

How Do You Reason? wondering how your students are reasoning?

The Development of Mathematical Reasoning



When given a problem, what kind of reasoning do you and your students employ? Answer each of the following questions. Then look at possible responses and kinds of reasoning.

What is 58 + 5?	 COUNTING STRATEGIES: counting by 1's Count out 58 objects (tallies, etc), count out 5. Put together, count the whole set. Start with 58 and count 59, 60, 61, 62, 63. Counting on. ADDITIVE THINKING: using jumps bigger than 1's From 58, add 2 to get to 60. Add the remaining 3 to get 63.
What is 16 × 9?	 COUNTING STRATEGIES: counting by 1's Count out 16 groups of 9 objects (tallies, etc) or 9 groups of 16 objects, one at a time. Put together, count the whole set. ADDITIVE THINKING: adding one group at a time Skip count by 16's or 9's. (16, 32, 48, 144 or 9, 18, 27,144) MULTIPLICATIVE REASONING: using bigger chunks than one group at a time Think about 16 9's as 10 9's and 6 more 9's. 10 x 9 + 6 x 9 = 90 + 54 = 144.
	Think about 9 16's as 10 16's subtract one 16. 10 × 16 − 1 × 16 = 160 − 16 = 144. Think about equivalent problems by doubling/halving. 16 × 9 = 8 × 18 = 4 × 36 = 2 × 72 = 144.
Solve for <i>x</i> : $\frac{5.5}{2.2} = \frac{1.25}{x}$	MULTIPLICATIVE REASONING: Find 2.2 × 1.25 ÷ 5.5. If done with traditional algorithms for multiplication and division, the multiplicative thinking can at best deal with single digits. PROPORTIONAL REASONING: Scale in tandem to find helpful ratios. $\frac{5.5}{2.2} = \frac{55}{22} = \frac{5}{2} = \frac{1.25}{0.5}$ so $x = 0.5$

Would you like to teach more complex ways of thinking and reasoning? Join us on #MathStratChat!

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Disappearing Sequences

On a scrap piece of paper, write the counting sequence that you want to practice. Make sure that you are **accurate** in your counting. For example, if you are working on counting by 2's, you would write:

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

Say the sequence while touching each number as you say its name. Repeat this step until you can say the sequence without hesitation. Write the sequence on the back of the paper for later reference if you get stuck.

After counting through the sequence, scratch out one number so that you can no longer read it. Count again, touching each number as you say the sequence. When you get to the number that "disappeared" touch the place where it used to be.

2, 4, 6, 8, 10, 🗰, 14, 16, 18, 20, 22, 24

After counting, "scratch out" another number. Count again. Be sure to touch each number or space as you count.

2, 4, 6, 🛤 10, 🛤, 14, 16, 18, 20, 22, 24

Repeat the process until all of the numbers have "disappeared." Count one final time, touching the spots in which each number used to be. If you get stuck, peek on the back of the paper.

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Blank numeral ladder

Numeral ladder graphic organizer

Cut along dotted line and tape together as desired for a longer ladder



Teaching Number in the Classroom with 4–8-Year-Olds Chapter 3 Supplementary Resource

4-Kings

Materials: Deck of Special 4-Kings Cards

How to Play:

- 1. Deal the card face down in four rows with nine cards each.
- 2. Place one start card beside each row. Each start card has a different color.
- 3. Place the four remaining cards in a draw pile below the rows.



- 4. Draw the first card from the draw pile. Determine the row by matching the start card color to the drawn card.
- 5. Count to determine where the card belongs in the row. Place the card in its spot while removing the card that was in that location.
- 6. Determine where the new card should go. Continue the procedure until all of the cards are face up.
- 7. The treasure chest card in the sequence is the stopper. When it is drawn, place it under the draw pile and draw a new card to continue play.
- 8. In order to win, you must turn over all of the cards before the fourth stopper is found.
- 9. To determine the counting sequence, read the bottom of the start cards.

Variation: This game was originally played as a form of solitaire with regular playing cards. Kings were stoppers.

Start:	Start:	Start:	Start:
2	<u>3</u>	4	5
Count by 2s	Count by 3s	Count by 4s	Count by 5s
4	<u>6</u>	8	10
6	<u>9</u>	12	15
8	<u>12</u>	16	20

10	<u>15</u>	20	25
12	<u>18</u>	24	30
14	<u>21</u>	28	35
16	<u>24</u>	32	40

18	<u>27</u>	36	45
20	<u>30</u>	40	50
Treasure Hunt Skip counting Set up: Deal 9 face-down cards per row.			

Quick Draw Multiples

Players 2

Materials

• Quick Draw Multiples card deck

A card deck consists of 4 cards of each of the first 10 numbers in a chosen multiples sequence. For example, for a Quick Draw Multiples deck by 4s, the card deck contains four cards of each of the following numbers: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40

How to play

- 1. Deal out fifteen cards from the Quick Draw Multiples card deck to each player. This becomes each player's draw pile.
- 2. Place two stacks of an additional five cards in the center of play.
- 3. Each player draws three cards from their individual draw pile. This becomes each player's *hand* during play.
- 4. To start play, each player turns over the top card from each stack of 5 cards in the center of play and places it face up.



- 5. Players choose a card from their hand that is either a multiple higher or lower than one of the cards in play. Players can play on either stack and place a card as soon as a number from the player's *hand* can be played. Players do not take turns.
- 6. As soon as a player places a card, that player draws a card from their draw pile so that each player holds three cards at all times.
- 7. If both players can not play one of the three cards in their *hand*, then each player turns over the top card from the 5 card pile and places that card face up in play. Play resumes.
- 8. The first player that uses all cards from their individual draw pile wins.



Multiples	Multiples	Multiples	Multiples
of	of	of	of
Two	Two	Two	Two



6	6	6	6
8	8	8	8

ΙΟ	ΙΟ	ΙΟ	ΙΟ
12	12	12	12



18	18	18	18
20	20	20	20

22	22	22	22
24	24	24	24



Multiples	Multiples	Multiples	Multiples
of	of	of	of
Three	Three	Three	Three





15	15	15	15
18	18	18	18

21	21	21	21
24	24	24	24

27	27	27	27
30	30	30	30

33	33	33	33
36	36	36	36



Multiples	Multiples	Multiples	Multiples
of	of	of	of
Four	Four	Four	Four



12	12	12	12
16	16	16	16

20	20	20	20
24	24	24	24

28	28	28	28
32	32	32	32

36	36	36	36
40	40	40	40

ЧЧ	ЧЧ	ЧЧ	ЧЧ
48	48	48	48

MULTIPLES	MULTIPLES	MULTIPLES	MULTIPLES
OF	OF	OF	OF
FIVE	FIVE	FIVE	FIVE

MULTIPLES	MULTIPLES	MULTIPLES	MULTIPLES
OF	OF	OF	OF
FIVE	FIVE	FIVE	FIVE

5	5	5	5
ΙΟ	ΙΟ	ΙΟ	ΙΟ

15	15	15	15
20	20	20	20

25	25	25	25
30	30	30	30

35	35	35	35
40	40	40	40

45	45	45	45
50	50	50	50

55	55	55	55
60	60	60	60
Multiples	Multiples	Multiples	Multiples
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of	of	of	of
Six	Six	Six	Six

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Six	Six	Six	Six

6	6	6	6
12	12	12	12

18	18	18	18
24	24	24	24

30	30	30	30
36	36	36	36

42	42	42	42
48	48	48	48

54	54	54	54
60	60	60	60

66	66	66	66
72	72	72	72

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Şeven	Seven	Seven	Seven

Multiples	Multiples	Multiples	Multiples
Of	of	of	of
Seven	Seven	Seven	Seven



21	21	21	21
28	28	28	28

35	35	35	35
42	42	42	42

49	49	49	49
56	56	56	56

63	63	63	63
70	70	70	70

77	77	77	77
84	84	84	84

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Eight	Eight	Eight	Eight

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Eight	Eight	Eight	Eight

8	8	8	8
I6	I6	16	I6

24	24	24	24
32	32	32	32

40	40	40	40
48	48	48	48

56	56	56	56
64	64	64	64

72	72	72	72
80	80	80	80

88	88	88	88
96	96	96	96

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Nine	Nine	Nine	Nine

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Nine	Nine	Nine	Nine

9	9	9	9
18	18	18	18

27	27	27	27
36	36	36	36

45	45	45	45
54	54	54	54

63	63	63	63
72	72	72	72

81	81	81	81
90	90	90	90

99	99	99	99
108	108	108	108

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Ten	Ten	Ten	Ten

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Ten	Ten	Ten	Ten

ΙΟ	ΙΟ	ΙΟ	ΙΟ
20	20	20	20

30	30	30	30
40	40	40	40

50	50	50	50
60	60	60	60

70	70	70	70
80	80	80	80

90	90	90	90
100	100	100	100

ΙΟ	ΙΟ	ΙΟ	ΙΟ
120	120	120	120

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Eleven	Eleven	Eleven	Eleven

Multiples	Multiples	Multiples	Multiples
of	of	of	of
Eleven	Eleven	Eleven	Eleven


33	33	33	33
LH LH	LH LH	L+L+	LH LH

55	55	55	55
66	66	66	66

77	77	77	77
88	88	88	88

99	99	99	99
IIO	IIO	ΙΟ	IIO

121	121	121	121
132	132	132	132

MULTIPLES	MULTIPLES	MULTIPLES	MULTIPLES
OF	OF	OF	OF
TWELVE	TWELVE	TWELVE	TWELVE

MULTIPLES	MULTIPLES	MULTIPLES	MULTIPLES
OF	OF	OF	OF
TWELVE	TWELVE	TWELVE	TWELVE

12	12	12	12
24	24	24	24

36	36	36	36
48	48	48	48

60	60	60	60
72	72	72	72

84	84	84	84
96	96	96	96

108	108	108	108
120	120	120	120

132	132	132	132
 -++-+	 -+++	 -+++	