Developing Multiplicative Thinking- 

Developing More Multiplication Strategies 

with Bonny Davenport
Welcome!

Your host

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Good News!
The KCM is hard at work to ensure Kentucky teachers have access to innovative professional development from home.

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Focus on Fractions - May 4 - May 8
Focus on Geometry - May 11 - May 15
More Multiplicative Thinking - May 18 - May 22
Today’s Agenda

• Standards
• Research
• Manipulatives: N-Tiles and L-Cover
• Strategies With Origo
  • Adding a Group
  • Subtracting a Group
• Resources
## Standards

### Operations and Algebraic Thinking

#### Standards for Mathematical Practice

- **MP.1.** Make sense of problems and persevere in solving them.
- **MP.2.** Reason abstractly and quantitatively.
- **MP.3.** Construct viable arguments and critique the reasoning of others.
- **MP.4.** Model with mathematics.
- **MP.5.** Use appropriate tools strategically.
- **MP.6.** Attend to precision.
- **MP.7.** Look for and make use of structure.
- **MP.8.** Look for and express regularity in repeated reasoning.

### Cluster: Multiply and divide within 100.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Clarifications</th>
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<tbody>
<tr>
<td>KY.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. <strong>MP.2, MP.8</strong></td>
<td>Students determine multiplication and division strategies efficiently, accurately, flexibly and appropriately. Being fluent means students choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and explain their approaches and they produce accurate answers efficiently. Knowing $8 \times 5 = 40$, one knows $40 \div 5 = 8$. Note: Reaching fluency is an ongoing process that will take much of the year.</td>
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### Attending to the Standards for Mathematical Practice

By studying patterns and relationships in multiplication facts, students develop fluency for multiplication facts (MP.8). For example, students notice $4 \times 6$ is equivalent to $2 \times 2 \times 6$ (doubling strategy). They know 9 facts can be found by thinking of the other factor $\times 10$ and subtracting one group. For example, recognizing $9 \times 8$ is equivalent to $10 \times 8 - 8$. For each fact, the student thinks, “What reasoning strategy can I use that is more efficient than skip counting?” (MP.2).
Standards

<table>
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<td><strong>MP.1.</strong> Make sense of problems and persevere in solving them.</td>
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Cluster: Understand properties of multiplication and the relationship between multiplication and division.

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<tr>
<td><strong>KY.3.OA.5</strong> Apply properties of operations as strategies to multiply and divide. <strong>MP.3, MP.4</strong></td>
<td>Students need not use formal terms for these properties. If $6 \times 4$ is known, then $4 \times 6 = 24$ is also known (Commutative property of multiplication). $3 \times 5$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$ (Associative property of multiplication). Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find $8 \times 7$ as $8 \times (5+2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ (Distributive property).</td>
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| **KY.3.OA.6** Understand division as an unknown-factor problem. **MP.2** | Find $32 \div 8$ by finding the number that makes $32$ when multiplied by $8$. |

**Attesting to the Standards for Mathematical Practice**

Students use strategies beyond skip counting to solve multiplication problems. They decide how to use known facts to solve facts like $6 \times 9$. Students use strategies like Adding a Group, thinking 5 groups of 9 (45) plus one more group (54) and Subtracting a Group, thinking $9 \times 6$ and reasoning 10 groups of 6 (60) minus one group of 6 (54) (**MP.7**). Students explain their selected reasoning strategy and listen and critique other students’ strategies, considering which strategies make sense and are efficient (**MP.3**). Students think about $84 \div 4$ as, “How many sets of 4 can be made from 84 items?” or “How many in a group, if there 84 items and 4 groups?” and use this relationship to solve the problem (**MP.2**).
The Development of Mathematical Reasoning

Counting Strategies, Additive Thinking, Multiplicative Reasoning, Proportional Reasoning, Functional Reasoning

Spatial Reasoning, Algebraic Reasoning

https://www.mathisfigureoutable.com/development/
Foundational Facts Must Precede Derived Fact Strategies


*We acknowledge that all the derived fact strategies are break apart (distributive property) strategies. We focus on specific ways to break apart (e.g., adding a group) and move towards generalizing the Break Apart strategy.*
Adding a Group (3s, 6s)

Start with a nearby 2s, 5s or 10s fact, then add the group.

Example: I don’t know 6x7, but I do know my 5s, so I can first find 5x7. I know 5 groups of 7 is 35. I have to add one more group of 7 to 35 and that equals 42.
Subtracting a Group (9s, 4s)

Start with a nearby 2s, 5s or 10s fact, then subtract the group.

Example: I don’t know 8x7, but I do know my 10s facts, so I can first find 10x7. I know ten groups of 7 is 70. That is two groups too many. I have to subtract two groups of 7 from 70 and that is 70-14=56. So, 8x7=56.
N Tiles and L Cover

Multiplication and Division Virtual Lesson and Resource Examples

Virtual intervention video using corresponding slide decks to facilitate lessons in Multiplication and Division (28:59)
L-Cover Jam Board
**Multiplication**
This week your student will work with the build-up strategy for multiplication. This strategy can be used anytime you multiply a number by 6 by building up from a known 5’s facts. For example, 5 groups of 8 is 40, so 6 groups of 8 must be 8 more, so the answer is 48. Watch the ORIGO ONE video about the build-up strategy before working with your student (also available in Spanish). This will help you to assist your student as they work through the activities this week. Encourage your student to look for patterns as they multiply by 6.

<table>
<thead>
<tr>
<th>Monday — Watch and Talk</th>
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<tbody>
<tr>
<td>Tuesday — Hands-on Math</td>
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<tr>
<td>Wednesday — Problem-solving</td>
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<tr>
<td>Thursday — Game Day</td>
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<tr>
<td>Friday — Practice</td>
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**Game board**

<table>
<thead>
<tr>
<th></th>
<th>36</th>
<th>18</th>
<th>42</th>
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<tbody>
<tr>
<td></td>
<td>12</td>
<td>6</td>
<td>24</td>
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<tr>
<td></td>
<td>30</td>
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<th>6 × 1</th>
<th>6 × 2</th>
<th>6 × 3</th>
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<tr>
<td></td>
<td>6 × 4</td>
<td>6 × 5</td>
<td>6 × 6</td>
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<tr>
<td></td>
<td>6 × 7</td>
<td>6 × 8</td>
<td>6 × 9</td>
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Add the cards then multiply by 9.
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