Developing Multiplicative Thinking - Habituation of Basic Facts for Multiplication and Division with Lisa Riggs
Welcome!

Your host

Lisa Riggs

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KCM Website

www.kentuckymathematics.org

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Focus on Fractions - May 4 - May 8
Focus on Geometry - May 11 - May 15
More Multiplicative Thinking - May 18 - May 22
Today’s Agenda

• Standards
• Research
• Strategies
  • Berkley Everett
• Resources
  • Orgio
  • nrich maths
## Standards

### Operations and Algebraic Thinking

#### Standards for Mathematical Practice

<table>
<thead>
<tr>
<th>MP.1</th>
<th>Make sense of problems and persevere in solving them.</th>
</tr>
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<tbody>
<tr>
<td>MP.2</td>
<td>Reason abstractly and quantitatively.</td>
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<td>Construct viable arguments and critique the reasoning of others.</td>
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### Cluster: Multiply and divide within 100.

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<th>Clarifications</th>
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<tr>
<td>KY.3.OA.7</td>
<td>Students determine multiplication and division strategies efficiently, accurately, flexibly and appropriately. Being fluent means students choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and explain their approaches and they produce accurate answers efficiently. Knowing $8 \times 5 = 40$, one knows $40 \div 5 = 8$. Note: Reaching fluency is an ongoing process that will take much of the year.</td>
</tr>
<tr>
<td>MP.2, MP.8</td>
<td></td>
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</table>

**Coherence** KY.3.OA.7 → KY.4.OA.4

### Attending to the Standards for Mathematical Practice

By studying patterns and relationships in multiplication facts, students develop fluency for multiplication facts (MP.8). For example, students notice $4 \times 6$ is equivalent to $2 \times 2 \times 6$ (doubling strategy). They know 9 facts can be found by thinking of the other factor $\times 10$ and subtracting one group. For example, recognizing $9 \times 8$ is equivalent to $10 \times 8 - 8$. For each fact, the student thinks, “What reasoning strategy can I use that is more efficient than skip counting?” (MP.2).
Standards

### Operations and Algebraic Thinking

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#### Cluster: Understand properties of multiplication and the relationship between multiplication and division.

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<td>KY.3.OA.5 Apply properties of operations as strategies to multiply and divide. MP.3, MP.4</td>
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Students need not use formal terms for these properties. If 6 x 4 is known, then 4 x 6 = 24 is also known (Commutative property of multiplication). 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30 (Associative property of multiplication). Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5+2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive property).

| KY.3.OA.6 Understand division as an unknown-factor problem. MP.2 |

Find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

#### Attending to the Standards for Mathematical Practice

Students use strategies beyond skip counting to solve multiplication problems. They decide how to use known facts to solve facts like 6 x 9. Students use strategies like Adding a Group, thinking 5 groups of 9 (45) plus one more group (54) and Subtracting a Group, thinking 9 x 6 and reasoning 10 groups of 6 (60) minus one group of 6 (54) (MP.7). Students explain their selected reasoning strategy and listen and critique other students’ strategies, considering which strategies make sense and are efficient (MP.3). Students think about 84 ÷ 4 as, “How many sets of 4 can be made from 84 items?” or “How many in a group, if there 84 items and 4 groups?” and use this relationship to solve the problem (MP.2).
The Development of Mathematical Reasoning

Counting Strategies  Additive Thinking  Multiplicative Reasoning  Proportional Reasoning  Functional Reasoning

Spatial Reasoning  Algebraic Reasoning

PAM HARRIS

https://www.mathisfigureoutable.com/development/
Foundational Facts Must Precede Derived Fact Strategies

Instructional Phases of Multiplication and Division

1. Building on students’ emergent strategies
2. Instruction on sequences of multiples
3. Structuring numbers multiplicatively
4. Developing strategies for 1-digit factors
5. Habituation of basic facts
6. Extending to multi-digit factors and beyond 100.
Examples of a strategy for a given multiplier, table 7.1, pg.154

<table>
<thead>
<tr>
<th>Multiplier</th>
<th>Strategy</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>Double Ex: $7 \times 2$ is $7 + 7$</td>
</tr>
<tr>
<td>3</td>
<td>Double and add again: $9 \times 3$ is double $9 + another 9$</td>
</tr>
<tr>
<td>4</td>
<td>Double and double again: $7 \times 4$ is $14 + 14$</td>
</tr>
<tr>
<td>5</td>
<td>Half of 10 times: $6 \times 5$ is half of 60</td>
</tr>
<tr>
<td>6</td>
<td>Double 3’s: $4 \times 6$ is $4 \times 3 + 4 \times 3$</td>
</tr>
<tr>
<td>7</td>
<td>Use 5 times or 6 times facts: $4 \times 7$ is $4 \times 5 + 4 \times 2$ or $4 \times 6 + 4$</td>
</tr>
<tr>
<td>8</td>
<td>Double and double and double a third time: $6 \times 8$, double 6 is 12, double 12 is 24, double 24 is 48.</td>
</tr>
<tr>
<td>9</td>
<td>10 times - 1 time: $4 \times 9$ is 40 - 4</td>
</tr>
<tr>
<td>10</td>
<td>The corresponding tens number: $7 \times 10$ is 70</td>
</tr>
</tbody>
</table>

Multiplication Relationships

$8 \times 6 =

\begin{array}{c}
4 \\
2 \\
2 \\
\hline
6
\end{array}

2 \times 6 = 12, \ 12 \times 2 = 24,
\ 24 \times 2 = 48

\begin{array}{c}
5 \\
\hline
8
\end{array}

40 + 8

\begin{array}{c}
3 \\
4 \\
\hline
3
\end{array}

3


3 \times 1 = 3

\rightarrow 3

\downarrow +1

\rightarrow 3

\downarrow +3

3 \times 2 = 6

\rightarrow 6

\downarrow +1

\rightarrow 3

\downarrow +3

3 \times 3 = 9

\rightarrow 3

\downarrow +3

3 \times 4 = 12

\rightarrow 12

\downarrow +3

3 \times 5 = 15

3 \times 6 = 18

\rightarrow 18

\downarrow +3

3 \times 7 = 21

3 \times 8 = 24

3 \times 9 = 27

harder for me

\rightarrow 27

\downarrow -3

3 \times 10 = 30

easy for me!
Origo: Pick a Product
Factors and Multiples

Click on a number to move it between the left and right squares. Numbers in the right grid can be dragged to reorder them. Aim to make the longest possible chain where each number is a factor or a multiple of its predecessor. Each number may be used once only. Chains are bracketed in green. Blue numbers are not part of a chain.
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