KCM

KENTUCKY CENTER
FOR MATHEMATICS

Focus on Fractions

Multiplication and Division

Welcome!

Your host

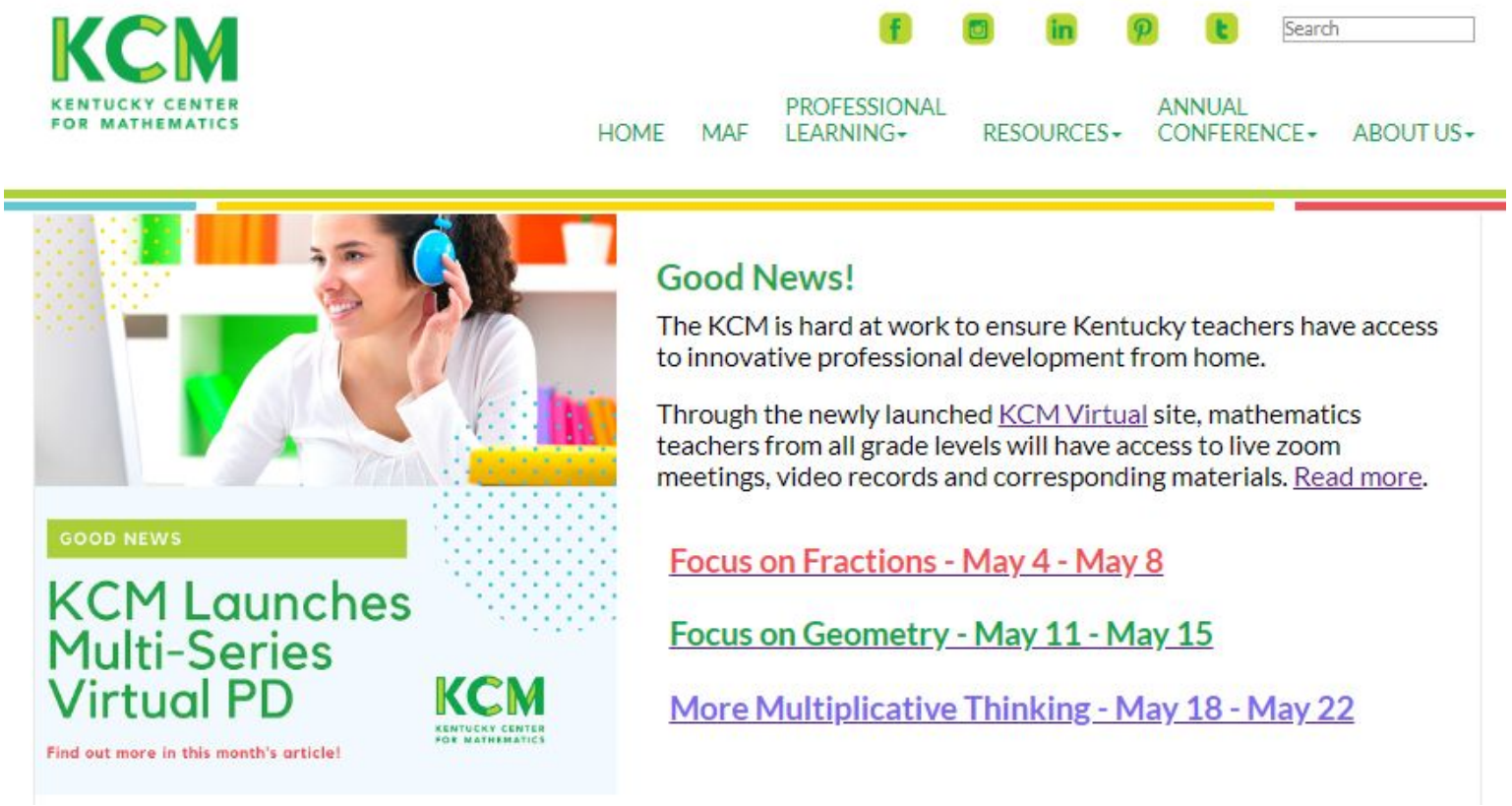
Kelly DeLong

Executive Director
Kentucky Center for Mathematics
delongk1@nku.edu

KCM
KENTUCKY CENTER
FOR MATHEMATICS

KCM Website

www.kentuckymathematics.org



KCM
KENTUCKY CENTER FOR MATHEMATICS

Agenda

- Standards
- Research
  - Neagoy
  - Lamon
  - Van de Walle
- Multiplication Progression
  - Fractions
- Division of Fractions
- Resources

KCM
KENTUCKY CENTER FOR MATHEMATICS

Activator

As we learn together today, write down 3 words that capture your learning experience today. I will ask you to share in the chat box at the end of our session.

KCM
KENTUCKY CENTER FOR MATHEMATICS

Standards

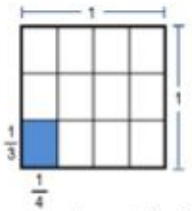
Progression- Building from 4th grade

Standard	Clarification
KY.4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. ✓ a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. ✓ b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ and use this understanding to multiply a fraction by a whole number. ✓ c. Solve word problems involving multiplication of a fraction by a whole number. **MP.5, MP.8**	Students refer this standard to *n* groups of a fraction (where *n* is a whole number) for example 3 groups of $\frac{1}{4}$, which can be seen as repeated addition. In grade 5 students will multiply a fraction by a whole number. a. Students use visual fraction models to represent $\frac{7}{5} = 7 \times \frac{1}{5}$ b. Students use the same thinking to see $3 \times \frac{2}{5}$ as $\frac{2}{5}+\frac{2}{5}+\frac{2}{5} = 3 \times \frac{2}{5} = \frac{6}{5}$ KY.4.OA.2 Coherence KY.3.NF.1→ KY.4.NF.4→KY.5.NF.4

KCM
KENTUCKY CENTER FOR MATHEMATICS

Standards

Progression- 5th grade

Standard	Clarification
KY.5.NF.4 Apply and extend previous understanding of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(\frac{a}{b}) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. b. Find the area of a rectangle with fractional side lengths by tiling it with squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas. **MP.1**	a. Students use a visual fraction model to show $(\frac{2}{3}) \times 4 = \frac{8}{3}$ and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.)  b. For example the shaded portion shows the rectangle with the appropriate unit fraction side lengths. Coherence KY.4.NF.4→ KY.5.NF.4→KY.6.G.1

KCM
KENTUCKY CENTER FOR MATHEMATICS

Standards

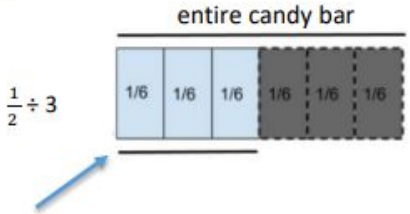
Progression- 5th grade

Standard	Clarification
KY.5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers. **MP.4, MP.5**	KY.5.MD.2 Coherence KY.4.NF.4→KY.5.NF.6

KCM
KENTUCKY CENTER FOR MATHEMATICS

Standards

Progression- 5th grade

Standard	Clarification
KY.5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. ✓ a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients. ✓ b. Interpret division of a whole number by a unit fraction and compute such quotients. ✓ c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions. **MP.1, MP.4, MP.8**	Students build upon the knowledge of division they gained in grades 3 and 4. Students connect previous understanding of division of whole numbers to divide whole numbers by unit fractions and unit fractions by whole numbers. Division of a fraction by a fraction is not a requirement at grade 5. a. Create a story context for $(\frac{1}{3}) \div 4$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(\frac{1}{3}) \div 4 = \frac{1}{12}$ because $(\frac{1}{12}) \times 4 = \frac{1}{3}$. b. Create a story context for $4 \div (\frac{1}{5})$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (\frac{1}{5}) = 20$, because $20 \times (\frac{1}{5}) = 4$. c. By using visual fraction models and equations to represent the problem.  Each child will get one piece. Half to be shared with 3 students. Coherence KY.4.NF.4→ KY.5.NF.7→KY.6.NS.1

KCM
KENTUCKY CENTER FOR MATHEMATICS

Standards

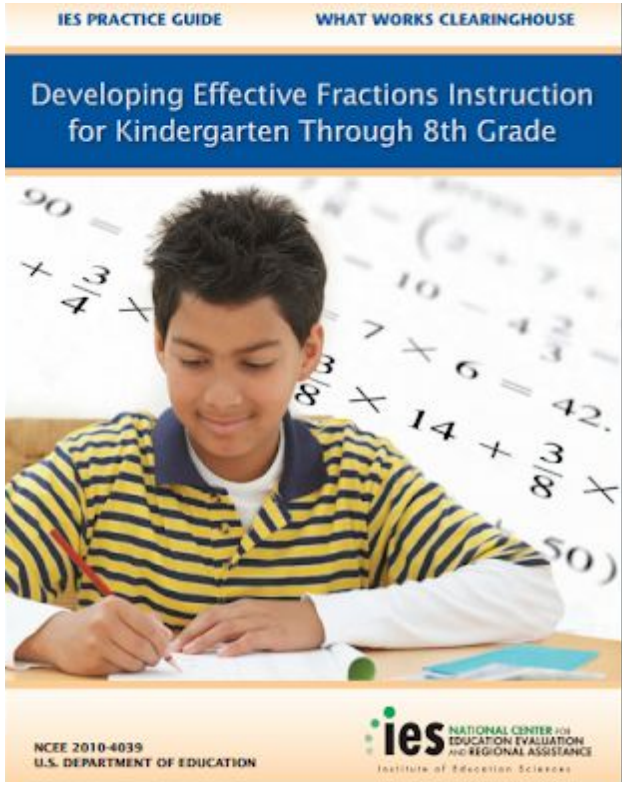
Apply and extend **previous understandings of…**

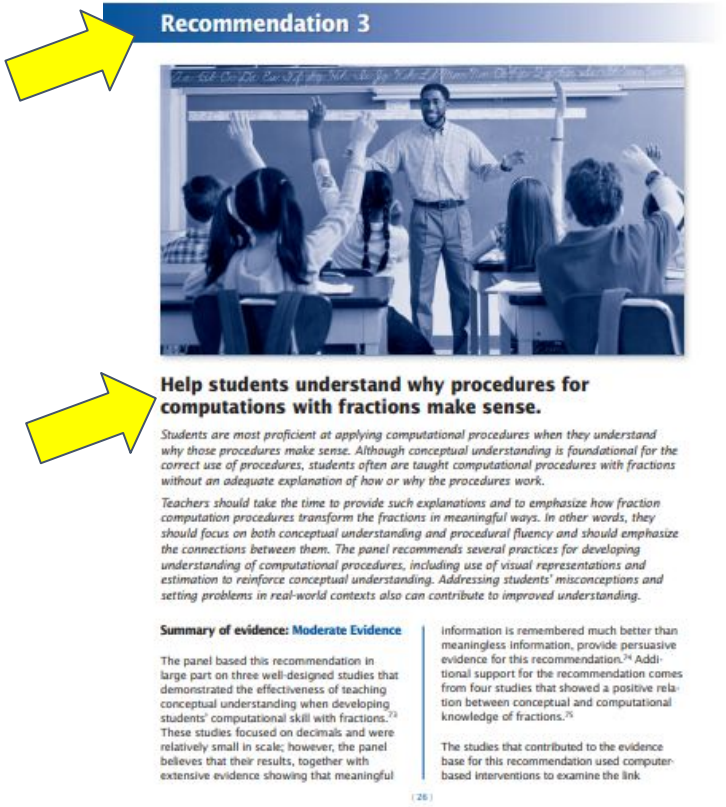
- multiplication
  - whole numbers to fractions
- division
  - whole numbers to fractions

KCM
KENTUCKY CENTER FOR MATHEMATICS

Research

Help students understand why procedures for computations with fractions make sense.





Recommendation 3

Help students understand why procedures for computations with fractions make sense.

Students are most proficient at applying computational procedures when they understand why those procedures make sense. Although conceptual understanding is foundational for the correct use of procedures, students often are taught computational procedures with fractions without an adequate explanation of how or why the procedures work.

Teachers should take the time to provide such explanations and to emphasize how fraction computation procedures transform the fractions in meaningful ways. In other words, they should focus on both conceptual understanding and procedural fluency and should emphasize the connections between them. The panel recommends several practices for developing understanding of computational procedures, including use of visual representations and estimation to reinforce conceptual understanding. Addressing students' misconceptions and setting problems in real-world contexts also can contribute to improved understanding.

Summary of evidence: Moderate Evidence

The panel based this recommendation in large part on three well-designed studies that demonstrated the effectiveness of teaching conceptual understanding when developing students' computational skill with fractions.[74] These studies focused on decimals and were relatively small in scale; however, the panel believes that their results, together with extensive evidence showing that meaningful information is remembered much better than meaningless information, provide persuasive evidence for this recommendation.[24] Additional support for the recommendation comes from four studies that showed a positive relation between conceptual and computational knowledge of fractions.[75]

The studies that contributed to the evidence base for this recommendation used computer-based interventions to examine the link

KCM
KENTUCKY CENTER FOR MATHEMATICS

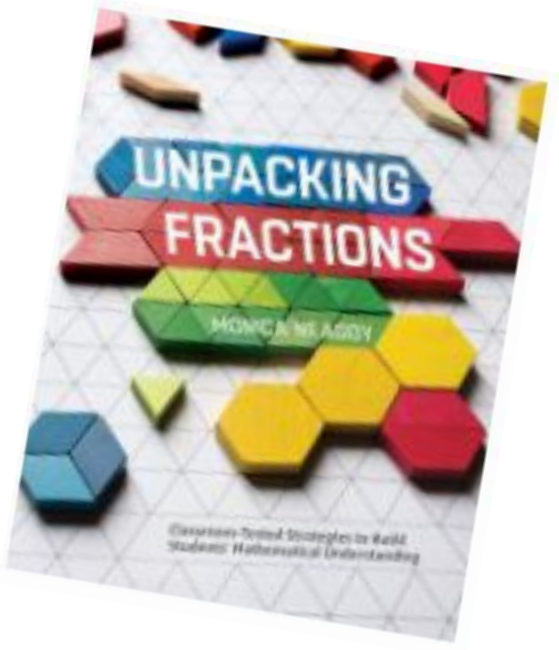
https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/fractions_pg_093010.pdf

Research

Dr. Monica Neagoy

- Teach meaning first, algorithms last.
- I have learned important lessons from practice and research about the long-standing rule-based approach to teaching fractions.
- Students need time to understand fractions, and teachers must be patient and give them time.
- In particular, student need ample experience with constructing new meaning for fractional symbols, working with concrete and visual representations of fractions, connecting these representations, and developing a good sense of the kinds of quantities the symbols may denote.

pg. 260



http://www.ascd.org/publications/books/115071.aspx

KCM
KENTUCKY CENTER FOR MATHEMATICS

Apply and Extend

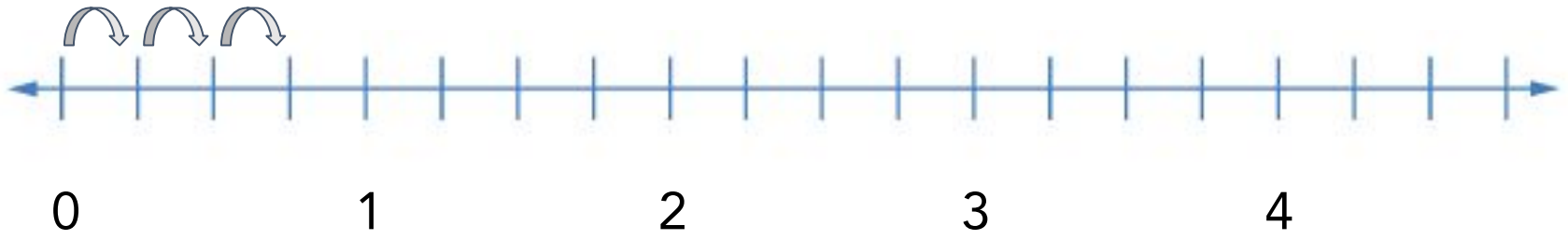
Meaning of Multiplication and Division	
Multiplication	**Division**
1. Adding equal groups.	1a. Equal or fair sharing (partitive concept)
	1b. Equal grouping or segmenting (quotative, measurement, or repeated subtraction).
2. Increasing or reducing quantities	
3. Moving from factors to product or product to factor	

KCM
KENTUCKY CENTER FOR MATHEMATICS

Unpacking Fractions, Neagoy, pg 201

Extending From Whole Number

Skip Counting

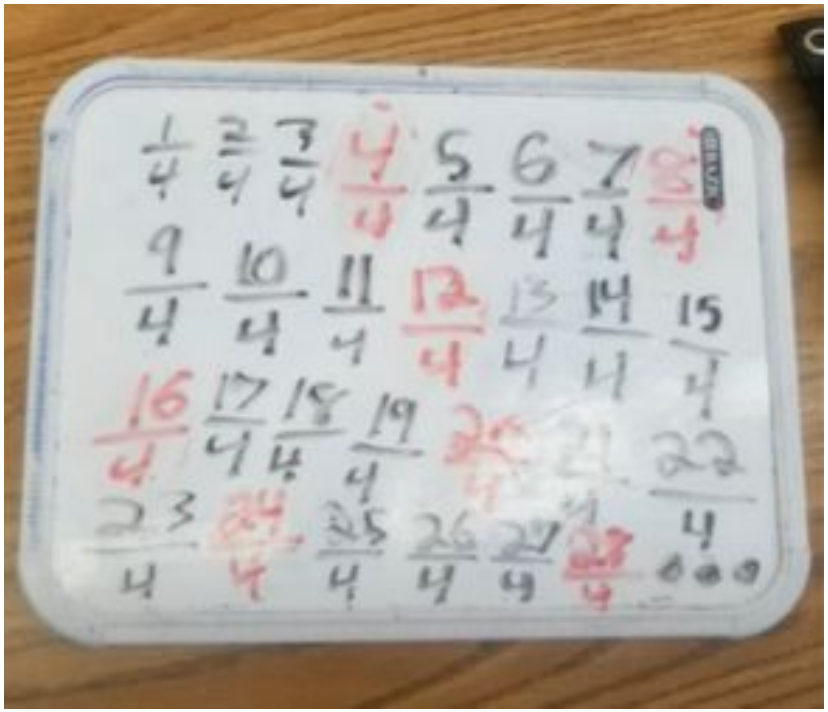


$\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

KCM
KENTUCKY CENTER FOR MATHEMATICS

Extending From Whole Number Multiplication

Skip Counting Activity

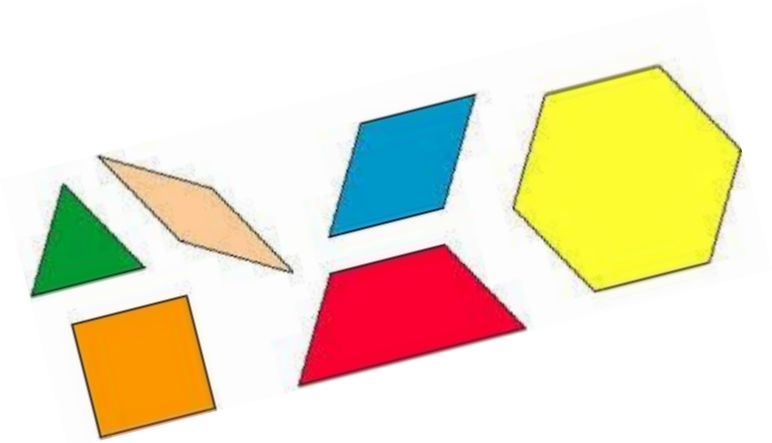


What evidence do you have that students are making sense the skip counting sequence?

KCM
KENTUCKY CENTER FOR MATHEMATICS

Extending From Whole Number Multiplication

Pattern blocks make an excellent tool for teaching multiplication of fractions.



http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

Multiplying Fractions with Pattern Blocks

The yellow hexagon is equal to 1 whole. Write an addition equation using unit fractions for each given set of pattern blocks. Next, write a multiplication equation to represent the problem and show the total value of each.

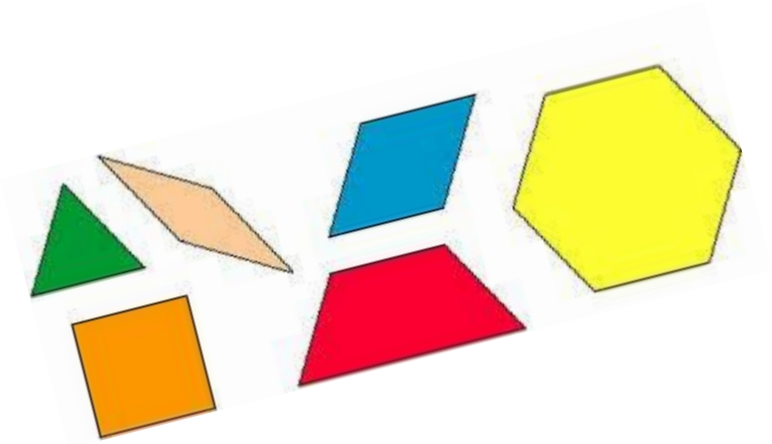
- 4 rhombuses
- 3 hexagons
- 9 triangles
- 4 triangles
- 11 trapezoids

KCM
KENTUCKY CENTER FOR MATHEMATICS

www.tools4ncteachers.com

Extending From Whole Number Multiplication

Pattern blocks make an excellent tool for teaching multiplication of fractions.



http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

Exit Ticket

1. If a trapezoid equals one whole, write an addition equation and multiplication equation for 4 green triangles.

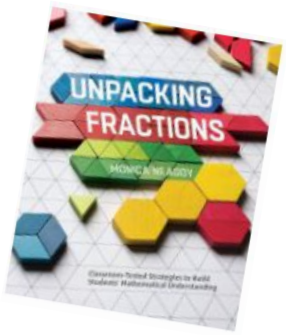
2. Write a multiplication sentence to match ¼ + ¼ + ¼.

KCM
KENTUCKY CENTER FOR MATHEMATICS

www.tools4ncteachers.com

Extending From Whole Number Multiplication

Using context to build understanding of fraction multiplication.



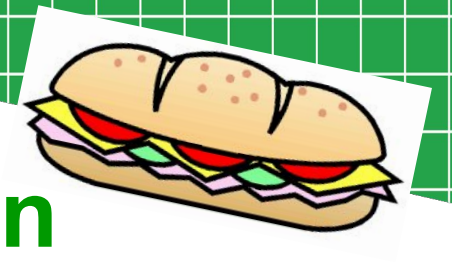
On a field trip, a teacher brought some large sandwiches to for her nine students. Each student got ⅔ of a sandwich. How many sandwiches did the teacher bring?

Neagoy, pg 195

KCM
KENTUCKY CENTER FOR MATHEMATICS

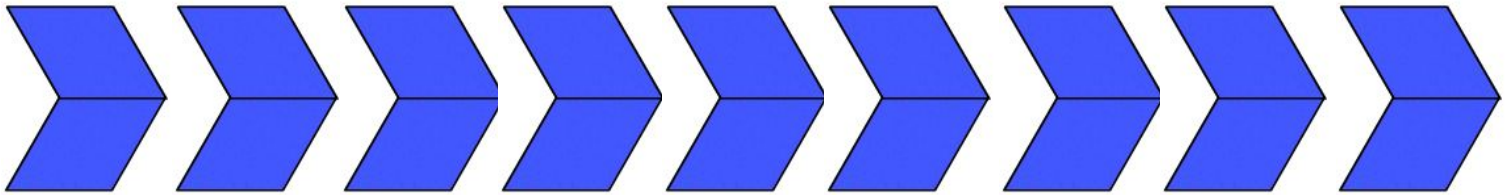
http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

Extending From Whole Number Multiplication



On a field trip, a teacher brought some large sandwiches to for her nine students. Each student got ⅔ of a sandwich. How many sandwiches did the teacher bring?

Neagoy, pg 195



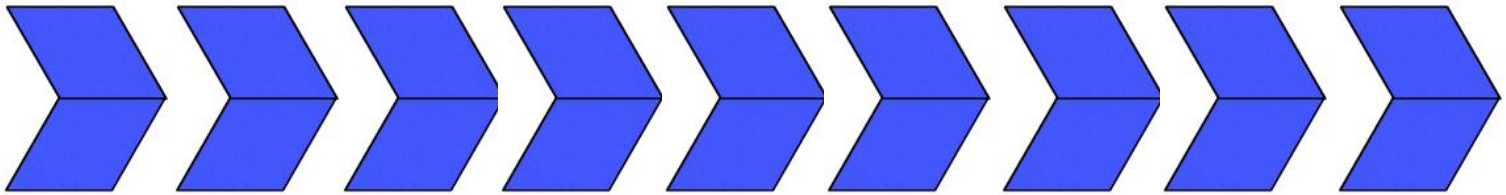
KCM
KENTUCKY CENTER FOR MATHEMATICS

http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

Extending From Whole Number Multiplication

On a field trip, a teacher brought some large sandwiches to for her nine students. Each student got ⅔ of a sandwich. How many sandwiches did the teacher bring?

Neagoy, pg 195



⅔ + ⅔ + ⅔ +⅔ + ⅔ + ⅔ + ⅔ + ⅔ + ⅔ = 18/3

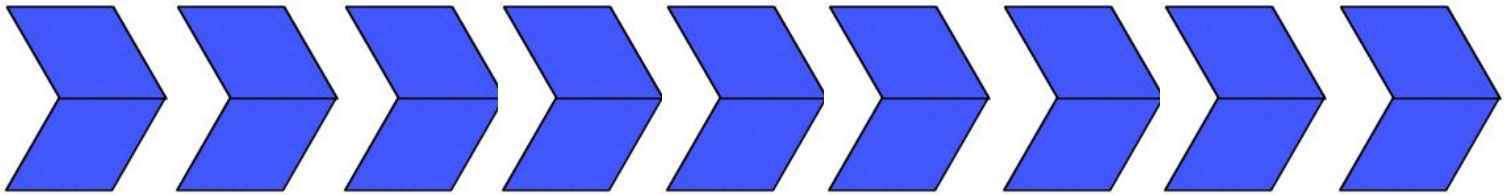
KCM
KENTUCKY CENTER FOR MATHEMATICS

http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

Extending From Whole Number Multiplication

On a field trip, a teacher brought some large sandwiches to for her nine students. Each student got ⅔ of a sandwich. How many sandwiches did the teacher bring?

Neagoy, pg 195



⅔ x 9 = 18/3

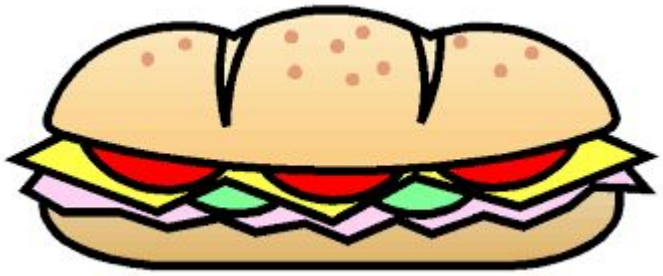
KCM
KENTUCKY CENTER FOR MATHEMATICS

http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

Extending From Whole Number Multiplication

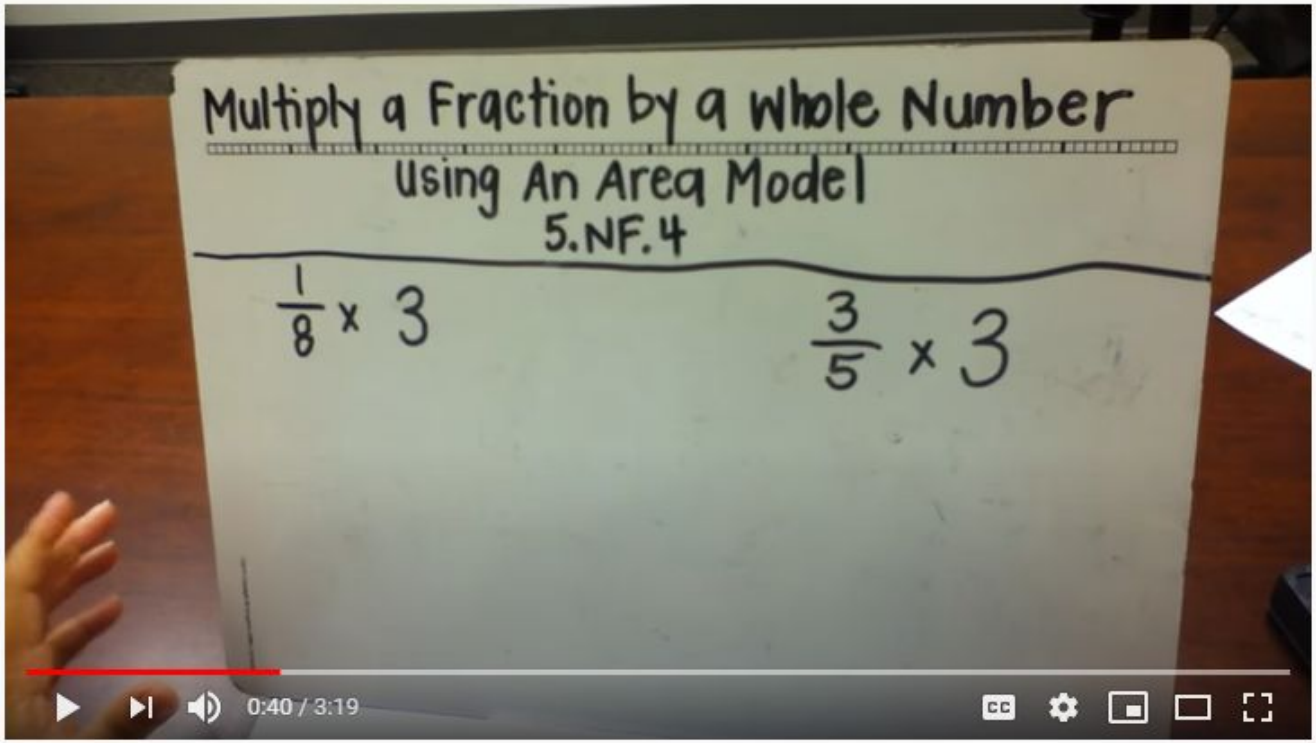
How does linking a real world context with the visual image help kids construct meaning of multiplication of fractions?

Type your answer in the chat or unmute to share your thinking.



KCM
KENTUCKY CENTER FOR MATHEMATICS

Multiplication of Fractions

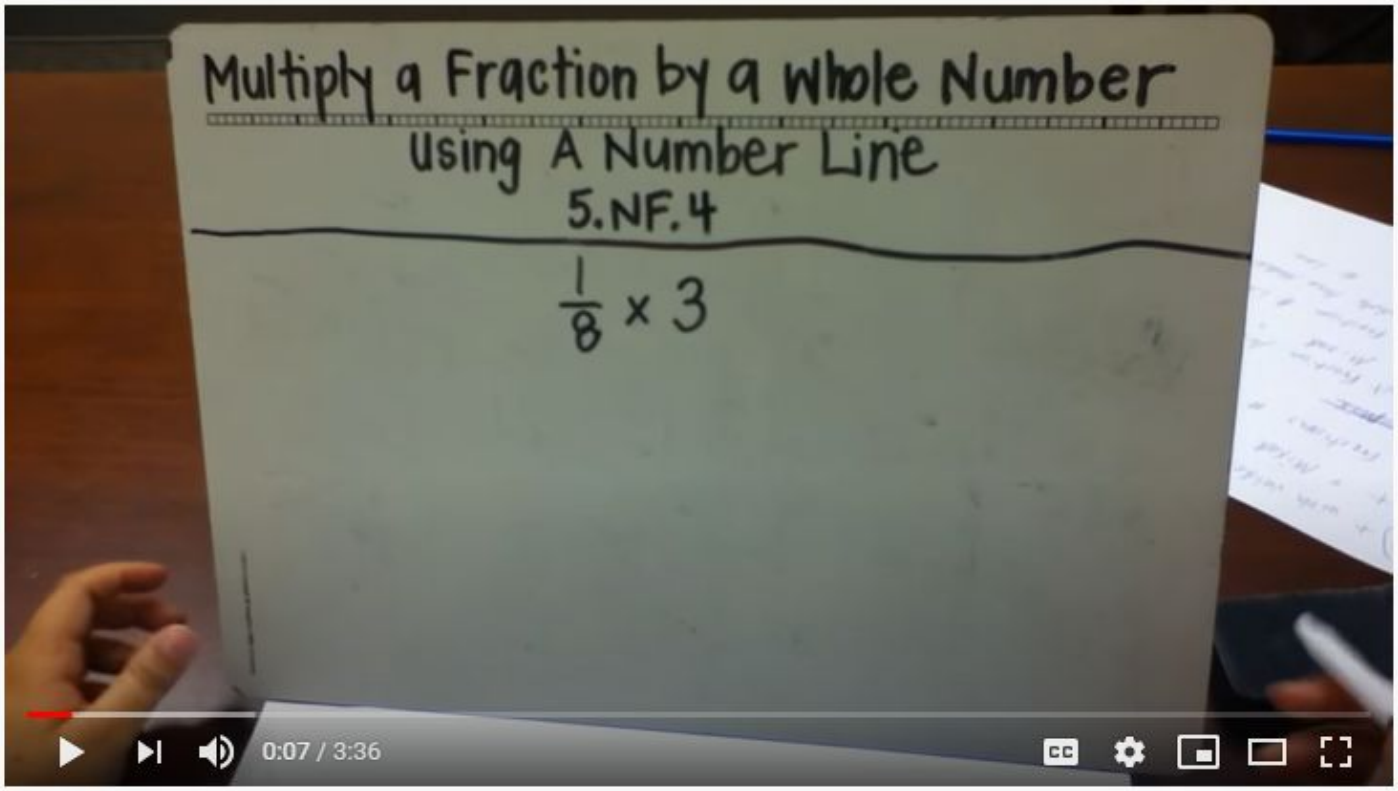


ES 5 Math Multiply Fraction X Whole with Area Models

https://www.youtube.com/watch?v=Cy7I-wT49Y4

KCM
KENTUCKY CENTER FOR MATHEMATICS

Multiplication of Fractions



ES 5 Math Multiply Fraction X Whole with Number Lines

https://www.youtube.com/watch?v=5Tx15t8jGZQ&t=86s

KCM
KENTUCKY CENTER FOR MATHEMATICS

Multiplication of Fractions

Blueberry Pancake Party
Activity Sheet

Todd is famous for his homemade blueberry pancakes. His recipe makes enough to serve 6 people. He has invited 24 people to enjoy his pancakes.

Recipe (http://allrecipes.com/recipe/20177/todds-famous-blueberry-pancakes/):

- $1\frac{1}{2}$ cup all-purpose flour
- $\frac{1}{2}$ teaspoon of salt
- 1 tablespoon of baking powder
- $1\frac{1}{4}$ tsp of sugar
- 1 egg
- 1 cup milk
- $\frac{1}{2}$ tablespoon of butter melted
- $\frac{1}{2}$ cup frozen blueberries (thawed)

How much of each ingredient would he need to give everyone a serving of pancakes?

fraction by a whole number

KCM
KENTUCKY CENTER FOR MATHEMATICS

www.tools4ncteachers.com

Multiplication of Fractions

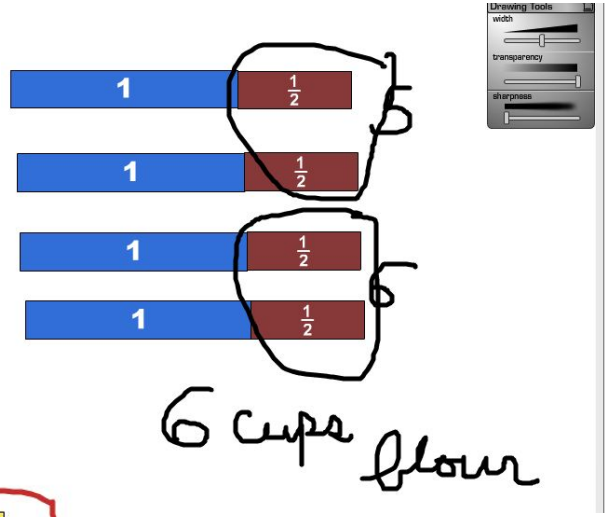
Blueberry Pancake Party
Activity Sheet

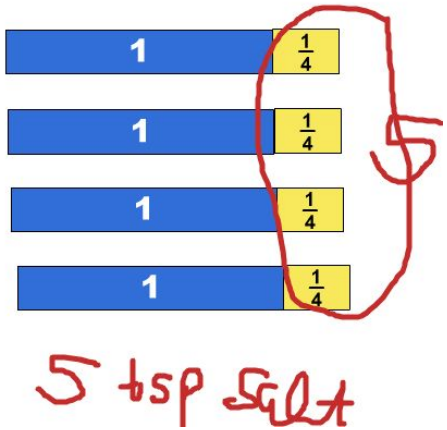
Todd is famous for his homemade blueberry pancakes. His recipe makes enough to serve 6 people. He has invited 24 people to enjoy his pancakes.

Recipe (http://allrecipes.com/recipe/20177/todds-famous-blueberry-pancakes/):

- $1\frac{1}{2}$ cup all-purpose flour
- $\frac{1}{2}$ teaspoon of salt
- 1 tablespoon of baking powder
- $1\frac{1}{4}$ tsp of sugar
- 1 egg
- 1 cup milk
- $\frac{1}{2}$ tablespoon of butter melted
- $\frac{1}{2}$ cup frozen blueberries (thawed)

How much of each ingredient would he need to give everyone a serving of pancakes?





fraction by a whole number

KCM
KENTUCKY CENTER FOR MATHEMATICS

www.tools4ncteachers.com

Multiplication of Fractions

Zach's Zoo Adventure

Building Fluency: multiply whole number by a fraction

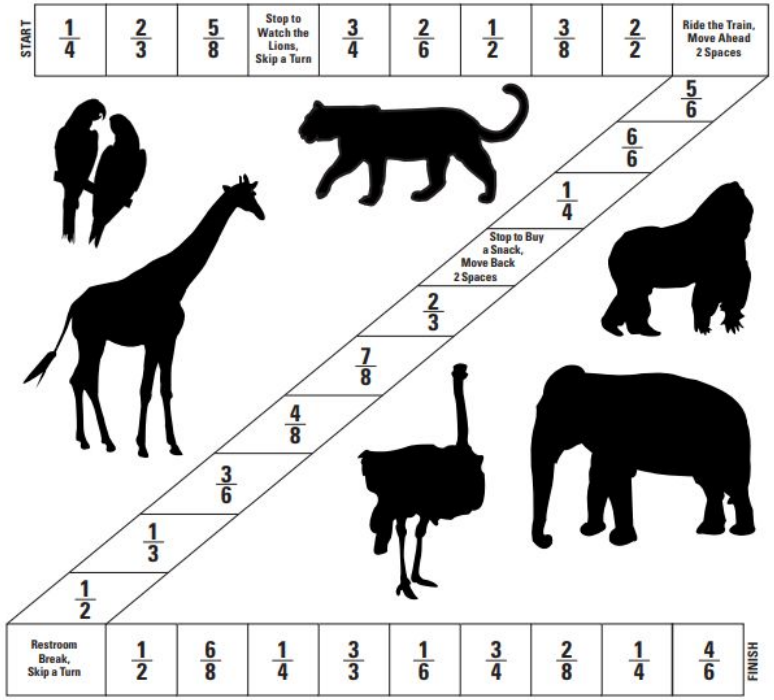
Materials: die, gameboard, game marker, calculator -optional

Number of Players: 2-4

Directions: Zach is visiting the Asheboro Zoo for the day. He needs your help to navigate his way through the zoo.

1. All players begin on "Start". Player 1 rolls the die and multiplies the digit on the die by the fraction their game piece is on.
2. If it is correct (may use calculator to check your work) Player 1 moves forward the number of spaces shown on the die.
3. Players take turns rolling the die and multiplying the digit on the die by the fraction their game piece is on.
4. The first player to cross the finish line wins. Play until every player crosses the finish line. You've helped Zach visit the entire zoo!

Variation/Extension: Students can create their own gameboard and/or use a die with larger numbers. Student can record their work on a piece of paper of in math notebook.



fraction by a whole number

https://tools4ncteachers.com/resources/district-leaders/documents/4thgrade-GAMES.pdf

KCM
KENTUCKY CENTER FOR MATHEMATICS

Multiplication of Fractions

Parts of a Whole

Building Fluency: multiplication of whole number by a fractions

Materials: whole number die (1-6), fraction circle, and fraction cards or fraction die or spinner

Number of Players: 2

Directions:

1. Player rolls a standard whole number die, and spins the spinner.
2. The standard die represents the number of groups, and the spinner represents the fraction in each group.
   Example: A roll of 3 on the standard die, and spin $\frac{1}{4}$ on the spinner would be represented 3 groups with $\frac{1}{4}$.
3. Use fraction circles to help determine the product for each round.
4. If your result is 1 or more, you receive a star.
5. Play several rounds and count the stars you have collected.
6. The player with the most stars collected is the winner.

Variation/Extension: Student may want to modify fractions on spinner or use a die 0-9. A blank spinner and fraction circles are added for your convenience. Teacher may also want students to add the products. Students may want to write coordinating problems to fit each equation.

PLAYER 1

ROLL	SPIN	EQUATION

PLAYER 2

ROLL	SPIN	EQUATION

fraction by a whole number

KCM
KENTUCKY CENTER FOR MATHEMATICS

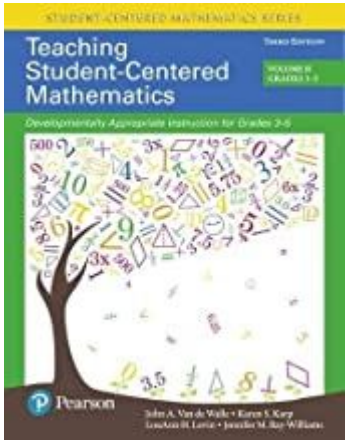
https://tools4ncteachers.com/resources/district-leaders/documents/4thgrade-GAMES.pdf

Multiplication of Fractions

How Big is the Banner?

6 ft

1 ft



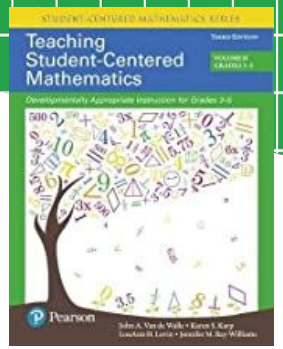
whole number by a fraction

Start with the known.
Numbers that are familiar.

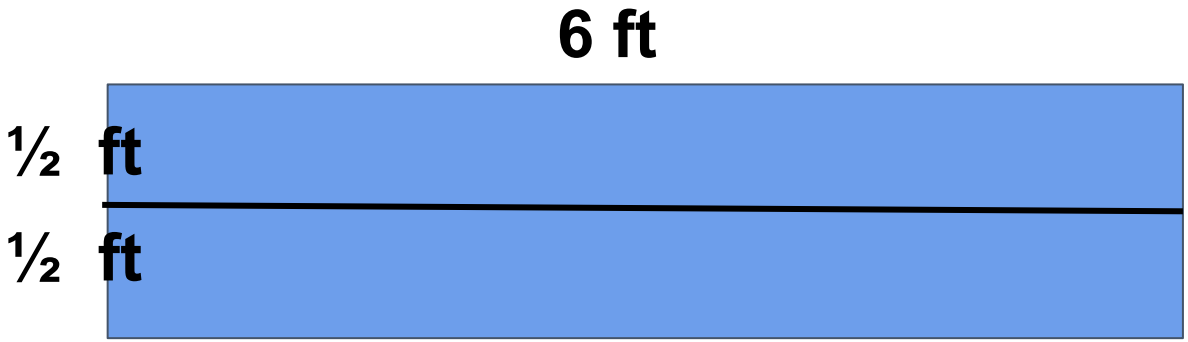
KCM
KENTUCKY CENTER FOR MATHEMATICS

Van de Walle, J. A., Karp, K. S., Lovin, L. H., & Bay-Williams, J. M. (2018). *Teaching student-centered mathematics*. New York, NY: Pearson Education.

Multiplication of Fractions



How Big is the Banner?



6 x ½

Start with the known.
Numbers that are familiar.

whole number by a fraction

KCM
KENTUCKY CENTER FOR MATHEMATICS

Van de Walle, J. A., Karp, K. S., Lovin, L. H., & Bay-Williams, J. M. (2018). *Teaching student-centered mathematics*. New York, NY: Pearson Education.

Multiplication of Fractions

whole number by a fraction

Formative Instructional and Assessment Tasks

Training for a 5K

Molly is training for a 5K. Her goal is to run 10 miles by the end of this week. On Monday, the distance she runs is ¼ of her goal.

Part 1: How far did Molly run on Monday? Draw a model to show your work.

Part 2: After Monday's run, how far is Molly away from her goal?

Part 3: If Molly runs this same distance each day, when will she reach her goal?

KCM
KENTUCKY CENTER FOR MATHEMATICS

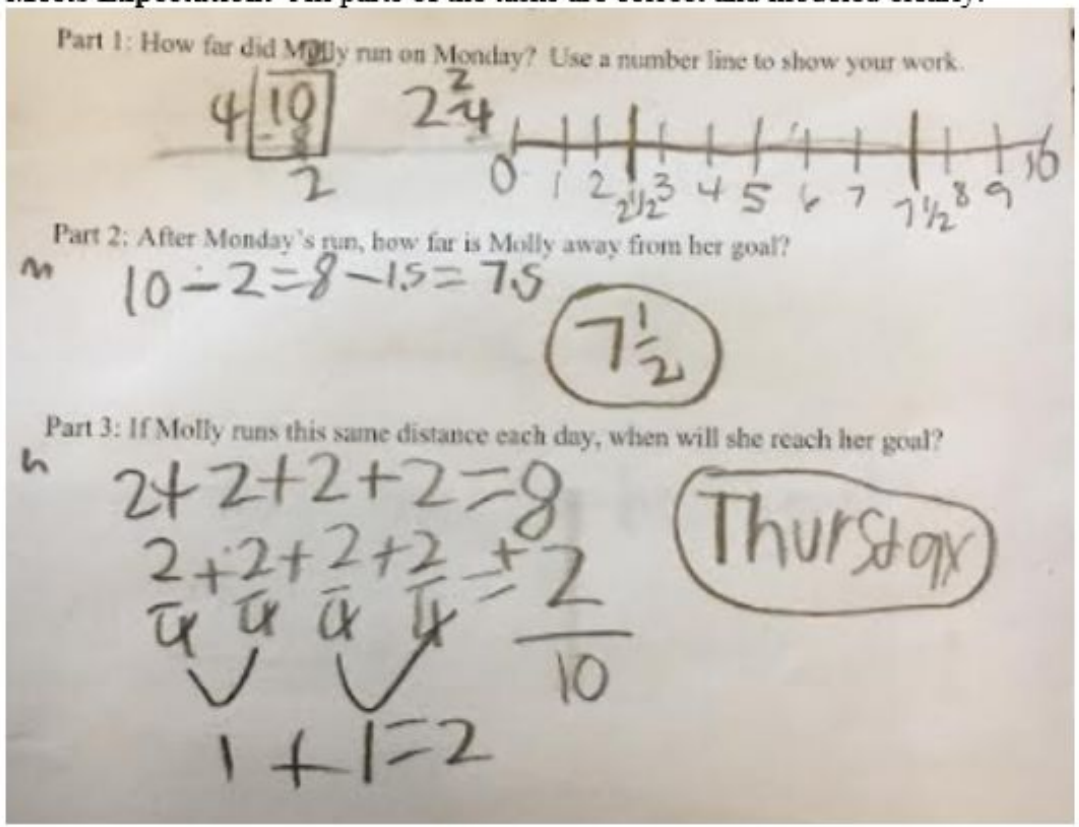
www.tools4ncteachers.com

whole number by a fraction

Multiplication of Fractions

Formative Instructional and Assessment Tasks

Meets Expectation: All parts of the tasks are correct and modeled clearly.



KCM KENTUCKY CENTER FOR MATHEMATICS

www.tools4ncteachers.com

whole number by a fraction

Multiplication of Fractions

Formative Instructional and Assessment Tasks

Chris's Cookies

On Saturday, Chris bakes a batch of 24 cookies. He uses ¾ of the batch of cookies in treat bags for his birthday party.

Part 1:
How many cookies did Chris use in treat bags? Draw a picture and write a sentence to explain your strategy.

KCM
KENTUCKY CENTER FOR MATHEMATICS

whole number by a fraction

Multiplication of Fractions

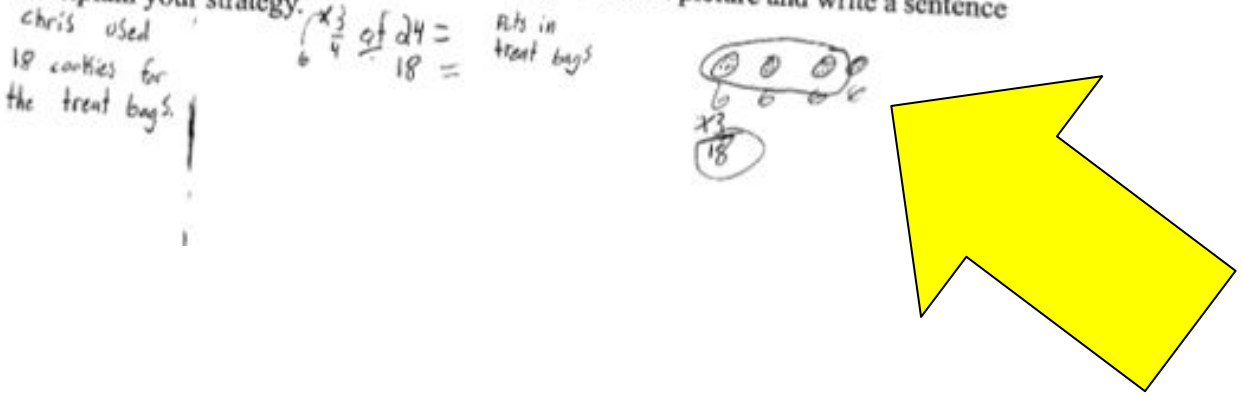
Formative Instructional and Assessment Tasks

Meets Expectation: The student correctly solves all aspects of the task, and explanation is clear and accurate.

On Saturday, Chris bakes a batch of 24 cookies. He uses ¾ of the batch of cookies in treat bags for his birthday party.

Part 1:

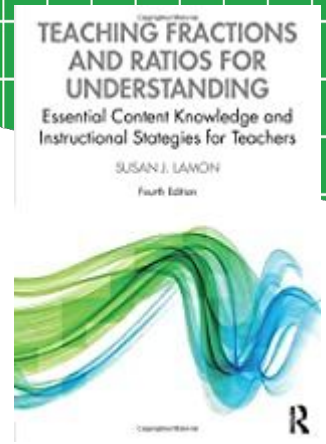
How many cookies did Chris use in treat bags? Draw a picture and write a sentence to explain your strategy.



KCM KENTUCKY CENTER FOR MATHEMATICS

www.tools4ncteachers.com

Multiplication of Fractions



Lamon calls this..
fractions as operators.

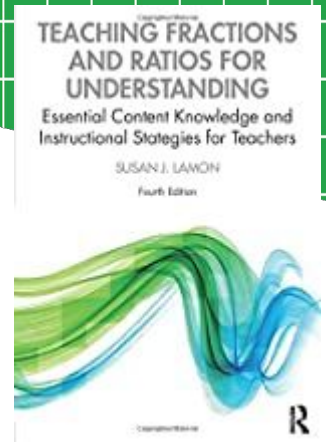
Operators are transformers that

- lengthen or shorten line segments.
- increase or decrease the number of items in a discrete set of objects
- take a figure in the geometric plane, such as a rectangle, and maps it onto a larger or smaller figure of the same shape.

Lamon, pg 201

KCM
KENTUCKY CENTER FOR MATHEMATICS

Multiplication of Fractions



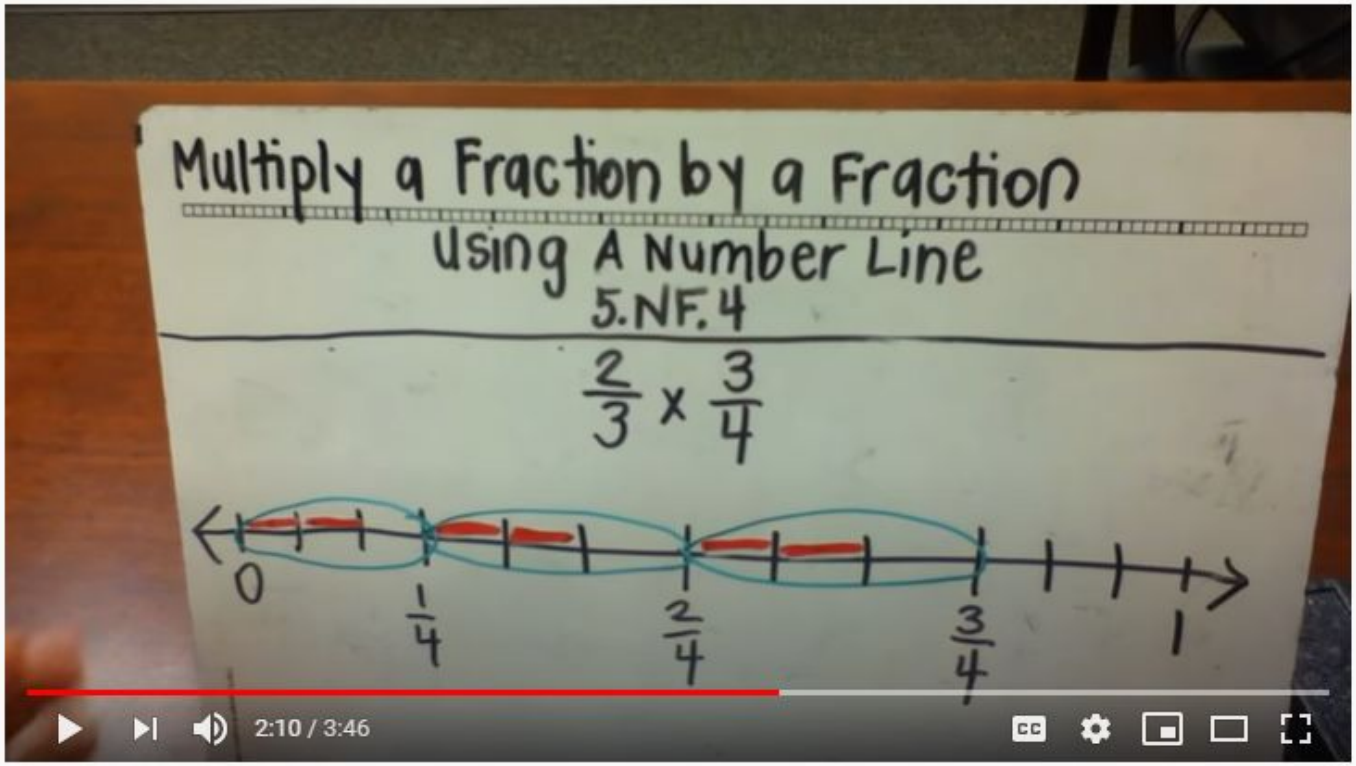
Troy has 1 ⅖ as many baseball cards as I have. I have 55 cards. How many does Troy have?

I canned 40 pounds of tomatoes last year. Jan did 5/6 as many. How many pounds did Jan can?

How did you use fractions as an operator?

KCM
KENTUCKY CENTER FOR MATHEMATICS

Multiplication of Fractions

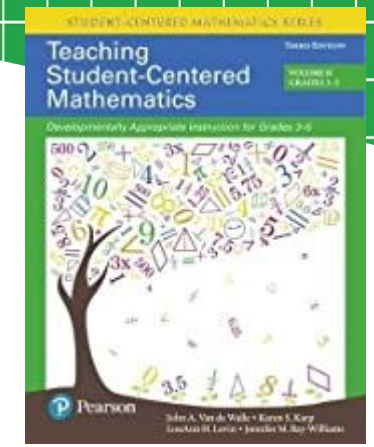


ES 5 Math Multiply Fraction X Fraction using Number Lines

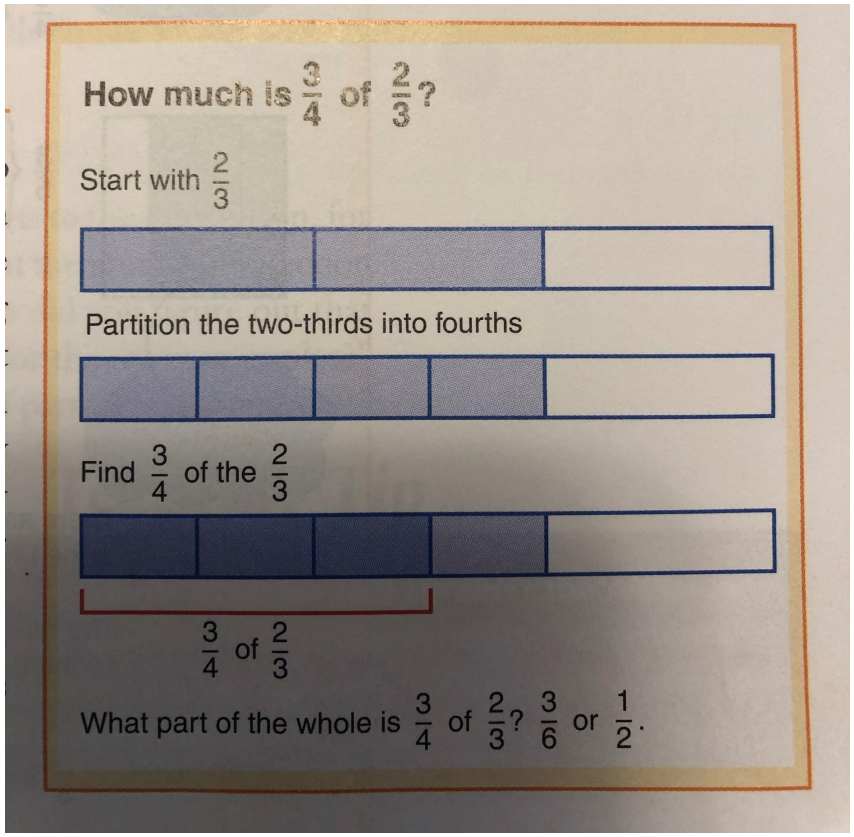
https://www.youtube.com/watch?v=2uDMVsAtvAw&t=92s

KCM
KENTUCKY CENTER FOR MATHEMATICS

Multiplication of Fractions



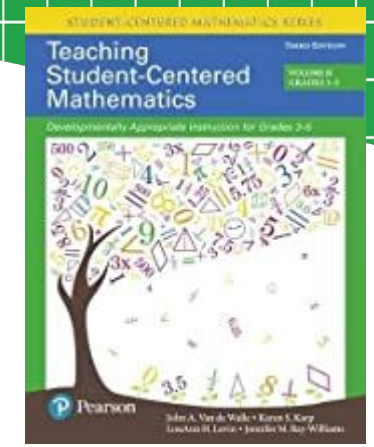
Area Model of Multiplication



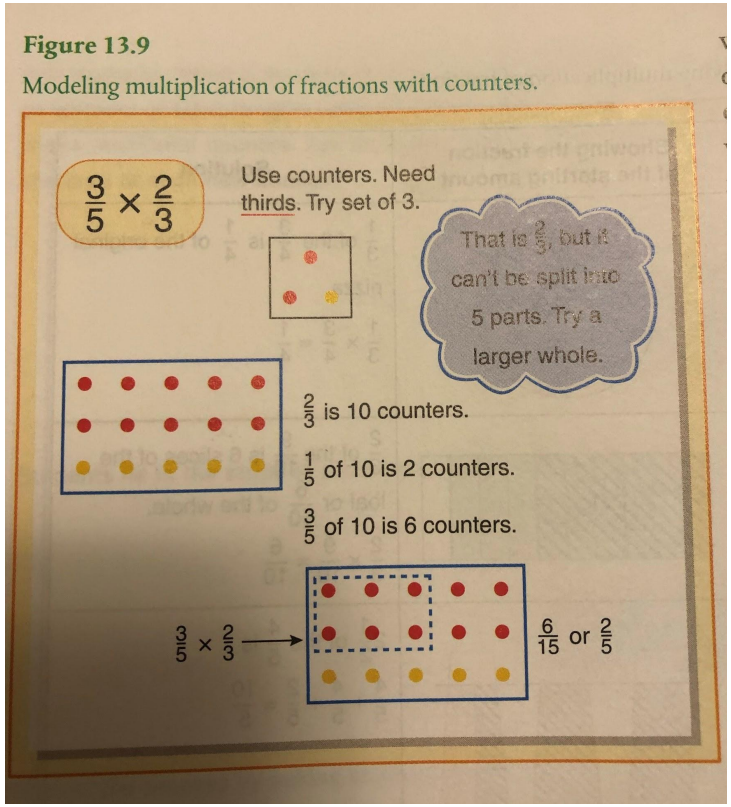
KCM
KENTUCKY CENTER FOR MATHEMATICS

Van de Walle, J. A., Karp, K. S., Lovin, L. H., & Bay-Williams, J. M. (2018). *Teaching student-centered mathematics*. New York, NY: Pearson Education.

Multiplication of Fractions



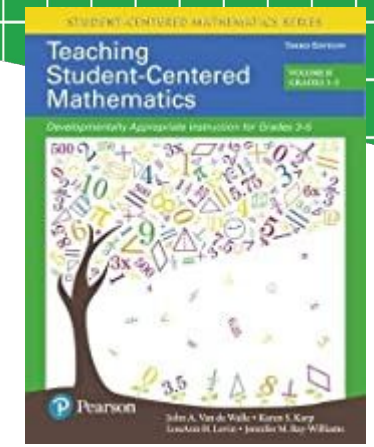
Area Model of Multiplication



KCM
KENTUCKY CENTER FOR MATHEMATICS

Van de Walle, J. A., Karp, K. S., Lovin, L. H., & Bay-Williams, J. M. (2018). *Teaching student-centered mathematics*. New York, NY: Pearson Education.

Multiplication of Fractions



Area Model of Multiplication

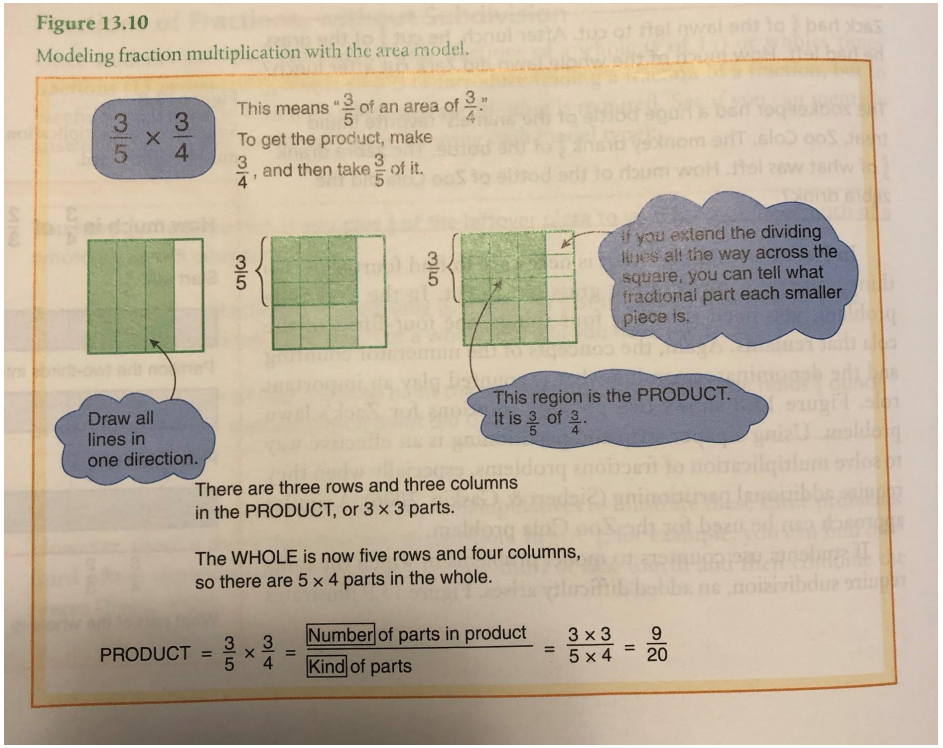


Figure 13.10

Modeling fraction multiplication with the area model.

$\frac{3}{5} \times \frac{3}{4}$

This means "$\frac{3}{5}$ of an area of $\frac{3}{4}$."

To get the product, make $\frac{3}{4}$, and then take $\frac{3}{5}$ of it.

$\frac{3}{5}$ $\frac{3}{5}$

If you extend the dividing lines all the way across the square, you can tell what fractional part each smaller piece is.

Draw all lines in one direction.

This region is the PRODUCT. It is $\frac{3}{5}$ of $\frac{3}{4}$.

There are three rows and three columns in the PRODUCT, or 3 × 3 parts.

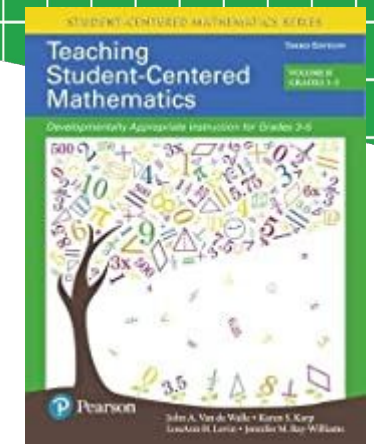
The WHOLE is now five rows and four columns, so there are 5 × 4 parts in the whole.

$$\text{PRODUCT} = \frac{3}{5} \times \frac{3}{4} = \frac{\boxed{\text{Number}}\text{ of parts in product}}{\boxed{\text{Kind}}\text{ of parts}} = \frac{3 \times 3}{5 \times 4} = \frac{9}{20}$$

Van de Walle, J. A., Karp, K. S., Lovin, L. H., & Bay-Williams, J. M. (2018). *Teaching student-centered mathematics*. New York, NY: Pearson Education.

KCM
KENTUCKY CENTER FOR MATHEMATICS

Multiplication of Fractions



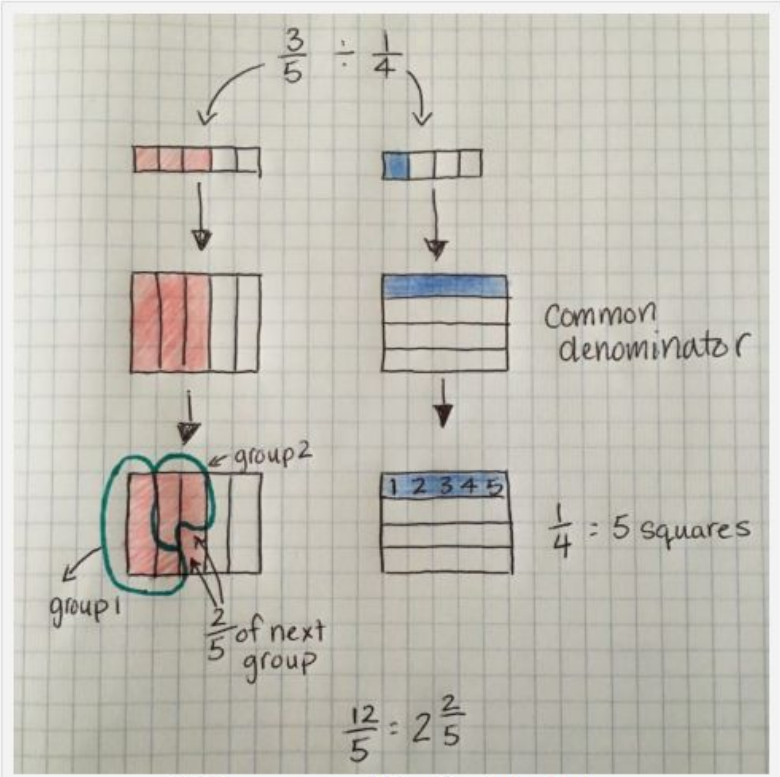
Area Model of Multiplication

Teaching Tip

Some textbooks make this sliced-rectangle approach mechanical, such that it actually becomes a meaningless algorithm in itself. Students are told to shade horizontally to show the first factor and shade vertically for the second factor. Without a rationale, they are told that the product is the region that is double-shaded. Such strategies are without meaning and the same as giving students rules to memorize.

Van de Walle, J. A., Karp, K. S., Lovin, L. H., & Bay-Williams, J. M. (2018). *Teaching student-centered mathematics*. New York, NY: Pearson Education.

Division of Fractions
Graham Fletcher

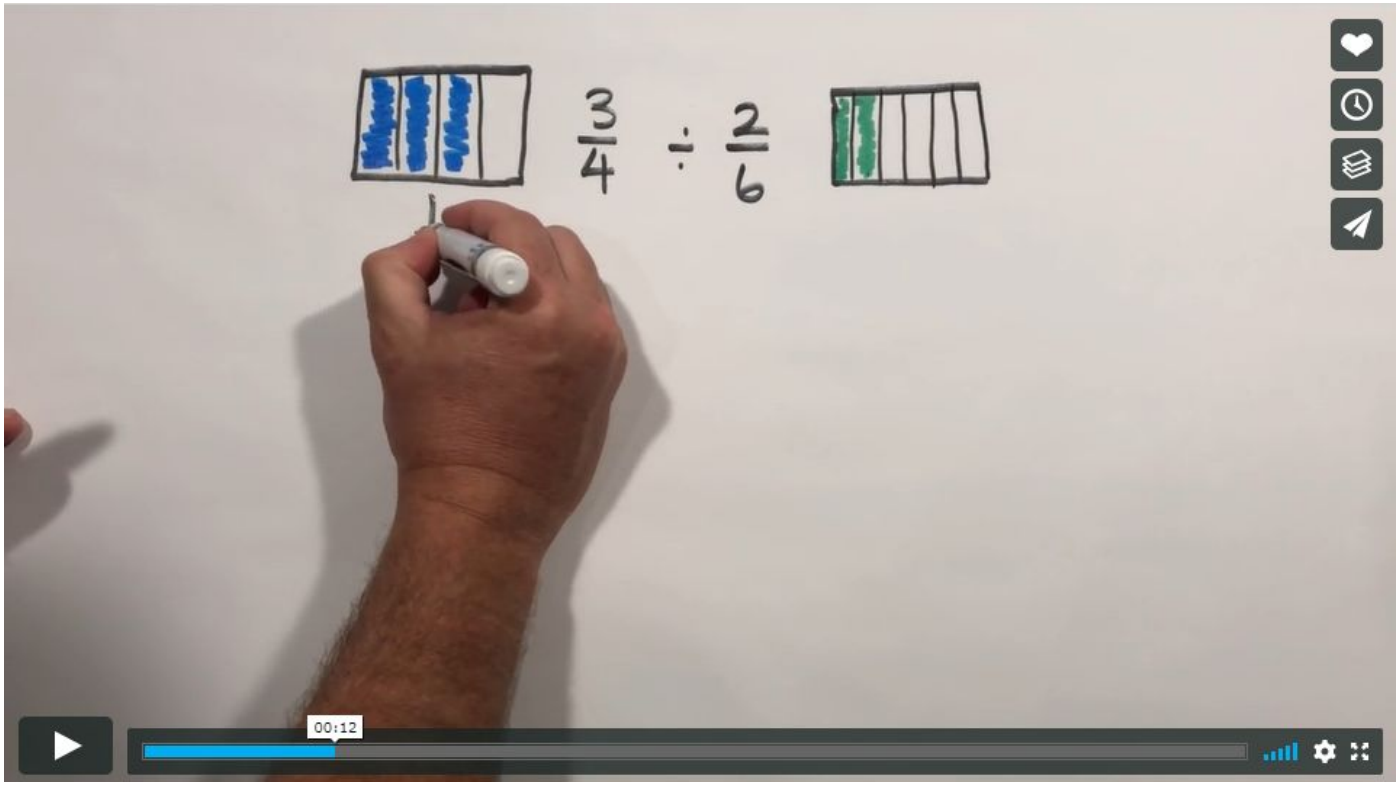


Modeling measurement division of fraction by a fraction.

https://gfletchy.com/2016/08/02/making-sense-of-invert-and-multiply/

KCM
KENTUCKY CENTER FOR MATHEMATICS

Division of Fractions
Graham Fletcher



https://vimeo.com/335263104

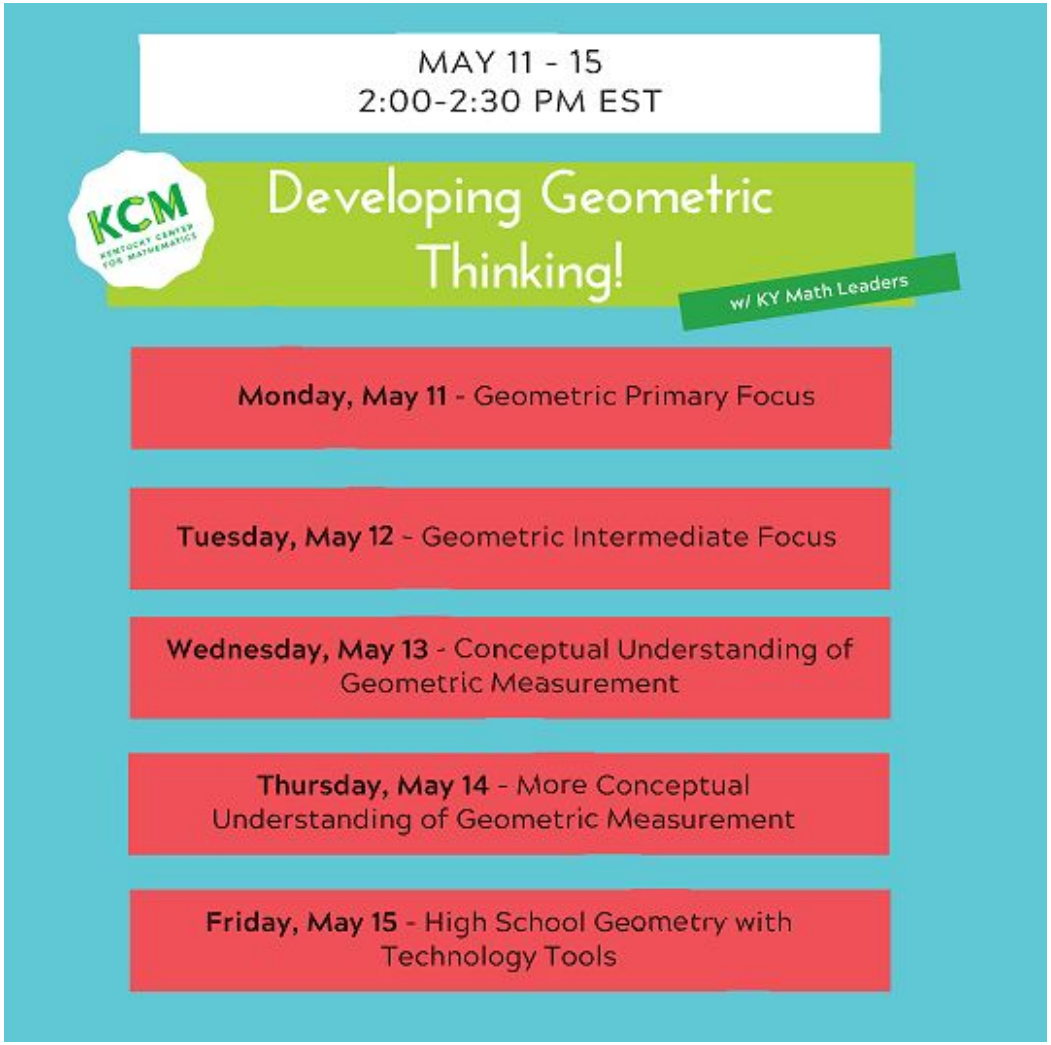
KCM
KENTUCKY CENTER
FOR MATHEMATICS

Summarizer

As we learn together today, write down 3 words that capture your learning experience today. I will ask you to share in the chat box at the end of our session.

KCM
KENTUCKY CENTER FOR MATHEMATICS

Developing Geometric Thinking



KCM
KENTUCKY CENTER FOR MATHEMATICS

Follow Us!

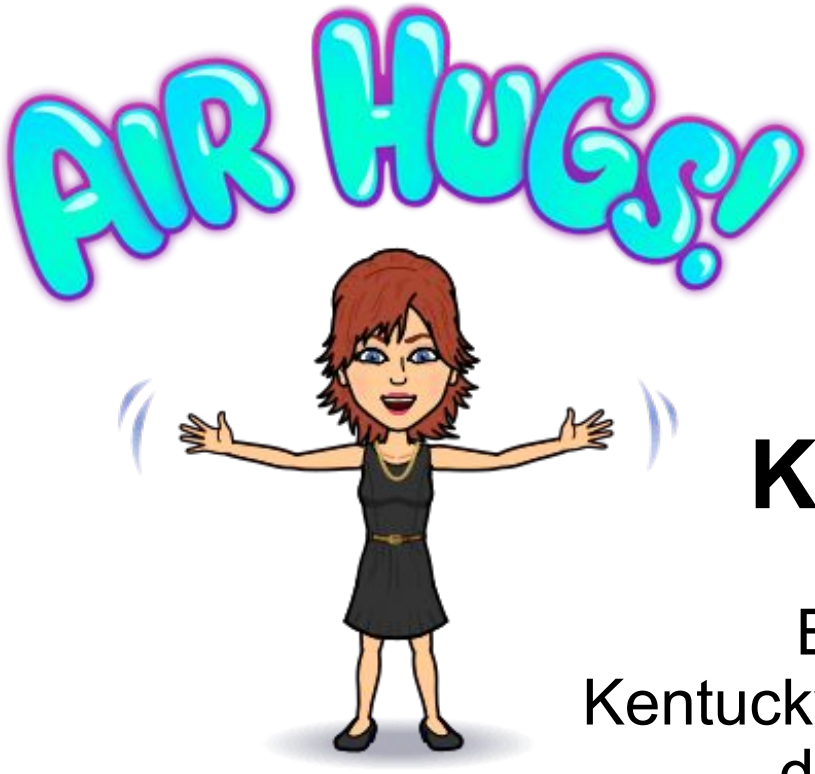
https://www.kentuckymathematics.org/kcm_virtual.php

@KyMath

@KyCenterforMath

KCM
KENTUCKY CENTER FOR MATHEMATICS

KCM is here to support you!



Contact me

Kelly DeLong

Executive Director
Kentucky Center for Mathematics
delongk1@nku.edu

KCM
KENTUCKY CENTER FOR MATHEMATICS