets as a possible strategy for calculation as well as formulas for volume and surface area, where appropriate. KY.6.G.1 KY.6.G.2 Coherence KY.6.G.4→ KY.7.G.6 → KY.8.G.6

Informal deduction!

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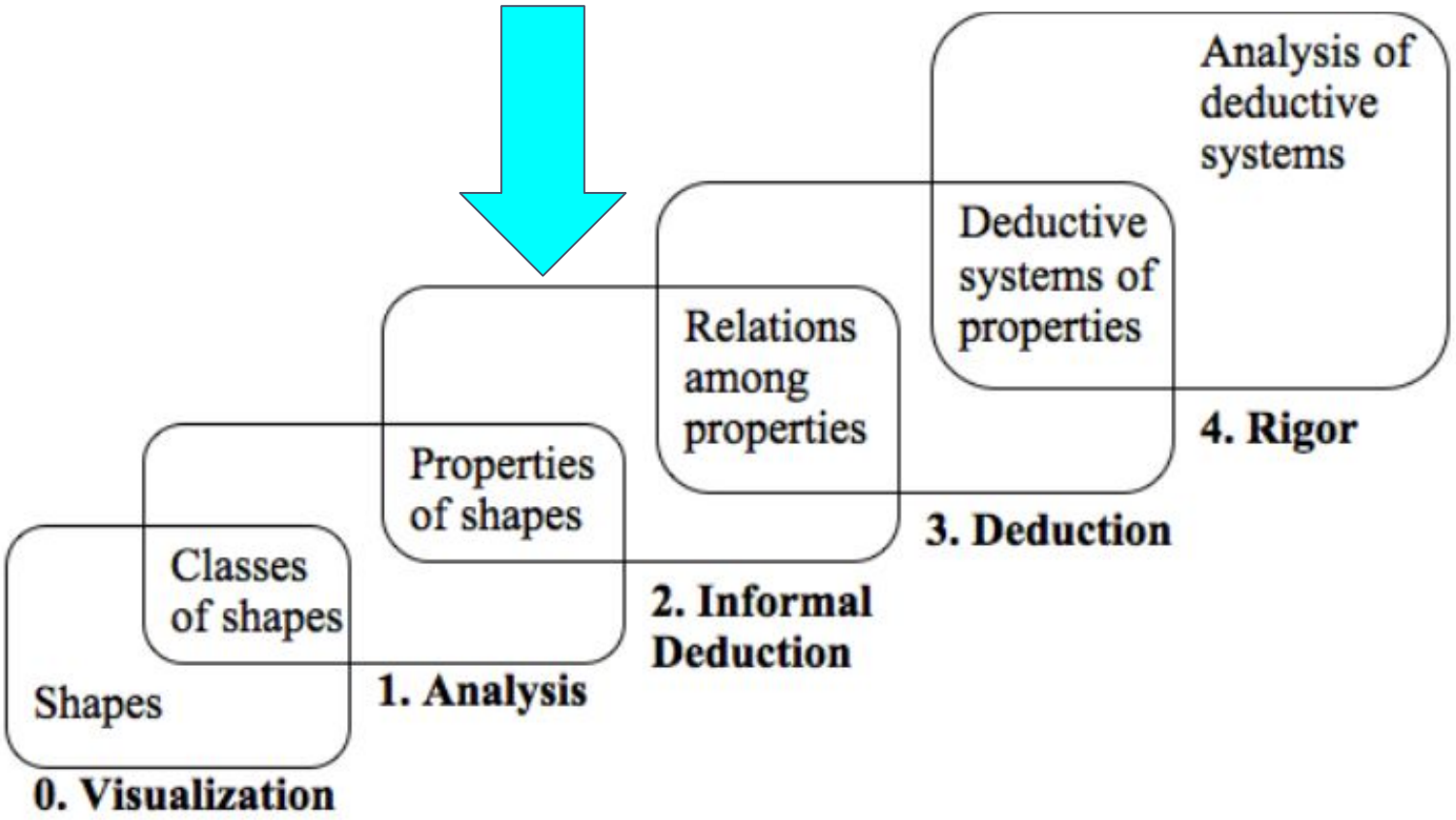

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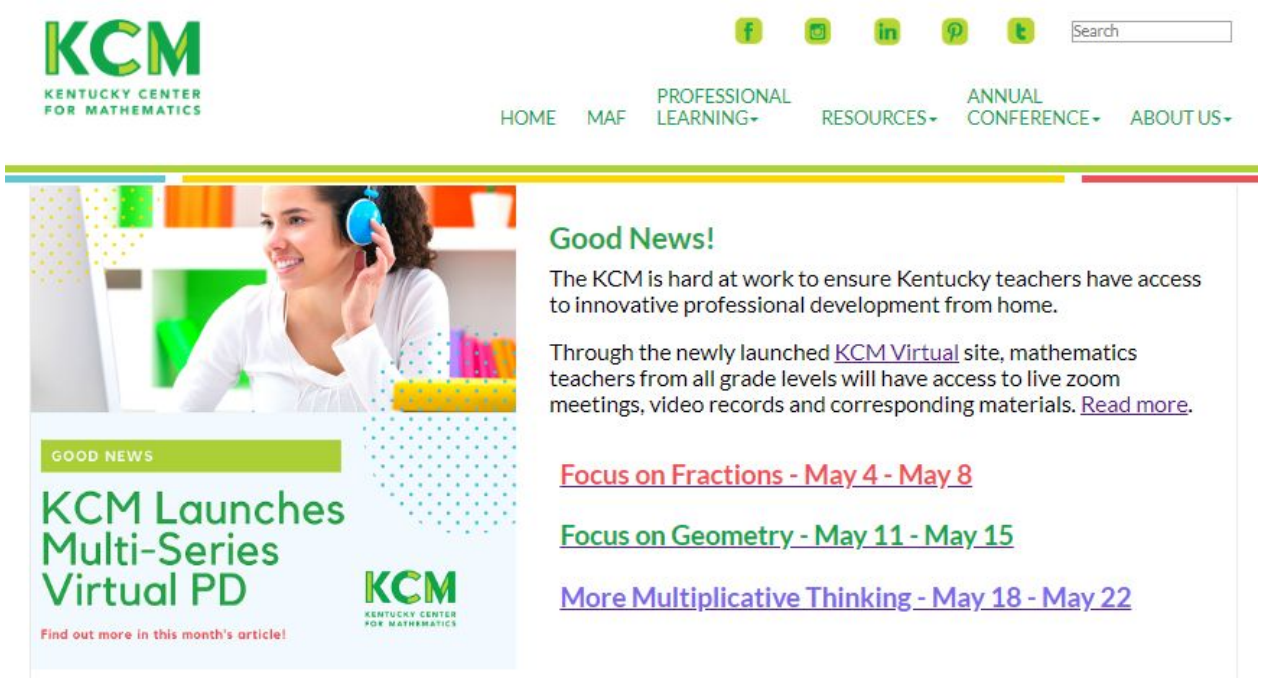

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KCM is here to support you!

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