



KENTUCKY CENTER  
FOR MATHEMATICS

## Developing Multiplicative Thinking

More Assessing and Monitoring  
Multiplicative Thinking

# Welcome



Your Host:

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### GOOD NEWS

## KCM Launches Multi-Series Virtual PD

Find out more in this month's article!



### Good News!

The KCM is hard at work to ensure Kentucky teachers have access to innovative professional development from home.

Through the newly launched [KCM Virtual](#) site, mathematics teachers from all grade levels will have access to live zoom meetings, video records and corresponding materials. [Read more.](#)

[Focus on Fractions - May 4 - May 8](#)

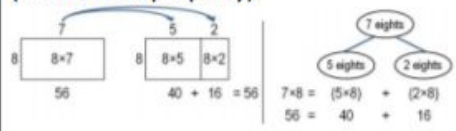
[Focus on Geometry - May 11 - May 15](#)

[More Multiplicative Thinking - May 18 - May 22](#)

# AGENDA

- Standards
- Research
- Assessment Task Groups
- Written Computation Methods
- Progression of Multiplication

# STANDARDS

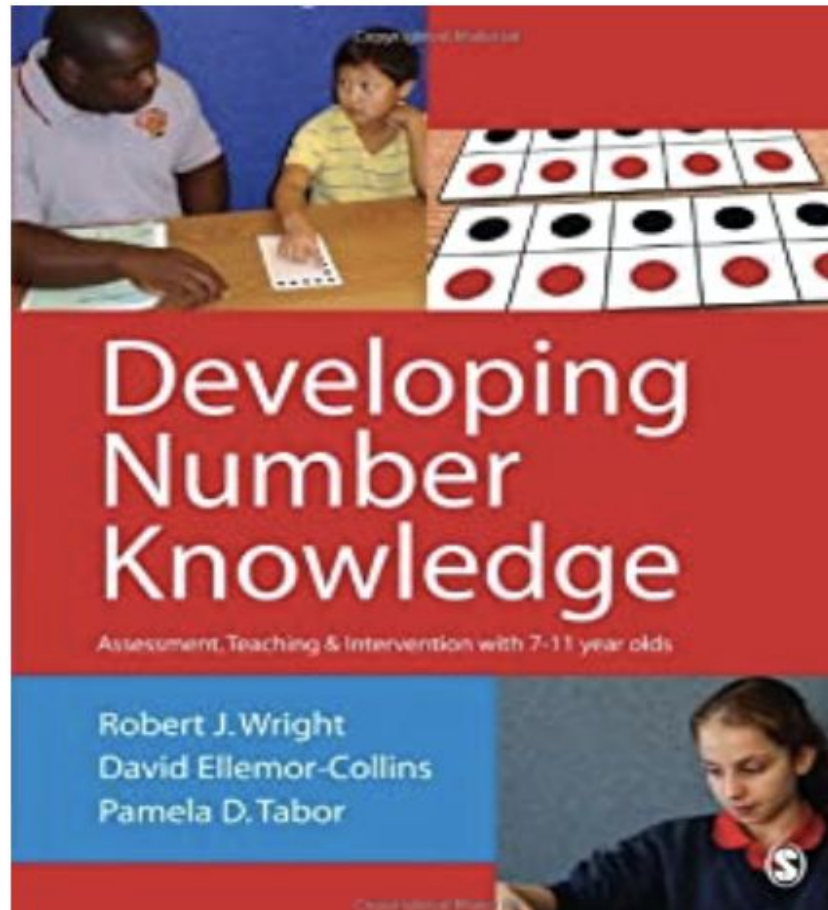
Operations and Algebraic Thinking	
Standards for Mathematical Practice	
<a href="#">MP.1.</a> Make sense of problems and persevere in solving them. <a href="#">MP.2.</a> Reason abstractly and quantitatively. <a href="#">MP.3.</a> Construct viable arguments and critique the reasoning of others. <a href="#">MP.4.</a> Model with mathematics.	<a href="#">MP.5.</a> Use appropriate tools strategically. <a href="#">MP.6.</a> Attend to precision. <a href="#">MP.7.</a> Look for and make use of structure. <a href="#">MP.8.</a> Look for and express regularity in repeated reasoning.
<b>Cluster: Understand properties of multiplication and the relationship between multiplication and division.</b>	
Standards	Clarifications
KY.3.OA.5 Apply properties of operations as strategies to multiply and divide. <b>MP.3, MP.4</b>	<p>Students need not use formal terms for these properties. If <math>6 \times 4</math> is known, then <math>4 \times 6 = 24</math> is also known (Commutative property of multiplication). <math>3 \times 5 \times 2</math> can be found by <math>3 \times 5 = 15</math>, then <math>15 \times 2 = 30</math>, or by <math>5 \times 2 = 10</math>, then <math>3 \times 10 = 30</math> (Associative property of multiplication). Knowing that <math>8 \times 5 = 40</math> and <math>8 \times 2 = 16</math>, one can find <math>8 \times 7</math> as <math>8 \times (5+2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56</math> (Distributive property).</p> 
KY.3.OA.6 Understand division as an unknown-factor problem. <b>MP.2</b>	<p>Find <math>32 \div 8</math> by finding the number that makes 32 when multiplied by 8.</p>
<b>Attending to the Standards for Mathematical Practice</b>	
<p>Students use strategies beyond skip counting to solve multiplication problems. They decide how to use known facts to solve facts like <math>6 \times 9</math>. Students use strategies like Adding a Group, thinking 5 groups of 9 (45) plus one more group (54) and Subtracting a Group, thinking <math>9 \times 6</math> and reasoning 10 groups of 6 (60) minus one group of 6 (54) (<b>MP.7</b>). Students explain their selected reasoning strategy and listen and critique other students' strategies, considering which strategies make sense and are efficient (<b>MP.3</b>). Students think about <math>84 \div 4</math> as, "How many sets of 4 can be made from 84 items?" or "How many in a group, if there 84 items and 4 groups?" and use this relationship to solve the problem (<b>MP.2</b>).</p>	



# STANDARDS

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
<a href="#">MP.1.</a> Make sense of problems and persevere in solving them.	<a href="#">MP.5.</a> Use appropriate tools strategically.
<a href="#">MP.2.</a> Reason abstractly and quantitatively.	<a href="#">MP.6.</a> Attend to precision.
<a href="#">MP.3.</a> Construct viable arguments and critique the reasoning of others.	<a href="#">MP.7.</a> Look for and make use of structure.
<a href="#">MP.4.</a> Model with mathematics.	<a href="#">MP.8.</a> Look for and express regularity in repeated reasoning.
<b>Cluster: Multiply and divide within 100.</b>	
Standards	Clarifications
<p>KY.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations.</p> <p><b>MP.2, MP.8</b></p>	<p>Students determine multiplication and division strategies efficiently, accurately, flexibly and appropriately. Being fluent means students choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and explain their approaches and they produce accurate answers efficiently. Knowing <math>8 \times 5 = 40</math>, one knows <math>40 \div 5 = 8</math>.</p> <p>Note: Reaching fluency is an ongoing process that will take much of the year.</p> <p style="text-align: right;">Coherence KY.3.OA.7 → <a href="#">KY.4.OA.4</a></p>
Attending to the Standards for Mathematical Practice	
<p>By studying patterns and relationships in multiplication facts, students develop fluency for multiplication facts (<b>MP.8</b>). For example, students notice <math>4 \times 6</math> is equivalent to <math>2 \times 2 \times 6</math> (doubling strategy). They know 9 facts can be found by thinking of the other factor <math>\times 10</math> and subtracting one group. For example, recognizing <math>9 \times 8</math> is equivalent to <math>10 \times 8 - 8</math>. For each fact, the student thinks, "What reasoning strategy can I use that is more efficient than skip counting?" (<b>MP.2</b>).</p>	

# Research



# List of Assessment Task Groups

1. Multiplication with Repeated Equal Groups
2. Grouping Division with Repeated Equal Groups
3. Sharing Division with Repeated Equal Groups
4. Multiplication with an Array
5. Grouping Division with an Array
6. Sharing Division with an Array
7. Multiplication Basic Facts
8. Multiplication with Bare Numbers - 2-digit X 1-digit
9. Division with Bare Numbers - 2 digit Quotients
10. Inverse Relationship of Multiplication and Division
11. Commutative Principle
12. Distributive Principle



# Distributive Principle

Materials: Cards as follows:  $23 \times 3 = 69$ ,  $2 \times 3 = 6$ ,  $25 \times 3$ ,  $49 \times 3$ ,


What to do and say: Display the first three cards. *Read the first three cards. Can you use those to help you work out  $25 \times 3$ ? Read this card ( $49 \times 3$ ). Can you think of an easy way to work this out?*

The purpose of this task is to gauge the student's facility with the distributive principle.

# Observation Tool: Multiplication Strategy

Bay-Williams, J.M. & Kling, G.G (2019) Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention.

## Observation Tool: Multiplication Strategy

Names 	Multiplication Strategy Selected						
	Foundational Fact (known)	Doubling	Adding a Group	Subtracting a Group	Near Square	Break Apart Strategy	Other (e.g., skip counting)

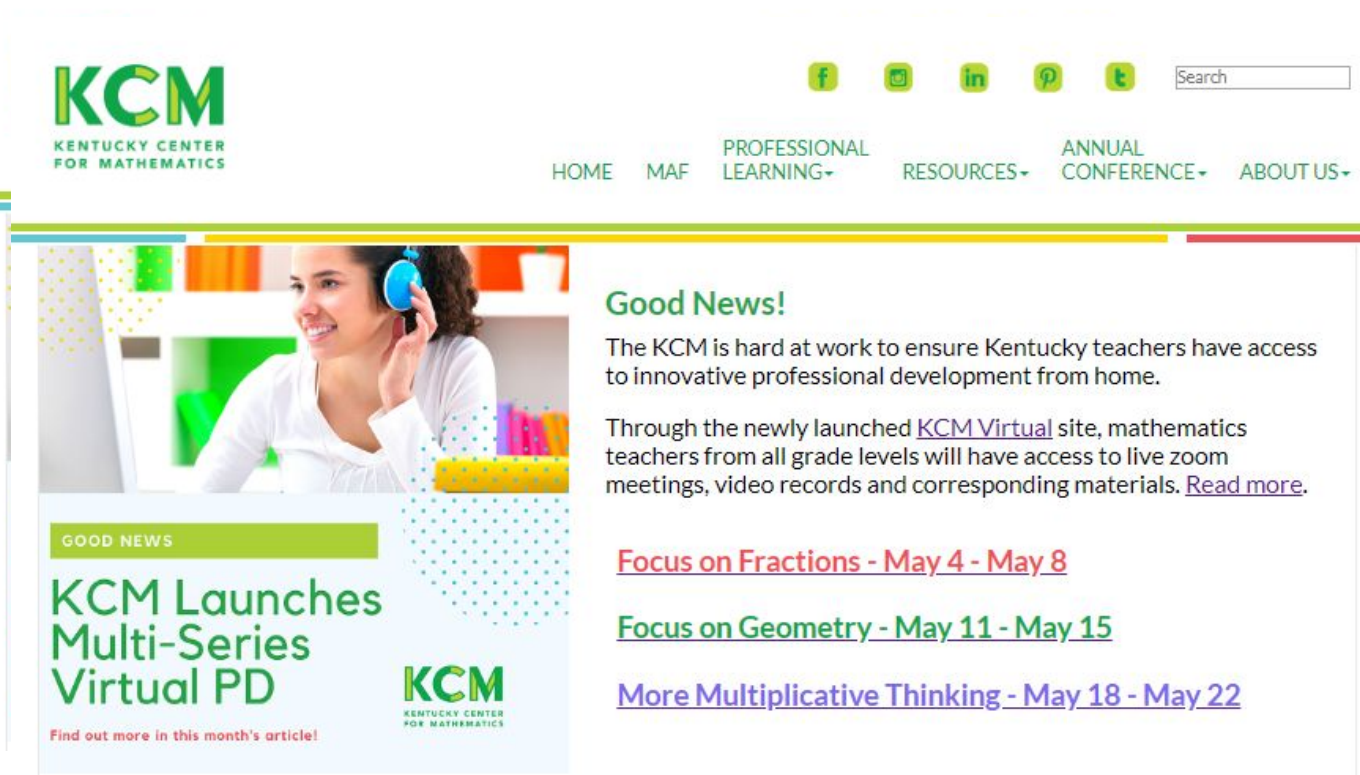
	Jotting	Semi-formal	Algorithm
<b>Add</b> $568 + 273$	$500 + 200 = 700$ $700 + 60 + 70 = 830$ $830 + 8 + 3 = 841$	$500 + 200 = 700$ $60 + 70 = 130$ $8 + 3 = 11$ $700 + 130 = 830$ $830 + 11 = 841$	$568 + 273 = 841$
<b>Subtract</b> $638 - 254$	$638 - 200 = 438$ $438 - 50 = 388$ $388 - 4 = 384$	$600 - 200 = 400$ $30 - 50 = -20$ $8 - 4 = 4$ $400 - 20 = 380$ $380 + 4 = 384$	$638 - 254 = 384$
<b>Multiply</b> $6 \times 143$	$43 \times 2 = 86$ $86 \times 2 = 172$ $172 \times 2 = 344$ $344 \times 2 = 688$ $688 \times 2 = 1376$ $1376 \times 2 = 2752$ $2752 \times 2 = 5504$ $5504 \times 2 = 11008$ $11008 \times 2 = 22016$ $22016 \times 2 = 44032$ $44032 \times 2 = 88064$ $88064 \times 2 = 176128$ $176128 \times 2 = 352256$ $352256 \times 2 = 704512$ $704512 \times 2 = 1409024$ $1409024 \times 2 = 2818048$ $2818048 \times 2 = 5636096$ $5636096 \times 2 = 11272192$ $11272192 \times 2 = 22544384$ $22544384 \times 2 = 45088768$ $45088768 \times 2 = 90177536$ $90177536 \times 2 = 180355072$ $180355072 \times 2 = 360710144$ $360710144 \times 2 = 721420288$ $721420288 \times 2 = 1442840576$ $1442840576 \times 2 = 2885681152$ $2885681152 \times 2 = 5771362304$ $5771362304 \times 2 = 11542724608$ $11542724608 \times 2 = 23085449216$ $23085449216 \times 2 = 46170898432$ $46170898432 \times 2 = 92341796864$ $92341796864 \times 2 = 184683593728$ $184683593728 \times 2 = 369367187456$ 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# Progression of Multiplication

<https://youtu.be/M7T0zNCCOQQ>

# Follow us!

## Kentucky Center for Mathematics: Home



The screenshot shows the KCM website home page. At the top left is the KCM logo with the text "KENTUCKY CENTER FOR MATHEMATICS". To the right are social media icons for Facebook, Instagram, LinkedIn, Pinterest, and Twitter, followed by a search bar. Below these are navigation links: HOME, MAF, PROFESSIONAL LEARNING, RESOURCES, ANNUAL CONFERENCE, and ABOUT US. The main content area features a large image of a woman wearing a headset, smiling. Below this image is a green banner with the text "GOOD NEWS". The main headline reads "KCM Launches Multi-Series Virtual PD" with the KCM logo to the right. Below the headline is a red link that says "Find out more in this month's article!". To the right of the image, there is a section titled "Good News!" followed by two paragraphs of text. The first paragraph states that KCM is working to ensure Kentucky teachers have access to innovative professional development from home. The second paragraph mentions the newly launched KCM Virtual site, which provides access to live zoom meetings, video records, and corresponding materials, with a red link to "Read more.". Below these paragraphs are three red links: "Focus on Fractions - May 4 - May 8", "Focus on Geometry - May 11 - May 15", and "More Multiplicative Thinking - May 18 - May 22".

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**GOOD NEWS**

**KCM Launches Multi-Series Virtual PD**

Find out more in this month's article!

**Good News!**

The KCM is hard at work to ensure Kentucky teachers have access to innovative professional development from home.

Through the newly launched [KCM Virtual](#) site, mathematics teachers from all grade levels will have access to live zoom meetings, video records and corresponding materials. [Read more.](#)

[Focus on Fractions - May 4 - May 8](#)

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Thank you for hanging out with KCM!

We are here to support all your math needs!

[Kentucky Center for Mathematics](#)

