Megan H. Wickstrom

Volume is a treat

Finding volume is not always an easy task. Students need hands-on experiences with volume units to make sense of three dimensions. Comparing the quantity of familiar objects helps students foster conceptions of volume because the task requires attention to attributes, counting strategies, and connections to multiplication and capacity.

Here's an example, using a familiar object—candies. Create two towers of Starburst™ candy. One tower should measure 4 × 4 × 14 candies; the second tower should measure 4 × 8 × 7 candies. Tape or wrap the towers so that students cannot disassemble them. Tell students that they can keep only one of the towers and will need to decide which one. Center student exploration and class discussion on students' counting and comparison strategies by asking the following questions:

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- Which tower should we keep? Why?
- How can we find the total number of candies in each tower? (Explain two different strategies.)
- Are there ways we can compare the two towers without counting all the candies?

Extensions and modifications

To modify this task for younger students, allow opportunity to build rectangular prisms with the candies and to decompose the towers if needed. To add difficulty and extend the task to capacity, fill a jar with irregular-shape candies. Have students use their counting strategies to estimate the number of candies in the jar. Ask the following questions:

- How can we estimate the number of candies in the jar?
- How is this problem similar or different from the Starburst problem?

Megan H. Wickstrom, **Megan.wickstrom@montana.edu**, is an assistant professor of mathematics education at Montana State University in Bozeman. She is interested in elementary students' thinking as they model and solve real-world problems. Edited by **Martha Hildebrandt**, **mhildebrandt@chatham.edu**, who teaches undergraduate and graduate mathematics education and mathematics courses at Chatham University in Pittsburgh, Pennsylvania; and **Theodore (Teddy) Chao**, **chao.160@osu.edu**, an assistant professor of mathematics education at The Ohio State University in Columbus. Submit your quick game, puzzle, activity, or instructional strategy along with suggestions for how teachers of different grade bands (K–1, 2–3, 4–6) can use this idea. Access **http://tcm.msubmit.net** to send submissions of no more than 250 words to this department. Find detailed submission guidelines for all departments at **http://www.nctm.org/tcmdepartments**.

Exploring Volume as Additive

In this lesson students will explore volume as additive.

NC Mathematics Standard(s):
Measurement and Data

NC.5.MD.5 - Relate volume to the operations of multiplication and addition.
• Find the volume of a rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths.
• Build understanding of the volume formula for rectangular prisms with whole-number edge lengths in the context of solving problems.
• Find volume of solid figures with one-digit dimensions composed of two non-overlapping rectangular prisms.

Additional/Supporting Standards:
NC.5.MD.4 - Recognize volume as an attribute of solid figures and measure volume by counting unit cubes, using cubic centimeters, cubic inches, cubic feet, and improvised units.

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
7. Look for and make use of structure.

Student Outcomes:

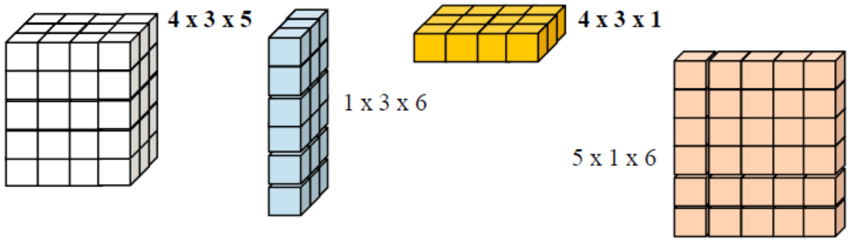
- I can use addition and/or multiplication to find volume.
- I can describe how to find the volume of a right rectangular prism.
- I can find the volume of a figure made from two right rectangular prisms.

Materials:

- Centimeter cubes, preferably interlocking; or other interlocking cubes

Advance Preparation:

- Use the interlocking cubes to assemble these rectangular solids. Each solid should be a different color.

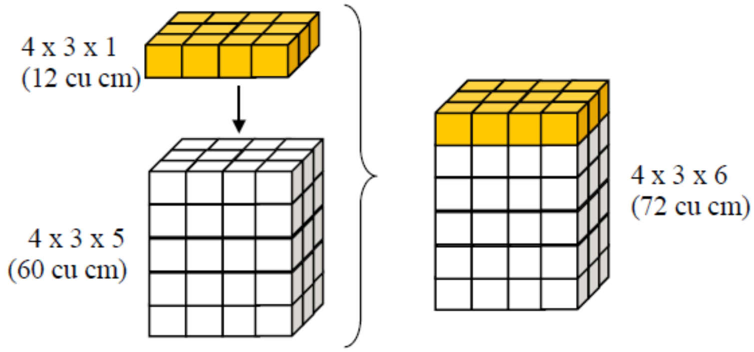


- Consider how you will pair or group students
- Students should be familiar with processes of multiplication
- Students should be comfortable with estimating
- Students should be comfortable explaining their reasoning

Directions:
Note: This lesson can be done along with the "Representing and Finding Volume" lesson, which develops the basic concept of volume measurement as filling a shape with cubic units (5.MD.5a); but it also develops the idea of volume as additive (5.MD.5c). It helps students to see how adding one more layer to any dimension makes a difference to the total volume, illustrating the difference one or more units in each dimension makes. It may also help students make better estimates of leftover space if the cubes do not exactly fill up the boxes in the "Representing and Finding Volume" lesson. This lesson is a whole group lesson. The class discussion is key to students understanding the concepts you are presenting.

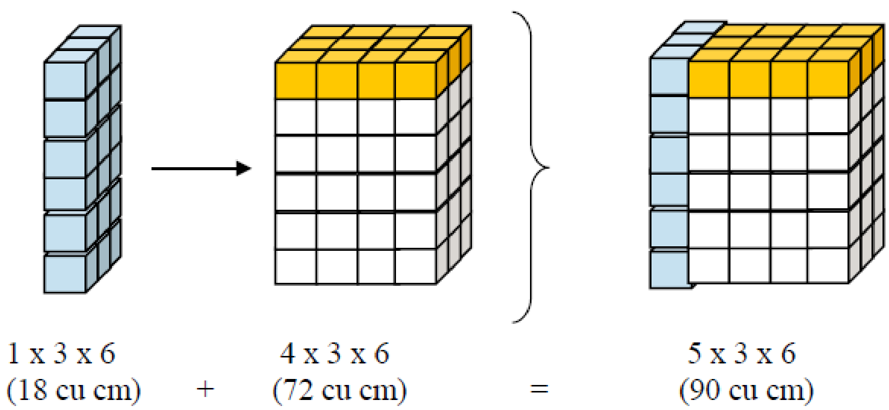
1. Pose this problem to the class: *Juanita measured a box's dimensions to the nearest whole number and its dimensions were: 4 centimeters long, 3 centimeters wide, and 5 centimeters high. What was its volume?* Show the 4 x 3 x 5 prism you made with the centimeter (or other) interlocking cubes. (If you have used some other unit cube rather than centimeter cubes, change the problem to reflect the units you used.) Have students consider the volume of the prism in pairs, then report their conclusions. They should figure out that the prism has 60 cubes in it, so Juanita's box would have a volume of 60 cubic centimeters. Ask how they figured it out. Some students will have multiplied 4 x 3, then multiplied 12 by 5. Others may have used the associative and commutative properties to multiply 5 x 4 first, then multiply 20 by 3. Here is another good opportunity for children to see that in multiplication, the order of the factors does not change the result.

   When all agree that Juanita's box would hold 60 cubic centimeters, ask *How much larger would the volume be if it were one centimeter higher?* Have the children make estimates of how many more cubic centimeters the box would hold. Then show them the original prism and the 4 x 3 x 1 prism you made (in a different color) and add it to the top of the 4 x 3 x 5 prism.

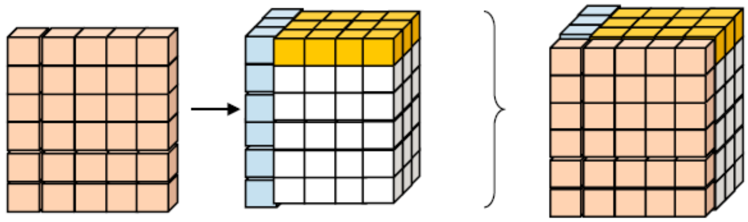


Students should see that adding one centimeter to the height of 5 cm, making the height 6 cm, adds another layer of 4 x 3, or 12 cubic centimeters, making the total volume of the new prism (box) 72 cubic centimeters.

2. Now ask the children to predict how much larger Juanita's box would be if not only the height were larger by one centimeter, but the length were also larger by one centimeter. Make sure they understand that a vertical layer as well as the top layer would be added. After the children have made predictions (and given some explanations of why they are making the predictions), show them the new 4 x 3 x 6 prism and the 1 x 3 x 6 prism which will be added to the length dimension of the prism.



3. Next ask for estimates of how much larger Juanita's box would be if all three dimensions were one centimeter longer. After the students give estimates and explanations for them, repeat the demonstration, adding the 5 x 1 x 6 prism to the "box."



The discussion following this demonstration should emphasize that adding just one more unit to each of the three dimensions can make quite a difference in the volume of a box. In this case, the volume grew from 60 cu cm to 120 cu cm, doubling the original volume. It is also important to emphasize that increasing a dimension by one unit means adding another layer of cubes to the volume, which demonstrates the “additive” nature of volume (5.MD.5c).

4. Students may make the conjecture that increasing each dimension by one unit would always double the volume. Provide pairs or groups a set of cubes and ask them to create a rectangular prism, find its volume, and then add one unit to each dimension as you did in the demonstration. Suggest that for each extra unit in one dimension, they use a different color for the cubes as you did in the demonstration. Have them record their results. Make a class chart and follow up with a class discussion of the results. They will find that adding one unit to each dimension will not always (or even often) double the total volume. Be sure they make note of any original dimensions which do result in a doubling of the total volume when each dimension increases by one unit. Are there patterns that exist between those prisms?

Questions to Pose:

<u>During class discussion:</u>

- How do you use multiplication when finding volume?
- How do you use addition when finding volume?
- Why does changing the measurement of a dimension by only one unit change the total volume by more than one?

<u>As students work together on #4:</u>

- What did you do to get started?
- What do you think the volume of your new shape will be after changing each dimension by one unit?
- Which dimension did you decide to change first?
- Why did you decide to…..?

Extension:

As students work in other lessons to find the volumes of various boxes or spaces (See lesson: “Finding Volume Using a Formula”), have them figure out the volume of some of the boxes or spaces if each dimension were increased by one unit.

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
• Students see the volume as only the cubes they can see on the surface of the prism	• Take the interlocking cubes apart if necessary so that students see the cubes that are "inside" the shape.

Special Notes:

- This lesson may help students make better estimates of leftover space when placing cubes in boxes whose dimensions are not whole numbers of the unit being used, but do not expect this one activity to greatly increase their ability to make appropriate estimates.

Name: ____ **Date:** _____________________

1. What did you notice?

2. What do you wonder?

3. Main Question:

4. Estimate

place your best estimate on the number line and label

5. What information would you like to know?

6. Answer

Show your work

Name________________________________

Saving Sir Cumference

Help Radius and Lady Di of Ameter save Sir Cumference! The answer to this problem lies in the poem. Can you solve the riddle?

Part A

Carefully read the poem, *The Circle's Measure*. Then answer the following questions:

1. What do you think is meant by "measure the middle and circle around?"

2. According to this poem, what two numbers should you "divide?"

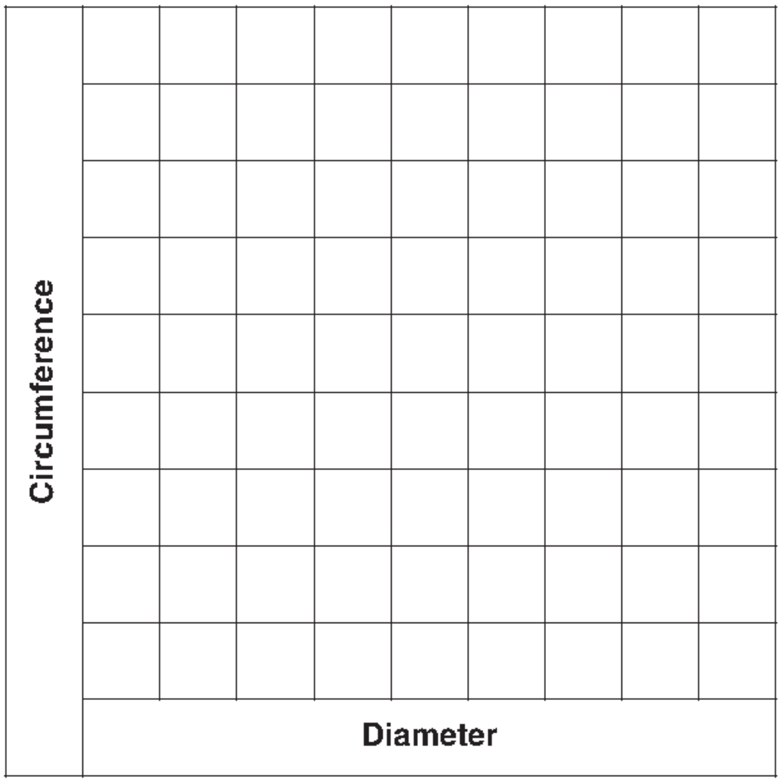
> ### *The Circle's Measure*
>
> Measure the middle and circle around,
> Divide so a number can be found.
> Every circle, great and small—
> The number is the same for all.
> It's also the dose, so be clever,
> Or a dragon he will stay…
> forever...

Part B

Use a measuring tape to find the diameter and circumference of several circular objects. Discuss with your group how to record your data. Create a table below.

Part C

Use a piece of centimeter grid paper to make a coordinate graph like the one below. Use the horizontal axis for diameter and the vertical axis for circumference. Plot the data points for each object your group measured on the graph.



Part D

Study your table and your graph. Look for patterns and relationships that allow you to predict the circumference using the diameter. Test your ideas on other circular objects. Once you think you have found a pattern, answer the questions below:

1. What is the relationship between the diameter and the circumference of a circle?

2. What dose should Radius give Sir Cumference? How do you know this is the correct dose?

The Circle's Measure Labels

Copy onto cardstock, cut out, and attach to Erlenmeyer flasks.

The Circle's Measure

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Teacher Assessment Sheet

As you observe students, add their name into the appropriate box below.

1	**2**	**3**	**4**	**5**
• *does not generalize from one situation to another* • *has difficulty explaining the activity* • *records answers to problems without checking for validity* • *does not see patterns or relationships* • *has difficulty getting correct answers and justifying answers*	• *can generalize to a few situations* • *can explain the procedures of the activity* • *rarely checks for validity or requires reminders to do so* • *has difficulty recognizing patterns and relationships* • *sometimes struggles to get correct answers and justify them*	• *can generalize to many situations* • *can explain the procedures and his/her reasoning* • *often checks for validity* • *recognizes patterns and relationships with guidance from others* • *gets correct answers but has difficulty justifying them*	• *can generalize to most situations* • *can explain the procedures and his/her reasoning* • *almost always checks for validity* • *independently recognizes patterns and relationships* • *gets correct answers and can justify them*	• *makes generalizations to all situations* • *can explain procedures and his/her reasoning* • *always checks for validity of results* • *independently recognizes patterns and relationships* • *gets correct answers and can justify them clearly to other students*

Centimeter Grid

Area of Circles

NAME ______________________________

DESCRIPTION OF THE OBJECT	YOUR ESTIMATE OF THE AREA (IN SQUARE CENTIMETERS)	RADIUS OF THE OBJECT	ACTUAL AREA

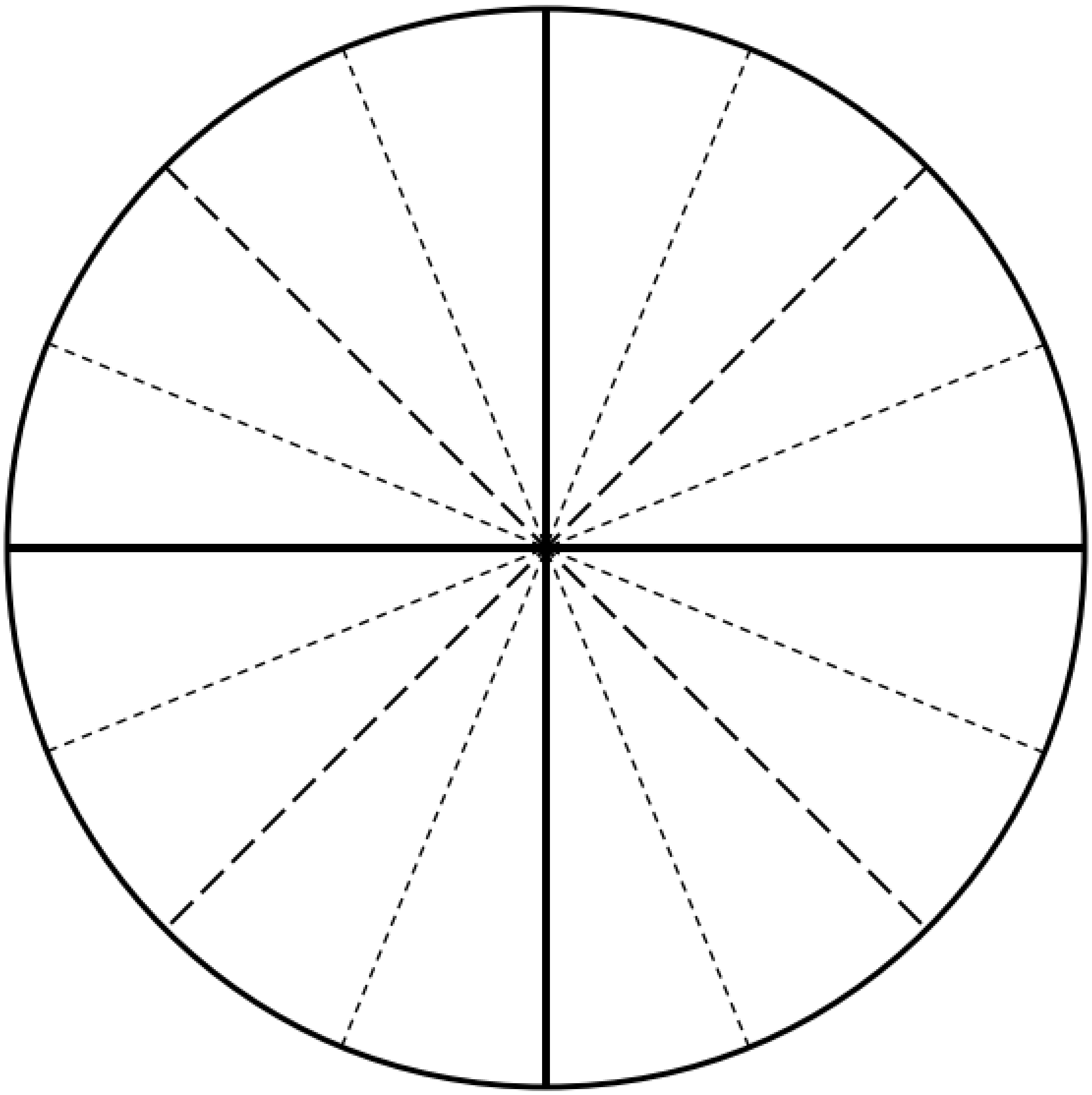
Remember: Include appropriate labels on all measurements!

https://www.nctm.org/Classroom-Resources/Illuminations/Lessons/Discovering-the-Area-Formula-for-Circles/

Fraction Circle

NAME________________________

Cut out the circle and carefully divide it into wedges, as shown.



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