



KENTUCKY CENTER  
FOR MATHEMATICS

# Developing Multiplicative Thinking

*Extending Multiplication and  
Division to Multi-Digit Factors  
with Belle Rush*

# Welcome!

Your host:

**Belle Rush**



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Kentucky Center for Mathematics  
[bsrush180@yahoo.com](mailto:bsrush180@yahoo.com)

# KCM Website

[www.kentuckymathematics.org](http://www.kentuckymathematics.org)



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GOOD NEWS

## KCM Launches Multi-Series Virtual PD

Find out more in this month's article!



## Good News!

The KCM is hard at work to ensure Kentucky teachers have access to innovative professional development from home.

Through the newly launched [KCM Virtual](#) site, mathematics teachers from all grade levels will have access to live zoom meetings, video records and corresponding materials. [Read more.](#)

[Focus on Fractions - May 4 - May 8](#)

[Focus on Geometry - May 11 - May 15](#)

[More Multiplicative Thinking - May 18 - May 22](#)

# Today's Agenda

Research

Standards

Getting there

- Strategies for Multiplication
- Strategies for Division

# Consider this...

Most of us learned to add whole numbers by lining numbers up vertically and adding each column, carrying/borrowing when necessary. But did we understand why it worked? It's most likely that the "how" of doing the computation was "drilled" into our heads, but the "why" was lost.

*"Knowledge not supported by understanding is fragile!"*

# Keep in mind...

The standard algorithm is not a bad thing; it has been in use for years and, when used correctly, works every time.

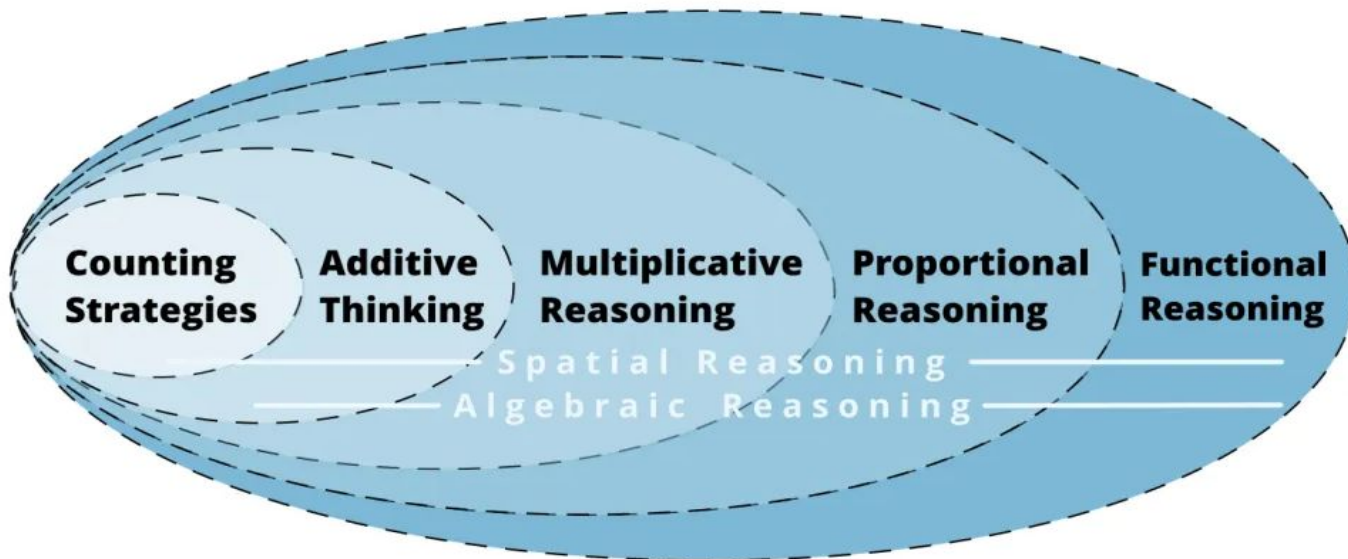
Although the standard algorithm is often introduced and used in primary grades, *the expectation* (Kentucky Academic Standards) is that students not formally use the an algorithm for addition and subtraction until grade 4, and multiplication and division until grade 5. Why do you think that is the case?

# Research from Pamela Weber Harris

<https://www.mathisfigureoutable.com/development/>

## The Development of Mathematical Reasoning

Register now for online

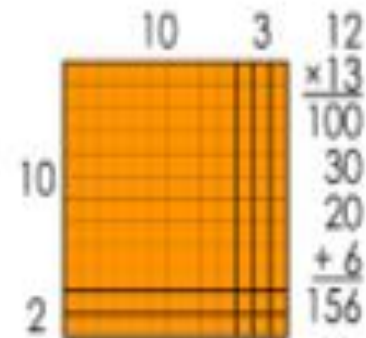
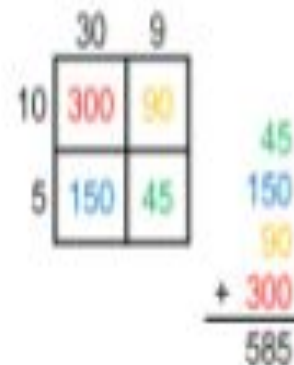
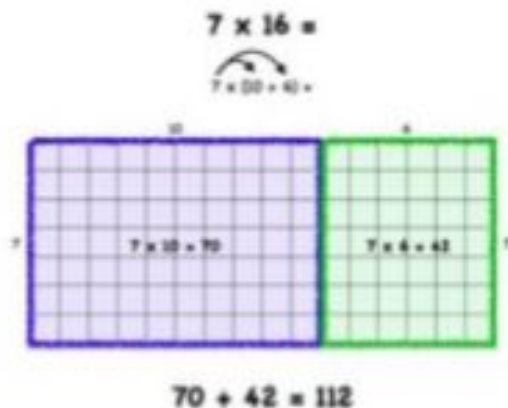


# What do the standards say?

KY.4.NBT.5 Multiply whole numbers

- Up to four digit number by a one-digit number
- Two-digit number by two-digit number

Multiply using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.





# What the standards say?

KY.4.NBT.6 Divide up to four-digit dividends by one-digit divisors. Find whole number quotients and remainders using

- strategies based on place value
- the properties of operations
- the relationship between multiplication and division

Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.

$$5,500 \div 4 = ?$$

	1,000	300	70	5	
4	$1,000 \times 4$	$300 \times 4$	$70 \times 4$	$5 \times 4$	1,000
	4,000	1,200	280	20	300
					70
					<u>+ 5</u>
					1,375

# What do the standards say?

KY.5.NBT.5 Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm.

	300	70	4	
50	15,000	3,500	200	$\approx 18,700$
3	900	210	12	$\approx 1,122$

19,822

$$\begin{array}{r} 374 \\ \times 53 \\ \hline 12 \text{ (} 3 \times 4 \text{)} \\ 210 \text{ (} 3 \times 70 \text{)} \\ 900 \text{ (} 3 \times 300 \text{)} \\ 200 \text{ (} 50 \times 4 \text{)} \\ 3,500 \text{ (} 50 \times 70 \text{)} \\ \underline{15,000 \text{ (} 50 \times 300 \text{)}} \\ 19,822 \end{array}$$

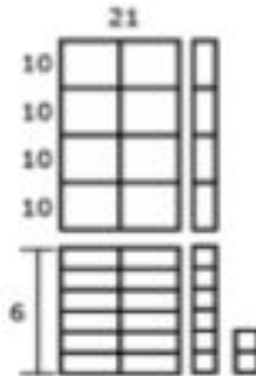
# What do the standards say

KY.5.NBT.6 Divide up to four-digit dividends by two-digit divisors.

a. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors using...

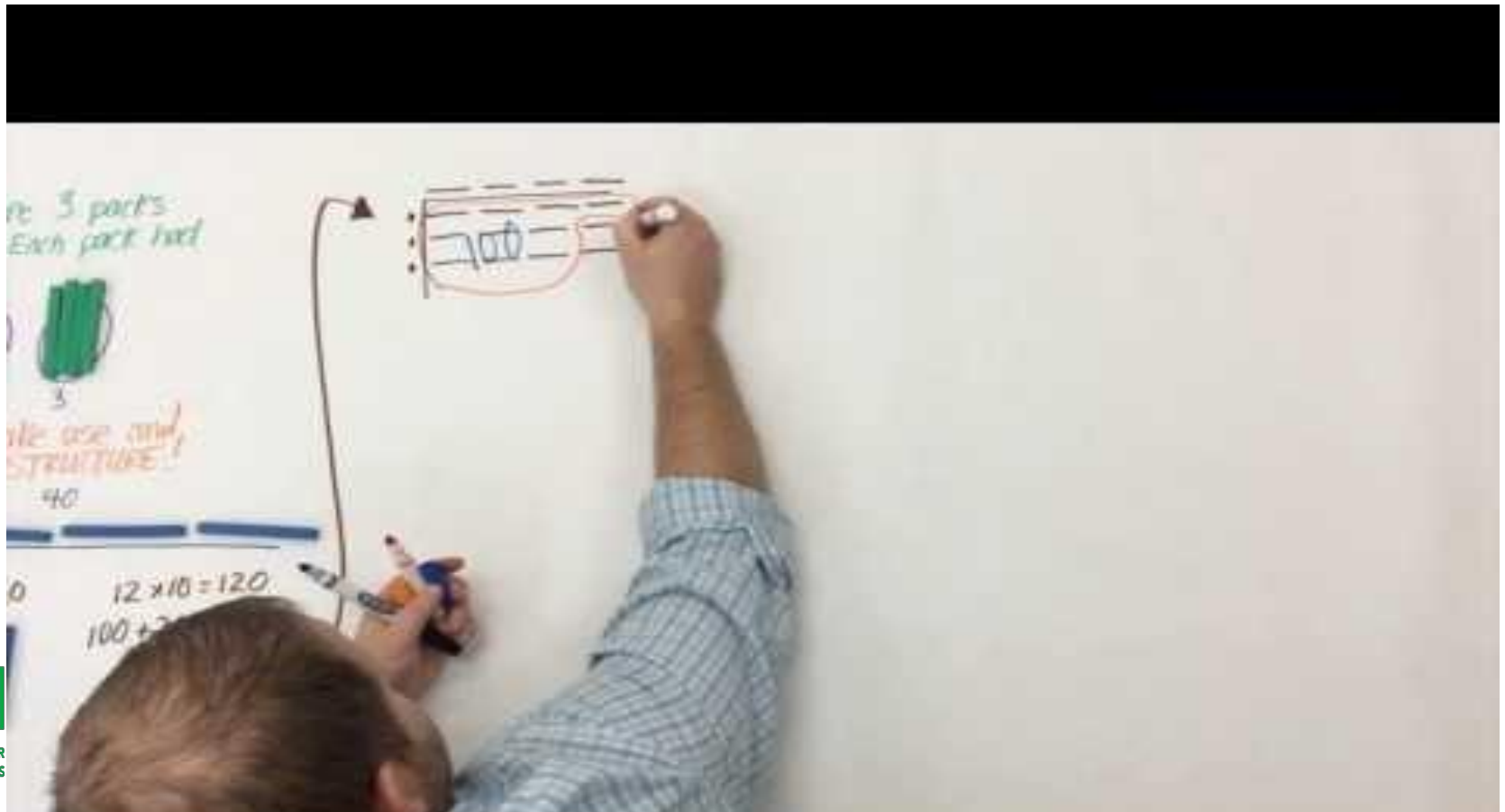
- strategies based on place value
- the properties of operations
- the relationship between multiplication and division

b. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.



# The Progression of Multiplication

[The Progression of Multiplication HD](#)



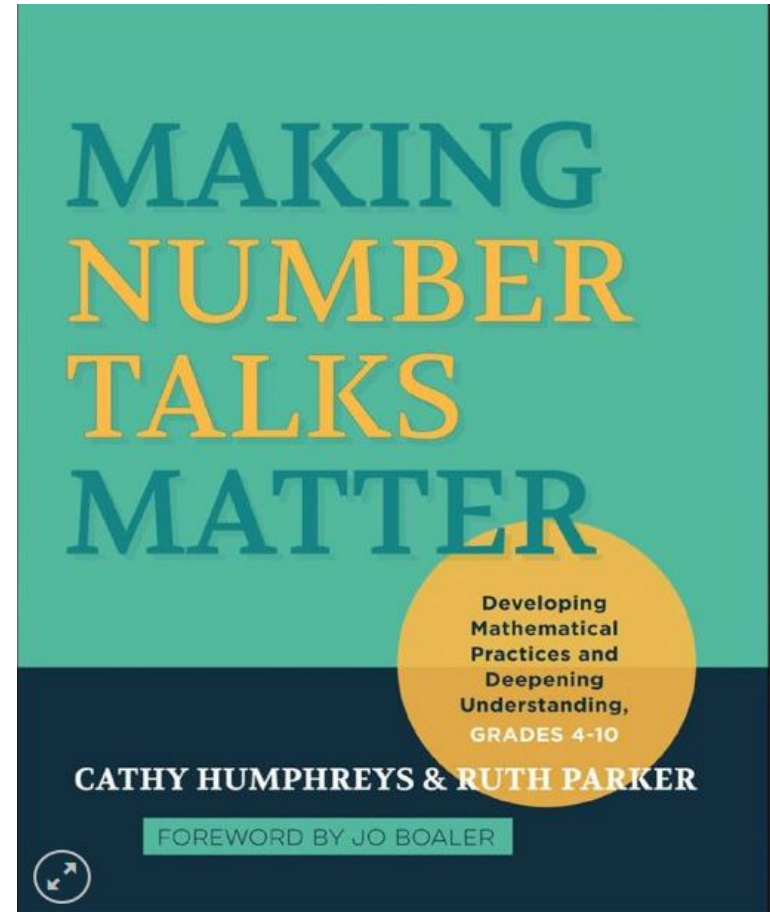
# Three Act Tasks

## [Sugar Packets](#)



# Four Strategies for Multiplication

- Break a factor into addends
- Factor a factor
- Round a factor and adjust
- Halving and doubling



# Break a Factor into two or more addends

$$12 \times 16$$

$$\begin{aligned} 12 \times 16 &= 12 \times (10 + 6) \\ &= (10 + 6) \times 12 \end{aligned}$$

$$10 \times 12 = 120$$

$$\begin{array}{r} 6 \times 12 = 72 \\ \hline 192 \end{array}$$

# Dreambox Teacher Tools

<https://www.dreambox.com/teachertools>

The screenshot displays the Dreambox Teacher Tools interface. At the top, the Dreambox logo and the user name "jennifer (teacher)" are visible. A "Sample Lesson" banner is present. The main instruction reads: "Build smaller arrays to help answer  $78 \times 97$ . You can create up to 6 arrays." Below this, the problem  $78 \times 97 = ?$  is shown, followed by a box containing  $70 \times 90 = ?$ . To the right, a large orange array with yellow dots is shown, with dimensions 90 and 7 indicated. A small character with eyes is on the array. The interface includes various control buttons like a lightbulb, a checkmark, and a refresh button. The bottom left corner features the KCM logo and the text "KENTUCKY CENTER FOR MATHEMATICS".



# Factors and Multiples Game

## Factors and Multiples Game

### Factors & Multiples Game



This is a game for two players.

You need:

a 100 square grid and some transparent counters.

#### What you have to do:

The first player chooses a positive even number that is less than 50, and covers it out on the grid with a counter.

The second player chooses a number to cover. The number must be a factor or multiple of the first number.

Players continue to take it in turns to cover numbers, at each stage choosing a number that is a factor or multiple of the number just covered by the other player.

The first person who is unable to cross out a number loses.

e.g. the following game started 12, 4, 44, 11, 77

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



1-100 Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

[nrich.maths.org/roadshow](http://nrich.maths.org/roadshow)

nrich.maths.org  
© University of Cambridge

# Factor a factor

$$12 \times 16$$

$$12 \times 16 = 12 \times (4 \times 2 \times 2)$$

$$12 \times 4 = 48$$

$$\begin{array}{r} \times 2 \\ \hline \end{array}$$

$$96 \times 2 = 192$$

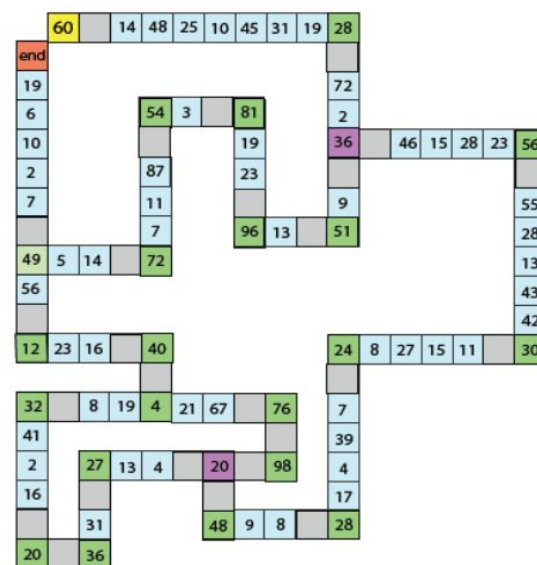
# Factor Track

## Factor Track



nrich

## Factor Track



### Rules:

You can move any factor, except 1, of the number you are on.

You start on the [yellow] 60 and make your way round to the [red] 'end' square.

You may not go round corners so you must get exactly to a green square.

<http://nrich.maths.org/7468>  
© University of Cambridge

# Round a factor and adjust

$$12 \times 16 \dots \{16 = 20 - 4\}$$

$$12 \times 20 = 240$$

$$12 \times 4 = 48 < \begin{array}{r} 40 \\ +8 \end{array}$$

$$240 - 40 = 200$$

$$200 - 8 = 192$$

# Multiplication Race

## Multiplication Race

### Multiplication Race

**Materials:** game board, number cube, one counter for each player, calculator

1. Each player places a counter on the box marked 'Start'.
2. Take turns to roll a number cube and move forward that number of spaces along the path. Solve the multiplication problem you land on or follow the instruction you land on.
3. Partners use a calculator to check each other's work. A player who gives an incorrect product must miss a turn.
4. Continue until one player reaches the box marked 'End'.

**Extension:** Create your own Multiplication Race board and try it out with a partner.

Multiplication Race

Go back 5	83 x 764	94 x 653	Roll again	End	Start
72 x 654			25 x 348	25 x 292	13 x 121
69 x 763		Miss a turn	36 x 896	Go back 8	21 x 242
58 x 982		47 x 358		94 x 695	34 x 615
47 x 884		58 x 312		83 x 772	45 x 672
35 x 653		Go back 5	69 x 467	72 x 563	Roll again
Go back 4					Go back 3
24 x 574	13 x 709	Roll again	92 x 772	89 x 346	Miss a turn
					78 x 524
					67 x 494

©K-5MathTeachingResources.com

### Multiplication Race

Go back 5	83 x 764	94 x 653	Roll again	End	Start
72 x 654			25 x 348	25 x 292	13 x 121
69 x 763		Miss a turn	36 x 896	Go back 8	21 x 242
58 x 982		47 x 358		94 x 695	34 x 615
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35 x 653		Go back 5	69 x 467	72 x 563	Roll again
Go back 4					Go back 3
24 x 574	13 x 709	Roll again	92 x 772	89 x 346	Miss a turn
					78 x 524
					67 x 494

©K-5MathTeachingResources.com

# Halving and doubling

$$12 \times 16$$

$$12 \times 16 = 24 \times 8$$

$$= 48 \times 4$$

$$= 96 \times 2$$

$$= 192$$

# Double and Halve

## Double and Halve

### Double and Halve

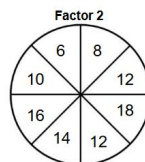
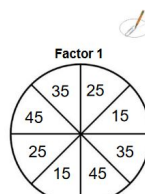
**Materials:** 10 counters per player, 2 paper clips, 2 pencils, Double and Halve board

1. Work with a partner. Collect 10 counters each.
2. Take turns to spin a paper clip on each spinner. Use the two numbers the paper clips land on to create a multiplication problem.
3. Double one factor and halve the other to change the problem to one with an equivalent product that is easy to solve mentally. Explain your strategy.
4. Place a counter on the multiplication fact on the board. If the multiplication fact is already covered play passes to the next player.
5. Continue playing until one player has placed all ten counters on the board.

©K-5MathTeachingResources.com

### Double and Halve

30 x 3	50 x 3	70 x 3	90 x 3
30 x 4	50 x 4	70 x 4	90 x 4
30 x 5	50 x 5	70 x 5	90 x 5
30 x 6	50 x 6	70 x 6	90 x 6
30 x 7	50 x 7	70 x 7	90 x 7
30 x 8	50 x 8	70 x 8	90 x 8
30 x 9	50 x 9	70 x 9	90 x 9



I know that the product of \_\_\_\_ multiplied by \_\_\_\_ is equivalent to the product of \_\_\_\_ multiplied by \_\_\_\_\_. The answer to both problems is \_\_\_\_\_.

I know that the product of \_\_\_\_ multiplied by \_\_\_\_ is equivalent to the product of \_\_\_\_ multiplied by \_\_\_\_\_. The answer to both problems is \_\_\_\_\_.

To solve \_\_\_\_ times \_\_\_\_ I doubled \_\_\_\_ and halved \_\_\_\_ to change the problem to \_\_\_\_ times \_\_\_\_\_. The product is \_\_\_\_\_.

To solve \_\_\_\_ times \_\_\_\_ I doubled \_\_\_\_ and halved \_\_\_\_ to change the problem to \_\_\_\_ times \_\_\_\_\_. The product is \_\_\_\_\_.

# Progression of Division

## The Progression of Division HD

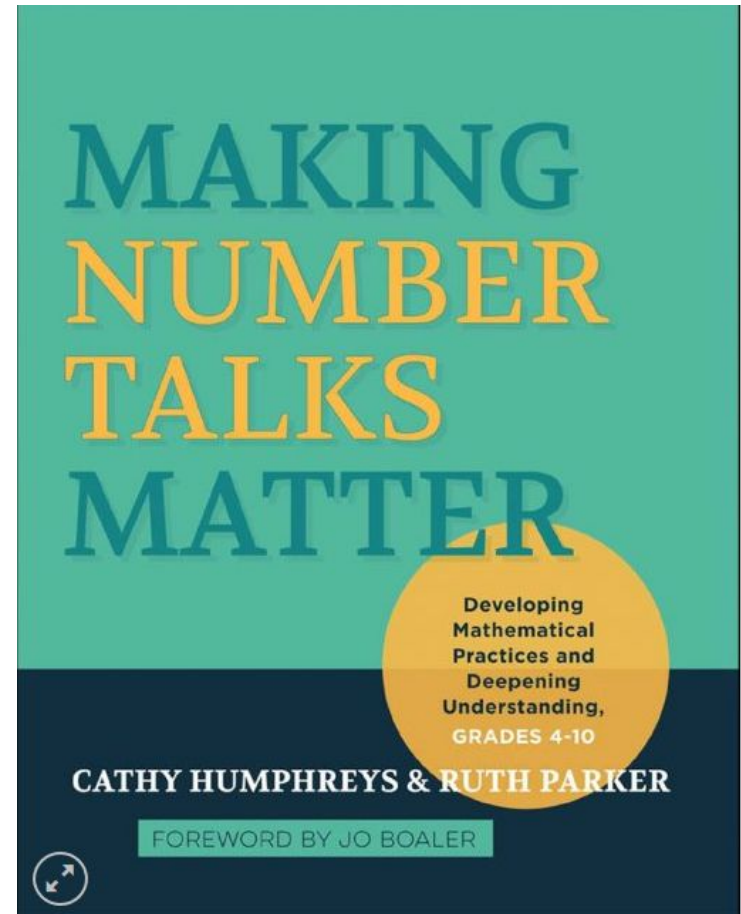
The whiteboard content includes:

- Left side:** A vertical list of division problems with their solutions:
  - $3672 \div 9$
  - $3672 - 9 = 3663$  (1 group)
  - $3663 - 9 = 3654$  (1 group)
  - $3654 - 9 = 3645$  (1 group)
  - $3645 - 9 = 3636$  (1 group)
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# Strategies for Division

- Multiply instead
- Chunk out
- Make a tower
- Halving and halving



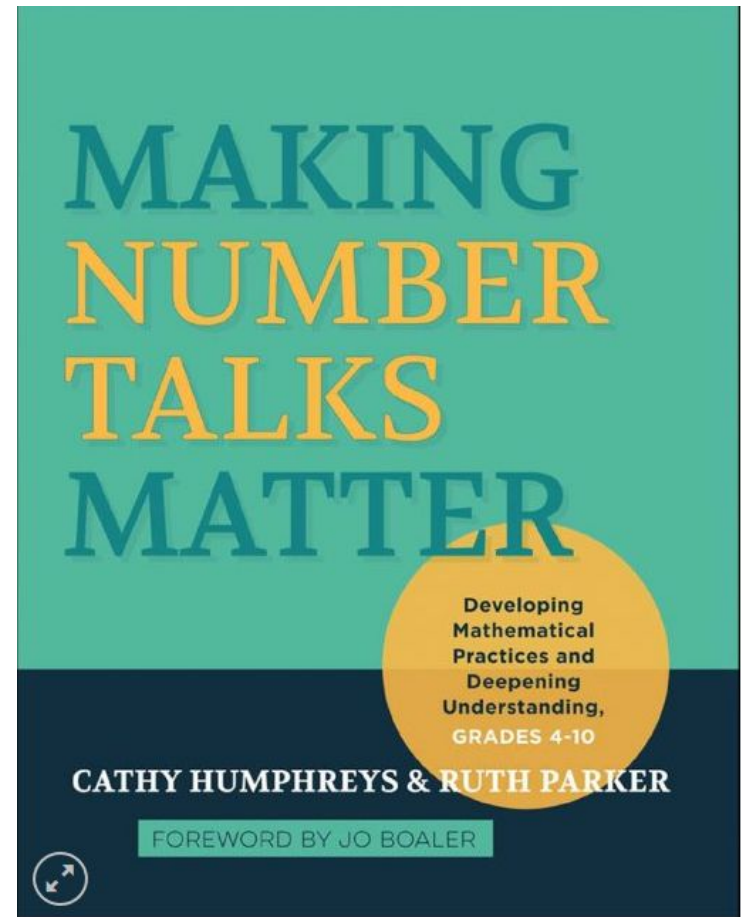
# Strategies for Division

"...when computers and calculators truly come of age in the schools, paper and pencil long division will probably be 'as dead as a dodo bird'".

Richard Anderson

"...there is no longer a job in the world--not one single job--where someone does long division with paper and pencil; not one job, that is, other than teaching,"

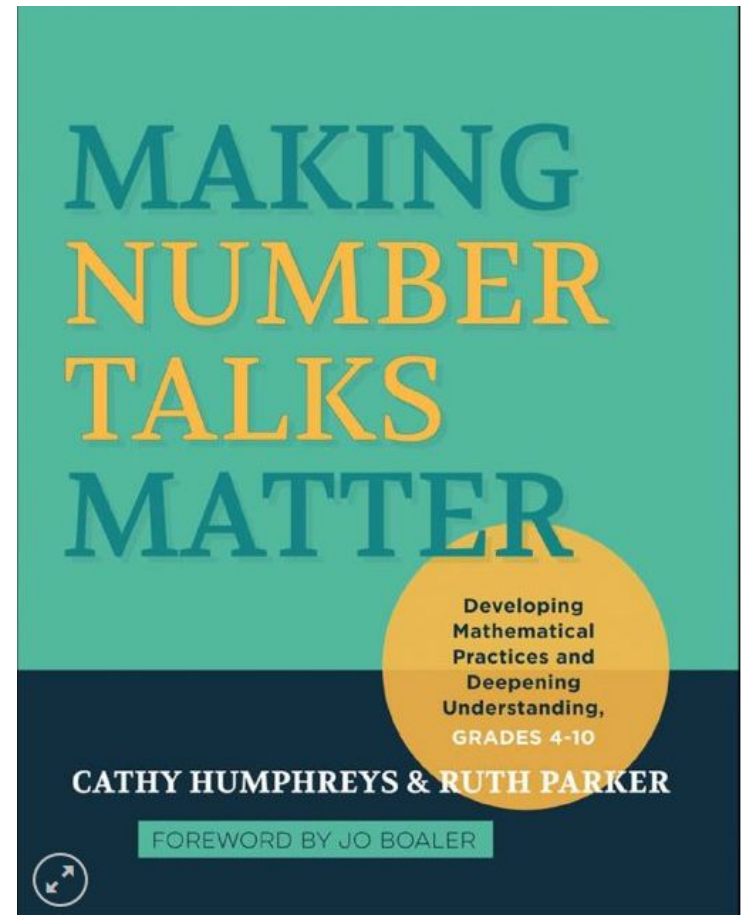
Maier 1982



# Strategies for Division

Through Number Talks, students can make sense of division (the operation--not the standard algorithm)--and, by maintaining a focus on the relationships between quantities, learn how to size up a problem to determine a reasonable “ballpark” answer.

Cathy Humphreys and Ruth Parker



# Multiply instead

$$17/3$$

"I know 3 times 5 is 15, so I have 5 groups of 3; then I have 2 left, so remainder of 2.

# Origo at Home

Each of the weeks below provides five daily activities organized around a specific math topic. To help establish a rhythm for learning at home, each day of the week has a focus:

- Monday — Read and Discuss (Grades K–2) or Watch and Talk (Grades 3–5)
- Tuesday — Hands-on Math
- Wednesday — Problem-solving
- Thursday — Game Day (digital practice)
- Friday — Practice

# Origo at Home

[ORIGO at Home - Interactive Digital K-6 Math plans & resouces for home](#)

The screenshot shows the 'Remainder Run' game interface. On the left is a large grid with numbers. The grid is divided into sections labeled 'Start' and 'Finish'. The 'Start' section contains a 2x8 grid of numbers, with the number 15 highlighted in blue. The 'Finish' section contains a 2x8 grid of numbers, with the number 47 highlighted in red. On the right is a control panel with a 'Cube' section showing a die with the number 8. Below the cube are two 'Counter' sections for Player 1 and Player 2. At the bottom right is a 'Fundamentals' section with icons for a calculator, a pencil, an eraser, and a hand cursor.

Start							
42	27	16	38	35	49	15	21
							9
11	50	18	47	43	13	32	29
22							
30	24	23	45	34	20	35	41
							42
10	39	36	15	22	12	33	19

Finish							

**Remainder Run**

Cube

8

Counter - Player 1

Counter - Player 2

Fundamentals

# Chunk out

$$643 \div 30$$
$$21 \frac{13}{30}$$

Handwritten long division of 643 by 30. The divisor 30 is on the left, and the dividend 643 is on the right. The quotient 21 is written above the dividend, with a remainder of 13. The division is shown in two steps: first, 600 is subtracted from 643, leaving 43; then, 30 is subtracted from 43, leaving 13.

30  $\overline{) 643}$

- 600

43

- 30

13

20 ... { Twenty 30s is 600 }

1 ... { one 30 is 30 }

... { 13 left out of a group of 30 }



# Chunk it out/ Partition the Dividend

## Division Strategy: Partition the Dividend

### Division Strategy: Partition the Dividend

**Materials:** Division Equations Board

1. Work with a partner. Choose a line of four problems from the board (vertically, horizontally or diagonally) that you will both solve.
2. Solve each problem by breaking the dividend into parts that are easy to divide. Solve the easier problems and then add the partial quotients.

**Example:**  $72 \div 5$

$$\begin{array}{r} 72 \\ \swarrow \searrow \\ 50 + 22 \text{ (+5)} \\ \downarrow \downarrow \\ 10 + 4R2 = 14 R2 \end{array} \quad \text{or} \quad \begin{array}{r} 10 + 4R2 = 14 R2 \\ 5 \overline{)50+22} \end{array}$$
  

$$\begin{array}{r} 256 \\ \swarrow \searrow \\ 210 + 46 \text{ (+7)} \\ \downarrow \downarrow \\ 30 + 6R4 = 36 R4 \end{array} \quad \text{or} \quad \begin{array}{r} 30 + 6R4 = 36 R4 \\ 7 \overline{)210+46} \end{array}$$

3. Check your work with your partner. Then repeat with another line of four problems

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ver. 1

$101 \div 9$	$91 \div 7$	$104 \div 8$	$125 \div 5$
$64 \div 4$	$58 \div 4$	$48 \div 3$	$79 \div 6$
$73 \div 6$	$97 \div 8$	$67 \div 5$	$41 \div 3$
$49 \div 3$	$120 \div 9$	$72 \div 5$	$84 \div 6$



# Make a tower

"Tower" of 139

x40	520
x4	52
x3	39
x2	26
x1	13

$$531 \div 13$$
$$40\frac{11}{13}$$

13	)	531	
		- 520	40
		<hr/>	
		11	

# Dreambox Teacher Tools

<https://www.dreambox.com/teachertools>

The screenshot displays the Dreambox Teacher Tools interface. At the top, the Dreambox logo and the user name "jennifer (teacher)" are visible. The interface is titled "Sample Lesson" and contains the instruction: "Use the Bag-O-Matic to compute how many bags of 18 gumballs can be made from 8224 gumballs." Below this, the division problem  $8224 \div 18 = ?$  is shown. To the left of the problem is a small icon of a bag with a star. To the right is a large, colorful illustration of a bag filled with gumballs, labeled "8224". A small red bag icon with the number "18" is positioned to the left of the gumball bag. The interface includes various interactive elements like a lightbulb icon for hints and a speaker icon for audio. The bottom of the screen shows a copyright notice: "© 2020 Dreambox Learning, Inc. All Rights Reserved. Legal | Privacy policy".

# Partial Quotients

<https://www.k-5mathteachingresources.com/support-files/division-strategy-partial-quotients-ver.3.pdf>

## Division Strategy: Partial Quotients

**Materials:** Division equations boards

1. Work with a partner. Choose a line of four problems from the board (horizontally, vertically or diagonally) that you will both solve using the partial quotients algorithm.

Step 1: Write a list of easy facts for the divisor.

Step 2: Subtract from the dividend an easy multiple of the divisor (e.g.  $\times 10$ ,  $\times 100$ ,  $\times 200$  etc.) Record the partial quotient in a column to the right of the problem.

Step 3: Repeat until the dividend has been reduced to zero, or the remainder is less than the divisor.

Step 4: Add the partial quotients to find the final quotient.

**Example:**  $3,863 \div 16$

### Easy Facts for 16

$10 \times 16 = 160$   
 $20 \times 16 = 320$   
 $30 \times 16 = 480$   
 $100 \times 16 = 1,600$   
 $200 \times 16 = 3,200$

$$\begin{array}{r}
 230 \text{ r}3 \\
 16 \overline{) 3863} \\
 \underline{- 3200} \quad 200 \times \\
 483 \\
 \underline{- 320} \quad 20 \times \\
 163 \\
 \underline{- 160} \quad 10 \times \\
 3
 \end{array}$$

3. Check your work with your partner.
4. Repeat with another line of four problems.

$3,292 \div 16 =$	$3,624 \div 17 =$	$3,155 \div 15 =$	$2,929 \div 14 =$
$6,835 \div 17 =$	$3,973 \div 13 =$	$4,836 \div 16 =$	$3,919 \div 19 =$
$4,591 \div 15 =$	$4,834 \div 16 =$	$3,828 \div 19 =$	$4,580 \div 15 =$
$4,984 \div 16 =$	$3,463 \div 34 =$	$1,831 \div 16 =$	$3,768 \div 18 =$

# Halving and halving

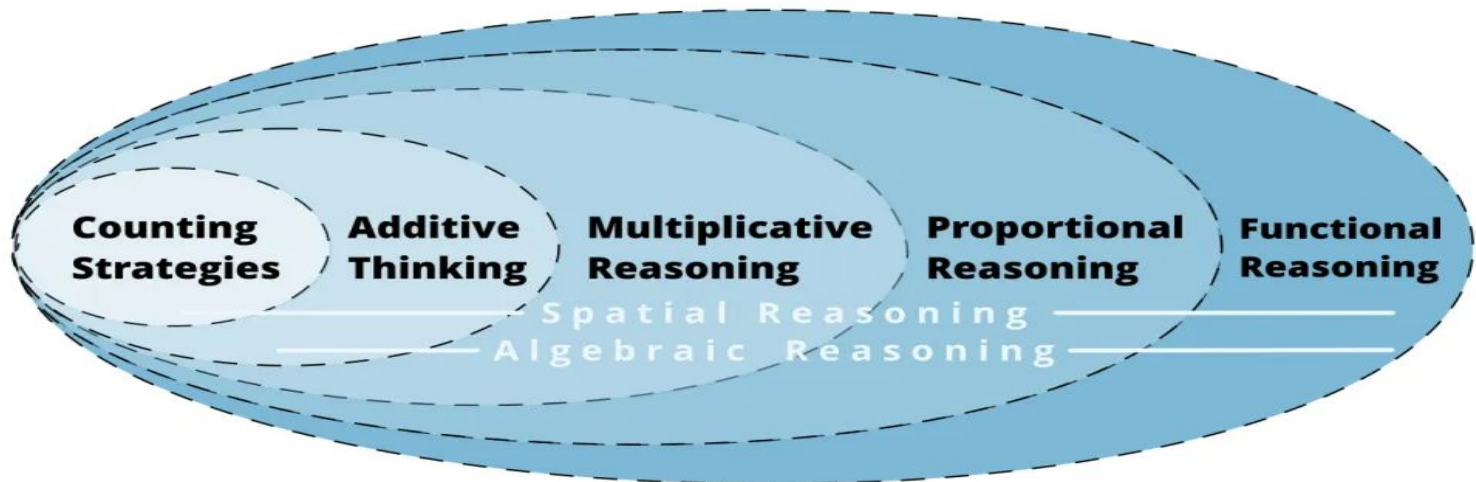
$$\begin{aligned}128 &\div 32 \\&= 64 \div 16 \\&= 32 \div 8 \\&= 16 \div 4 \\&\quad \textcircled{4}\end{aligned}$$

All of these strategies are useful with any numbers, including fractions, decimals, and negative numbers!

These strategies help students make use of properties of operations, which leads to algebraic understanding.

## The Development of Mathematical Reasoning

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"The best teachers are those who show you where to look, but don't tell you what to see."

~Alexandra K. Trenfor



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