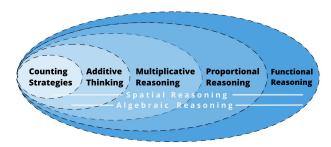
# How Do You Reason?

#### **WONDERING HOW YOUR STUDENTS ARE REASONING?**

#### The Development of **Mathematical Reasoning**



When given a problem, what kind of reasoning do you and your students employ? Answer each of the following questions. Then look at possible responses and kinds of reasoning.

What is 58 + 5?

**COUNTING STRATEGIES:** counting by 1's

Count out 58 objects (tallies, etc), count out 5. Put together, count the whole set. Start with 58 and count 59, 60, 61, 62, 63. Counting on.

**ADDITIVE THINKING: using jumps bigger than 1's** From 58, add 2 to get to 60. Add the remaining 3 to get 63.

What is  $16 \times 9$ ?

**COUNTING STRATEGIES:** counting by 1's

Count out 16 groups of 9 objects (tallies, etc) or 9 groups of 16 objects, one at a time. Put together, count the whole set.

**ADDITIVE THINKING:** adding one group at a time Skip count by 16's or 9's. (16, 32, 48, ... 144 or 9, 18, 27, ... 144)

MULTIPLICATIVE REASONING: using bigger chunks than one group at a time Think about 16 9's as 10 9's and 6 more 9's.  $10 \times 9 + 6 \times 9 = 90 + 54 = 144$ . Think about 9 16's as 10 16's subtract one 16.  $10 \times 16 - 1 \times 16 = 160 - 16 = 144$ . Think about equivalent problems by doubling/halving.  $16 \times 9 = 8 \times 18 = 4 \times 36 = 2 \times 72 = 144$ .

Solve for *x*:

**MULTIPLICATIVE REASONING:** 

Find  $2.2 \times 1.25 \div 5.5$ . If done with traditional algorithms for multiplication and division, the multiplicative thinking can at best deal with single digits.

**PROPORTIONAL REASONING:** 

Scale in tandem to find helpful ratios.

$$\frac{5.5}{2.2} = \frac{55}{22} = \frac{5}{2} = \frac{1.25}{0.5}$$
 so  $x = 0.5$ 

Would you like to teach more complex ways of thinking and reasoning? Join us on #MathStratChat!

## **Equal Addends**

I can count the total number of hidden cubes (within 20) for two equal rows when told the number in each row and write the addition sentence showing the number in each row and the total.

KNP # M 4402.4 - Equal Addends, PURPLE

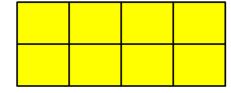
Fluency Standard: 3.OA.7

Standards: 2.OA.3

Materials: 20 snap cubes, cover, writing space

#### **Directions:**

- 1. Get 20 snap cubes, cover and writing material.
- 2. Player 1 hides his eyes.
- 3. Player 2 creates a block figure with two equal rows.
- Ex.



- 4. Player 2 covers the whole figure.
- 5. Player 2 tells how many blocks there are in each of the two rows. Player 1 imagines the figure and finds the total number of blocks.
- 6. Player 1 writes the number sentence showing the addition of two equal rows.
- 7. Players switch places and begin again.

### **Shark Attack**

# I can count groups of items using skip counting, when total is hidden, to find the product.

KNP # M 4403.3 - Shark Attack, Purple

Fluency Standard: 3.OA.7

Standard: 2.OA.4, 3.OA.4, 3.OA.1,

Materials: - Ocean Game Card: 150 cards total, 5 sheets: Sea Snail Cards: 1 shell; Clownfish Cards: 3 stripes; Sea Star Cards: 5 arms; Clam Shell Cards: 7 spots; Jellyfish Cards: 9 tentacles; Clownfish Recording Sheet: one per person

#### **Directions:**

#### Set Up:

- 1. Each player gets a clownfish recording sheet.
- 2. Sort the ocean game cards by animal types. Place those cards in four different stacks face up in the middle of the group.

#### Game Play - On your turn:

- 1. Select ten ocean game cards from one of the four piles.
- 2. Record your animal type and value on to your recording sheet. (Hint: This value will be the number in each group.)
- 3. Say "Shark Attack!" and toss the ocean cards up into the air.
- 4. Count the number of animal cards that are lying face down. (Hint: This will be the number of groups.)
- 5. Record your equation on to your recording sheet.
- 6. Using skip counting, find your total and record it on your recording sheet.
- 7. Play will continue until all players have played the activity with all five sets of ocean game card.

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#### Printables for "Shark Attack"

#### KNPIG ID # M4403.3 - PURPLE

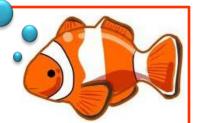
#### This file contains printables for up to five students.

For each additional group of students print one new file.

- 5 Swimmin' Out Clownfish Recording Sheets
- Ocean Game Card: 150 cards total, 5 sheets
  - -Sea Snail Cards: 1 shell -Clownfish Cards: 3 stripes - Sea Star Cards: 5 arms
  - -Clam Shell Cards: 7 spots -Jellyfish Cards: 9 tentacles

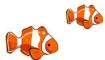
The teacher note for this activity can be found on the activity lesson plan.

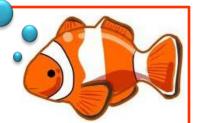
Created by Jordan Rhude & Emily Westerling, 2015



Turn:	Animal Type:	Equation:	Total:
1			
2			
3			
4			
5			
6			
7			
8			

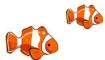


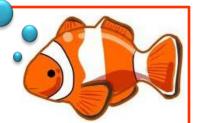




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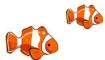


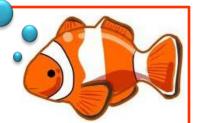




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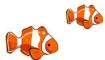


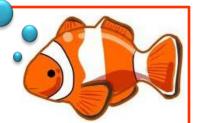




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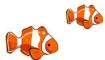


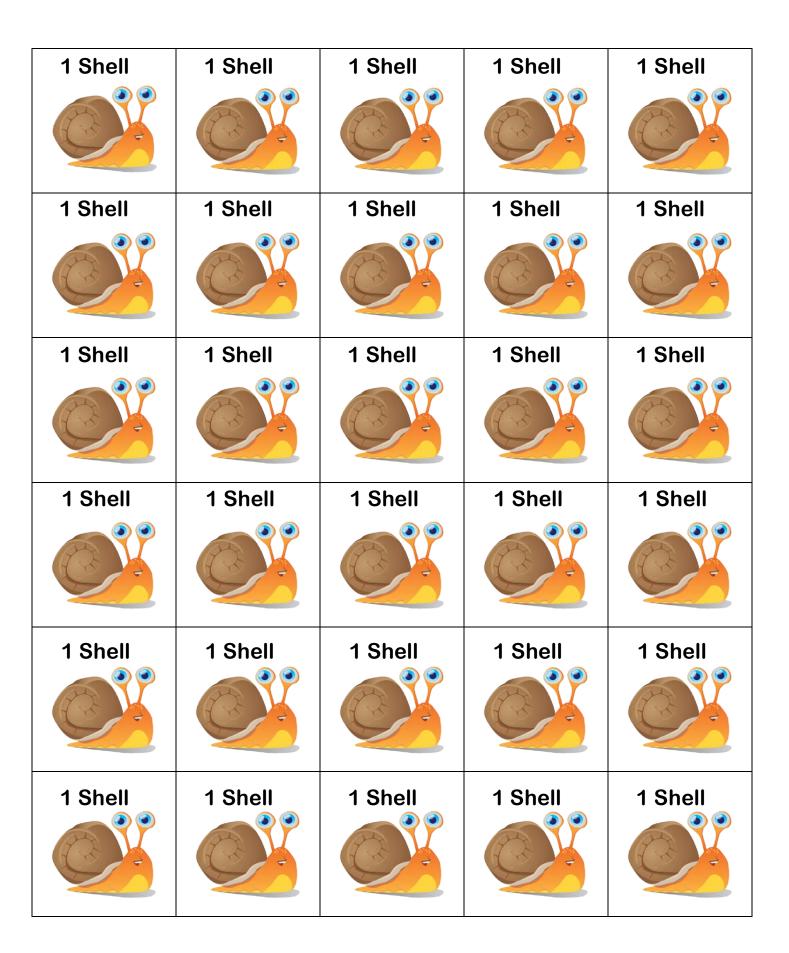


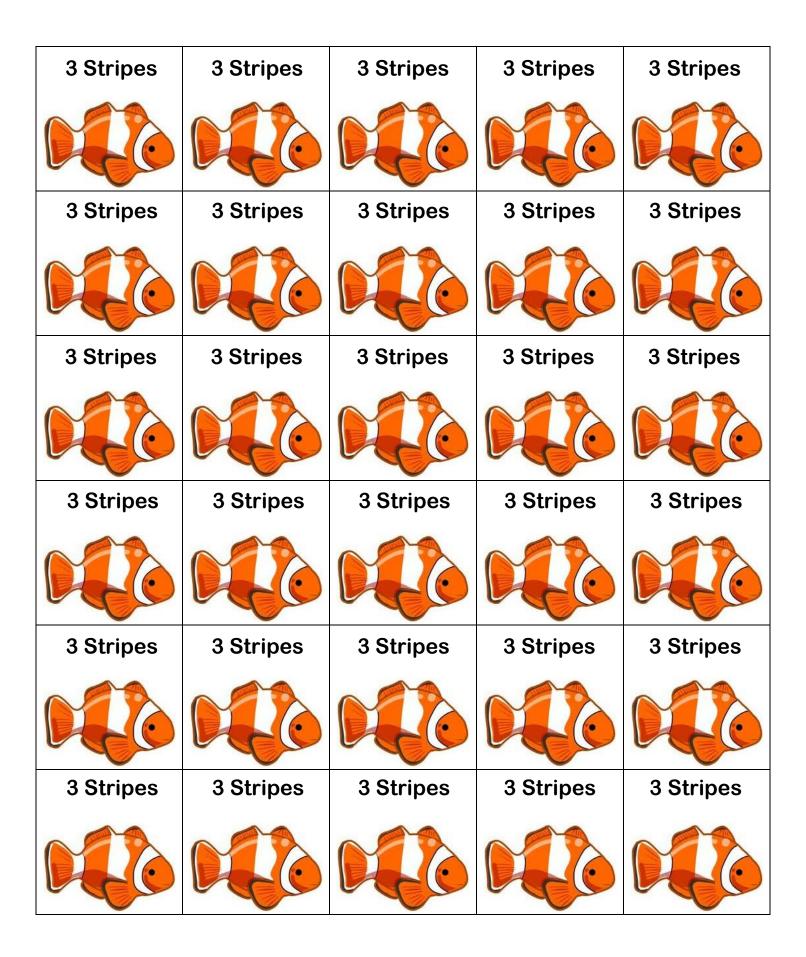


Turn:	Animal Type:	Equation:	Total:
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2			
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5			
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7			
8			



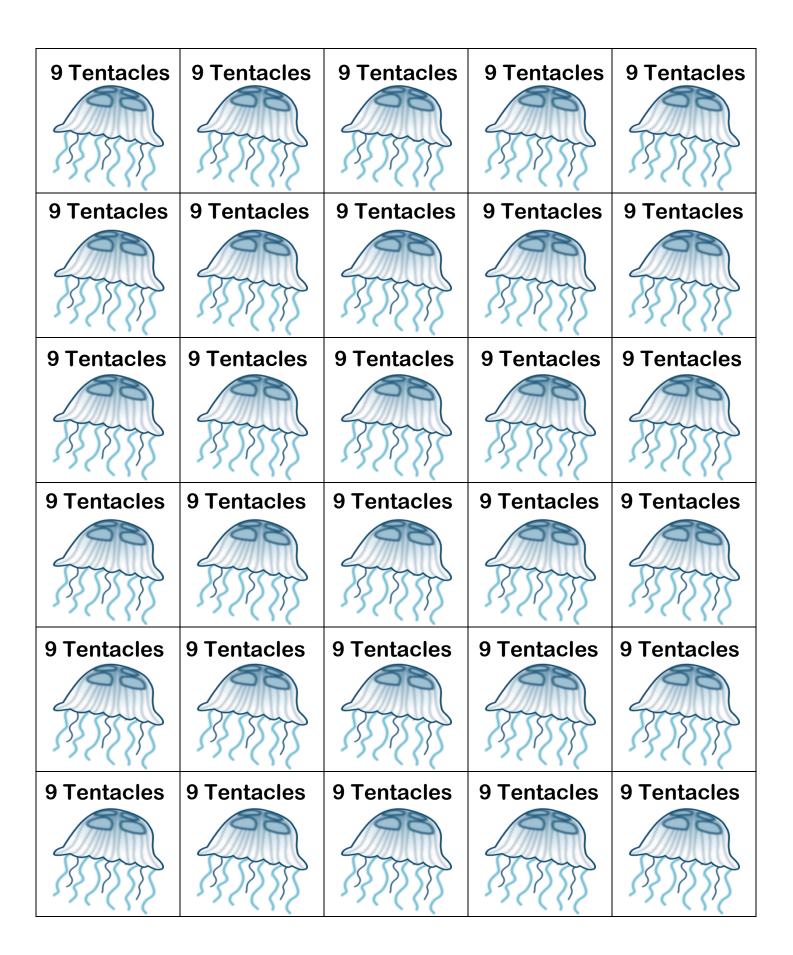






| 5 Arms |
|--------|--------|--------|--------|--------|
| 5 Arms |
| 5 Arms |
| 5 Arms |
| 5 Arms |
| 5 Arms |

| 7 Spots |
|---------|---------|---------|---------|---------|
|         |         |         |         |         |
| 7 Spots |
|         |         |         |         |         |
| 7 Spots |
|         |         |         |         |         |
| 7 Spots |
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| 7 Spots |
|         |         |         |         |         |
| 7 Spots |
|         |         |         |         |         |



M 4443.1



Entry M 4443.1 – Describing Groups and Arrays
Part of Task Group M 4443 – Multiplication Arrays PowerPoint

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#### **Teacher Directions**

- Display an image, leaving the image available for students to see and discuss
- Give discussion prompts such as
  - Tell me something about this picture?
  - How many groups are there?
  - How many dots (items) are in each group? (Bring out words such as group, set, row, column and array and support students in using words correctly.)
  - Initially, do not ask students about the total number of dots. The purpose of
    this activity is to help students attend to and be able to communicate about
    the group structure of the image. If students are asked too soon about the
    total number of dots, students will likely resort to counting dots by ones with
    little or no attention to the structure.



Click HERE to skip to first image.

The Kentucky Center for Mathematics KNP: http://kymath.org

M 4443.1

#### I can...

... describe quantities arranged in equal groups or arrays.



M 4443.1



Click HERE to skip to first image.

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M 4443.1

#### **Teacher Directions**

Additional activity prompts may include

- Use counters to create what you see. (In place of counters, students could use dotted popsicle sticks or (for arrays) or dot strips.)
- Draw what you see. (For the arrays, students could be given the option of recording images on a 10 by 10 empty dot array. Click here for pdf of the empty array. The array can be laminated or slipped into a page protector for repeated use. As an alternative to coloring in the array, students can "frame" the array using an L shaped paper. See next slide.)
- Use a 10x10 bead rack to create what you see. (See next slide.)

These additional activities may help struggling students attend to the group structure and will give the teacher insight into what the student is attending to in the image.



Click HERE to skip to first image.

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#### Connections to CCSS

This activity is foundational for the following standards:

- 2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and 5 columns; write an equation to express the total as a sum of equal addends.
- 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

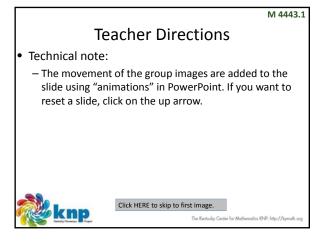


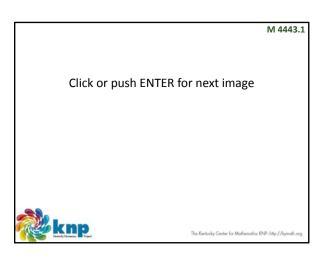
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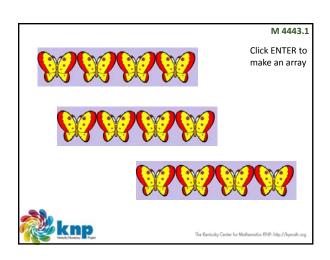
# Pictures •(top) 10 x 10 bead rack •(right) L shaped screen on a 10 x 10 array grid Click HERE to skip to first image. The Karbody Carlor for Mathematica DSP- high/Fuyneth. arg

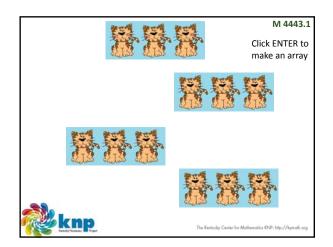
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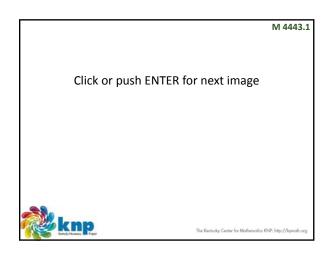


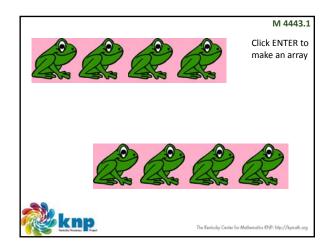


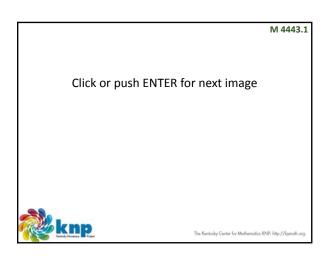




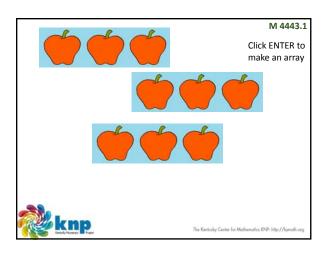


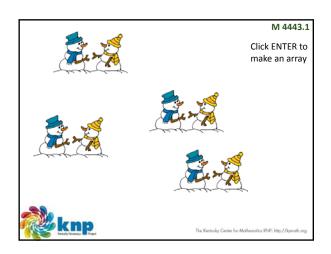




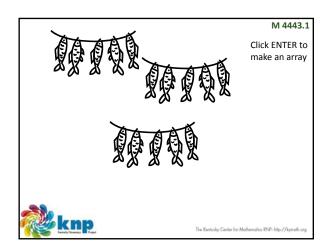


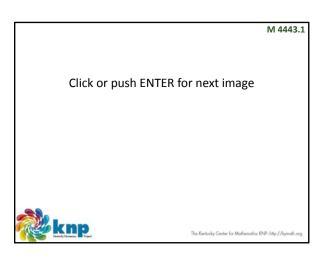


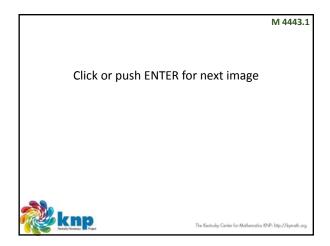


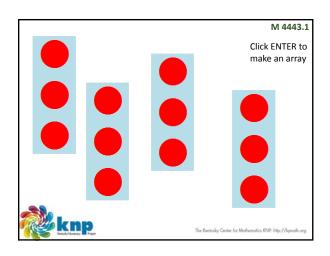




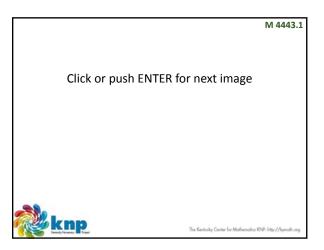


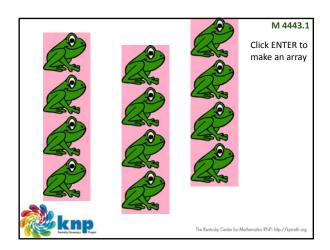


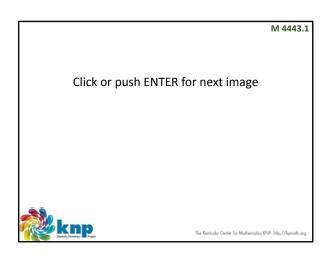


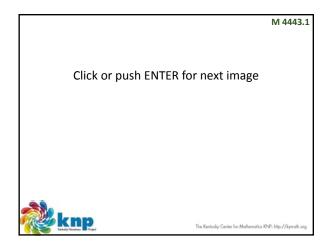


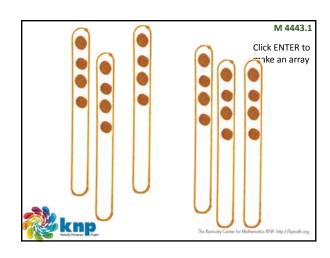


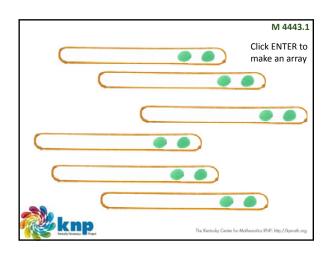




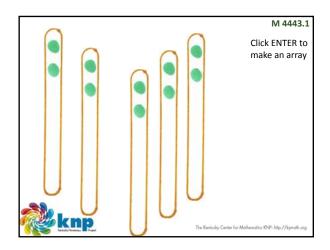


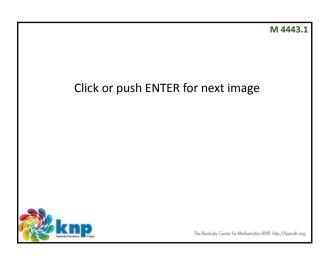




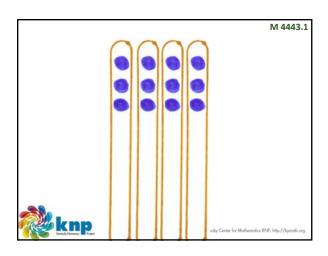


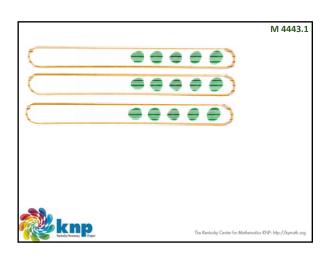


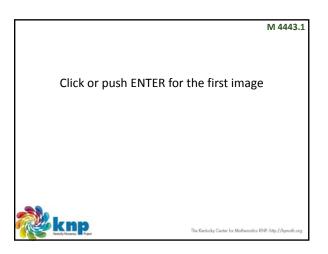


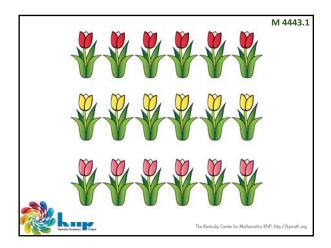


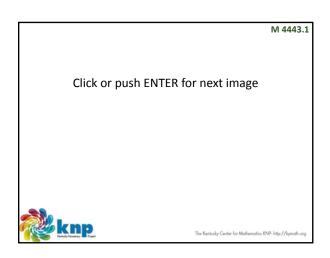


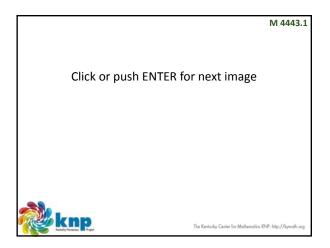


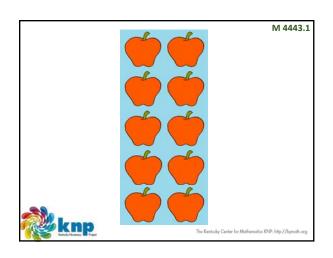


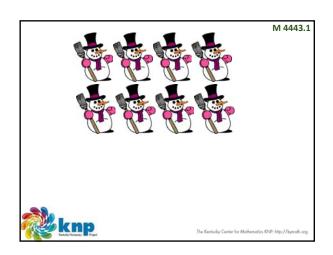


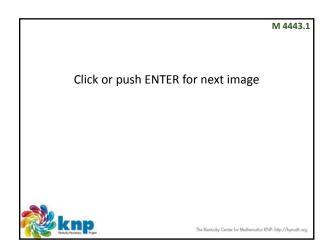




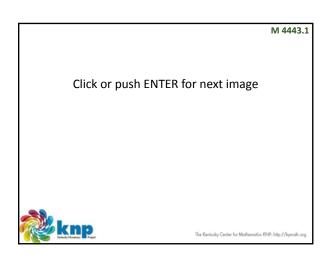


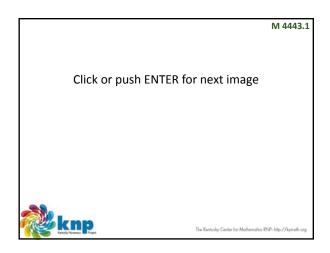


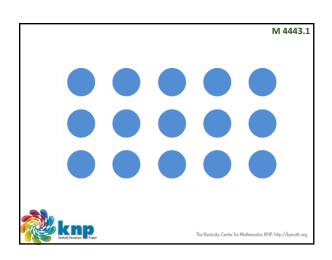


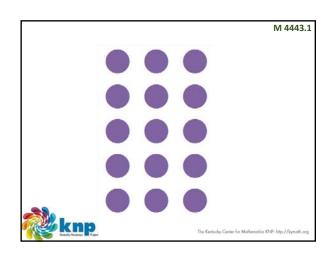




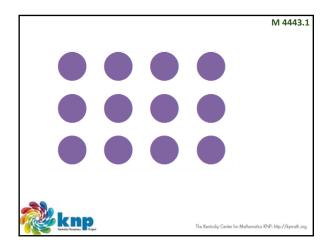


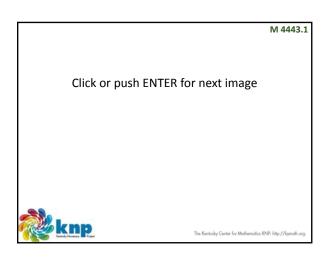




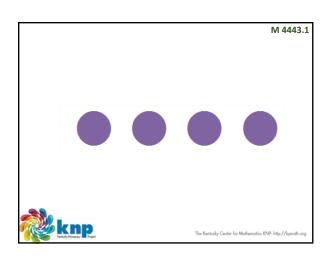


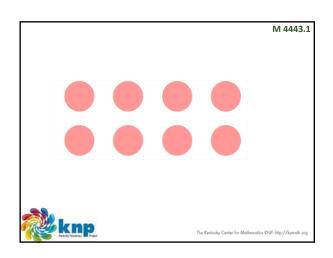


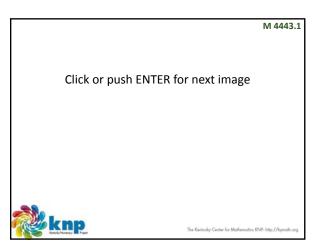


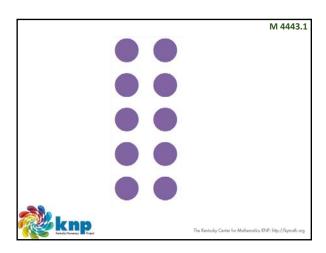




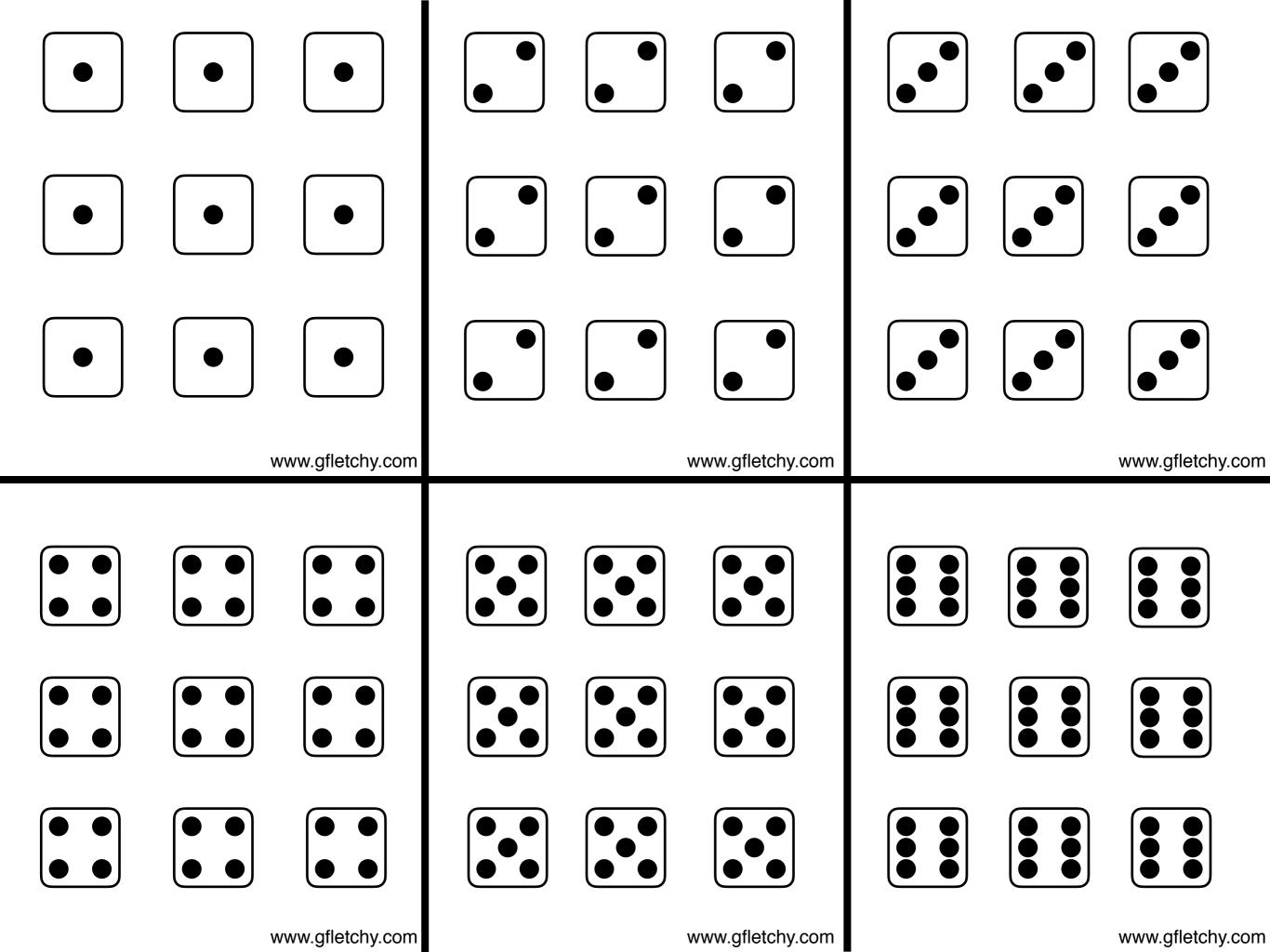


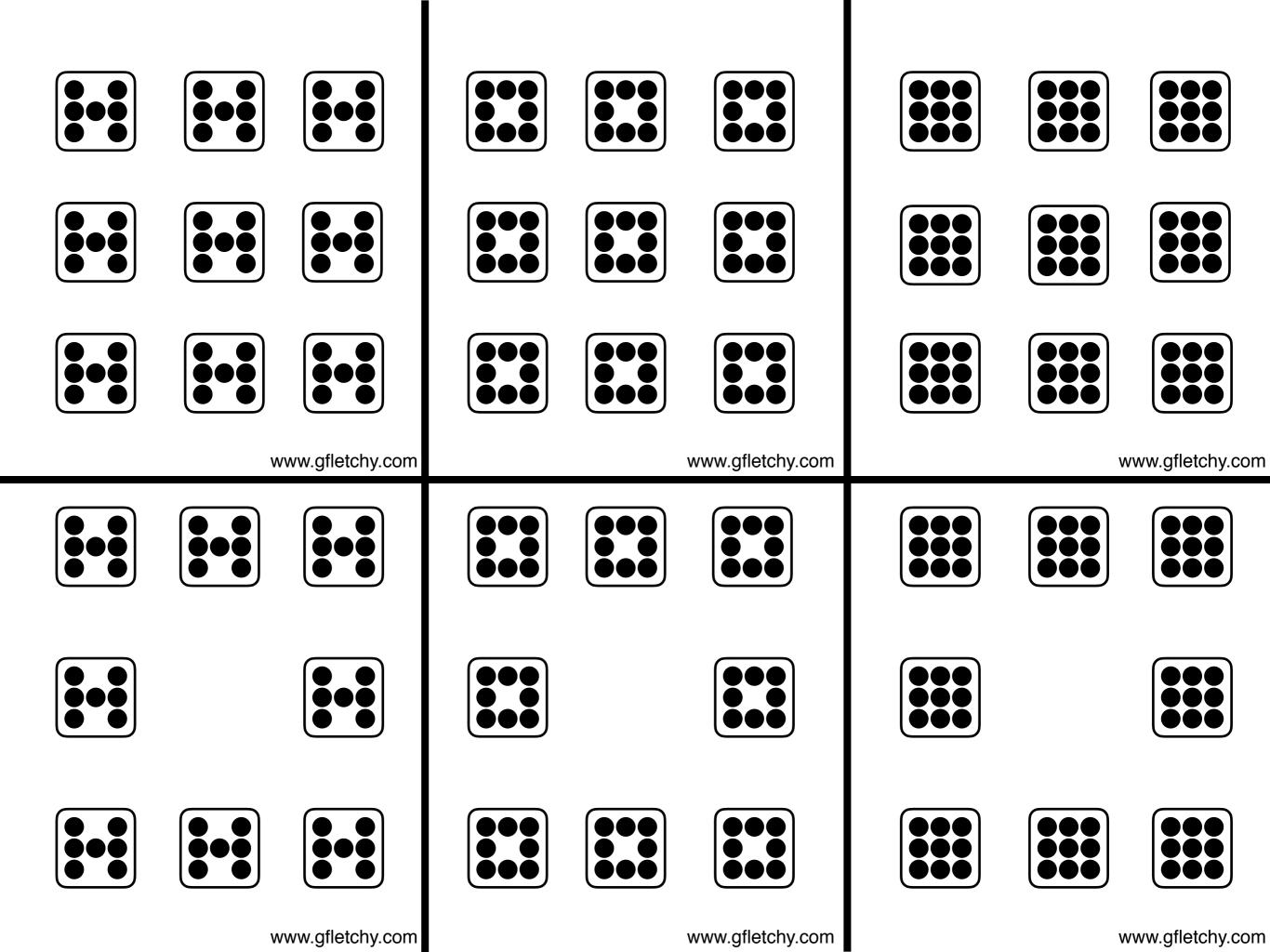


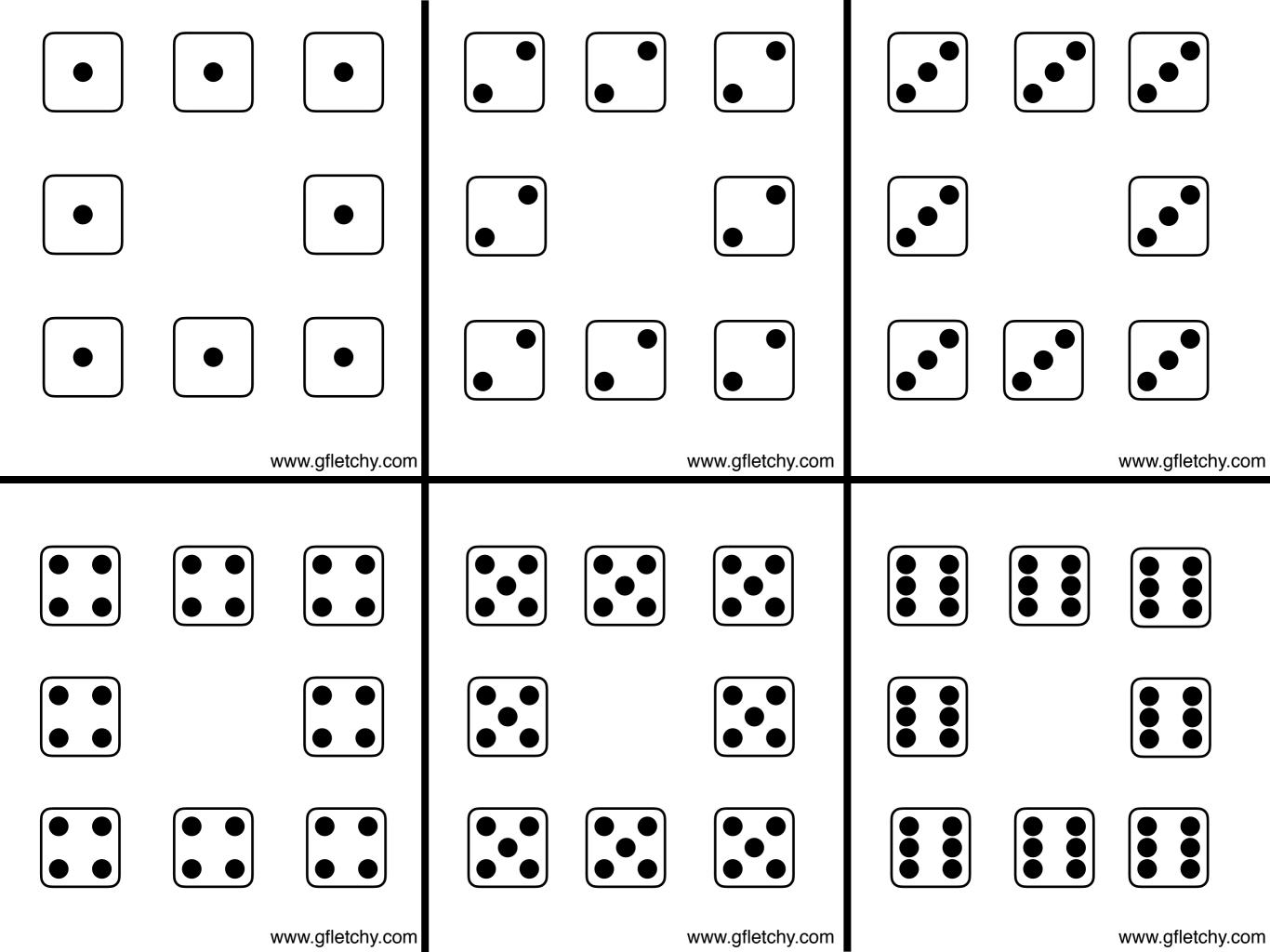


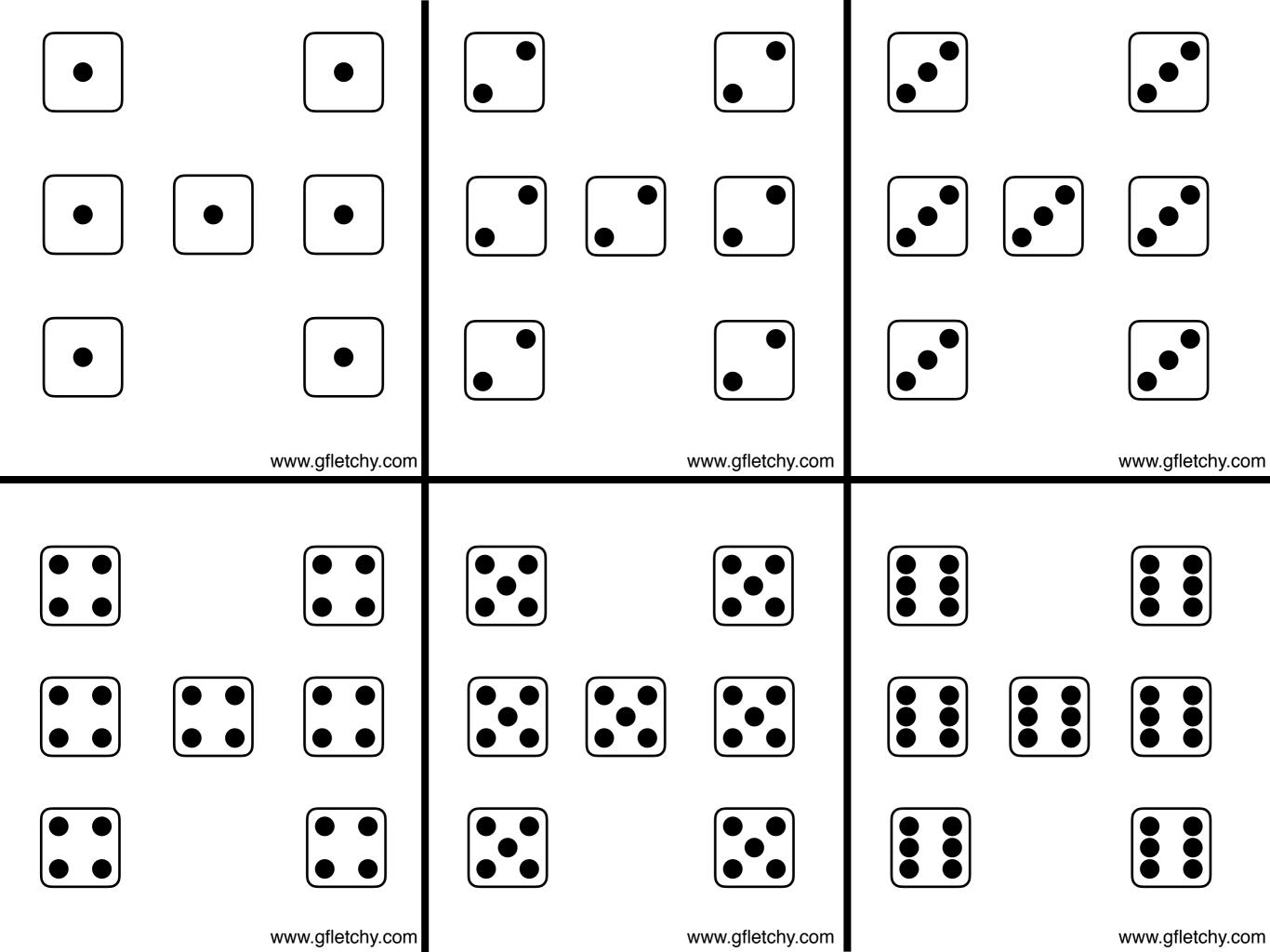


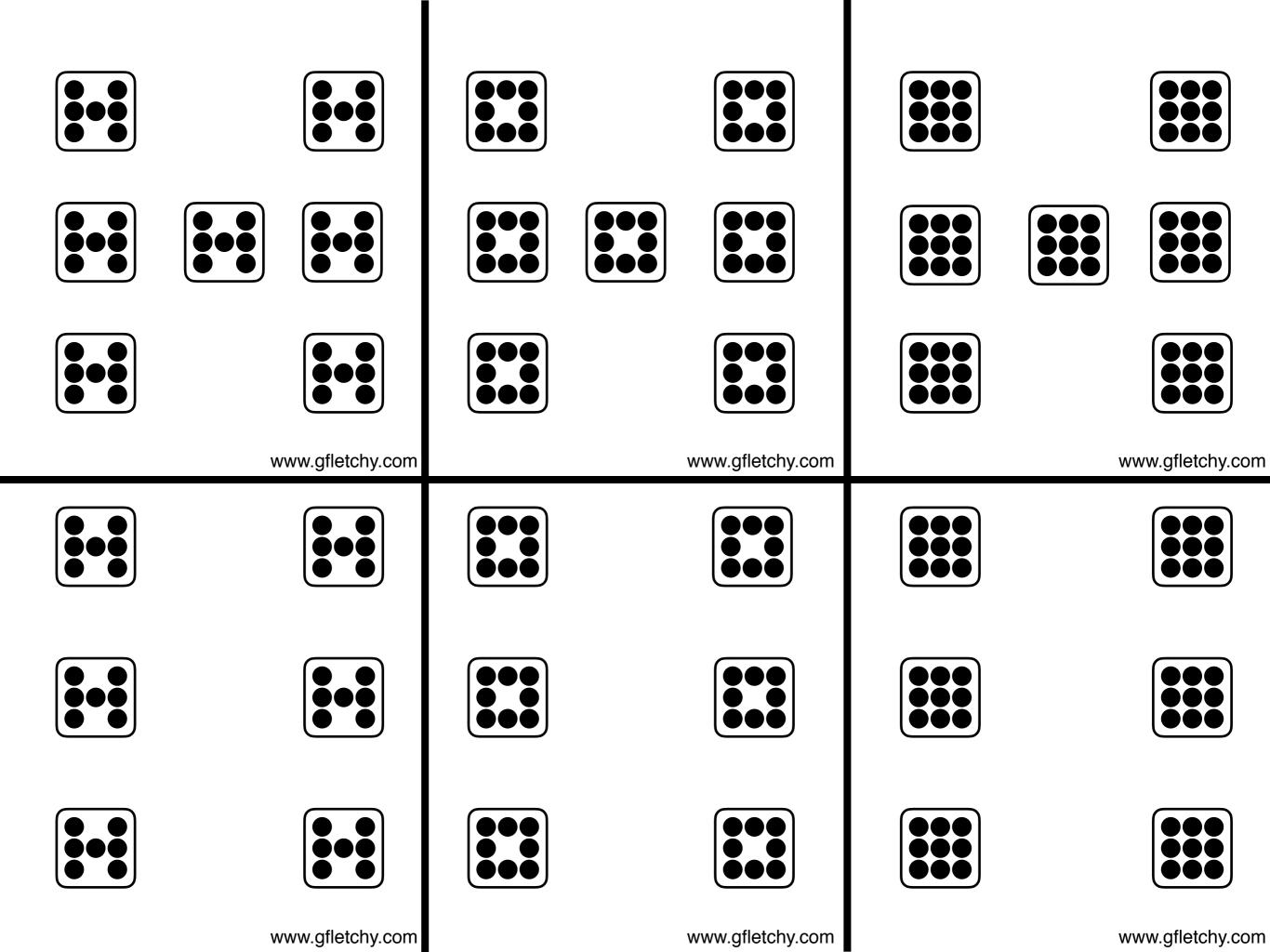


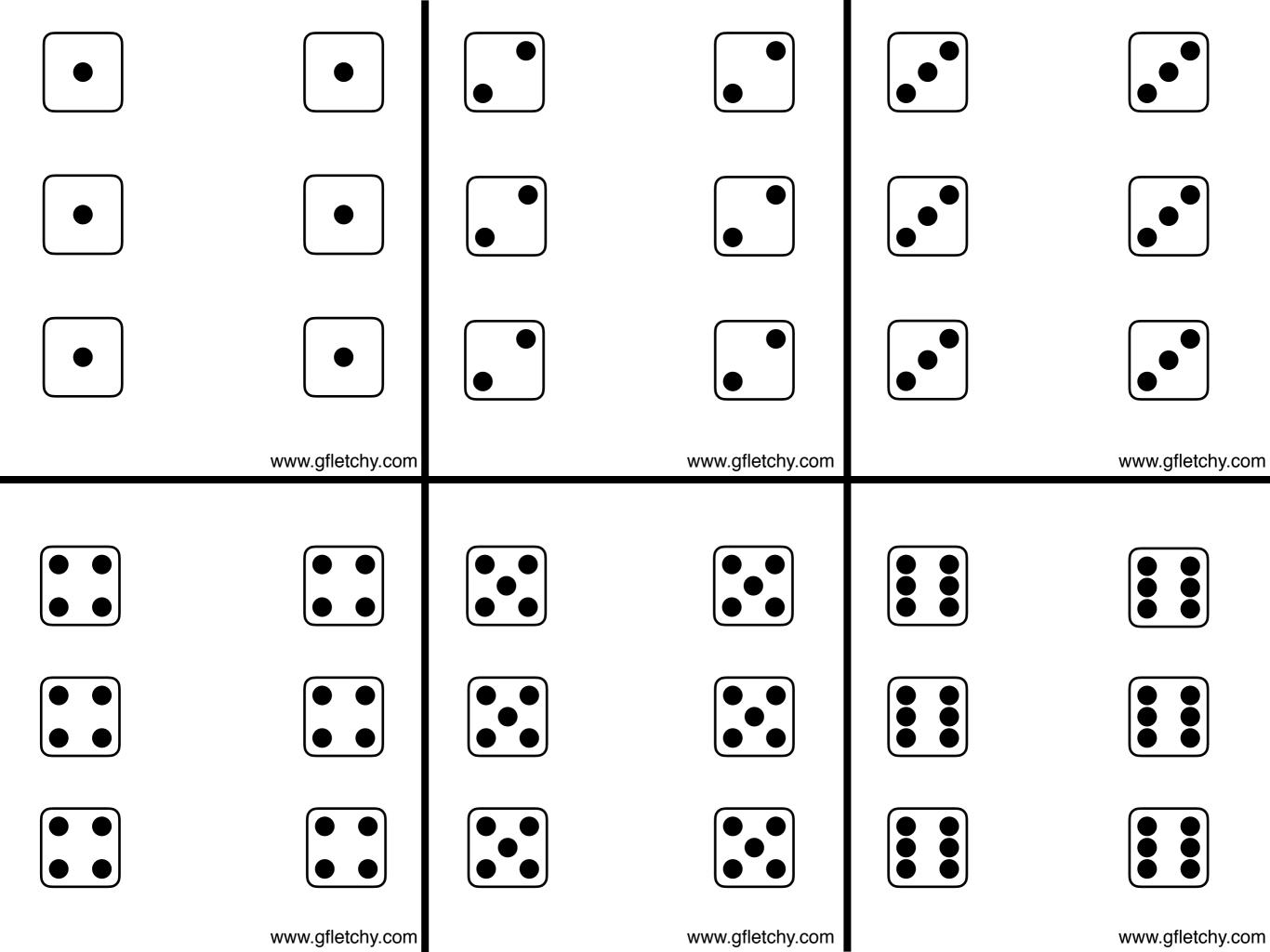


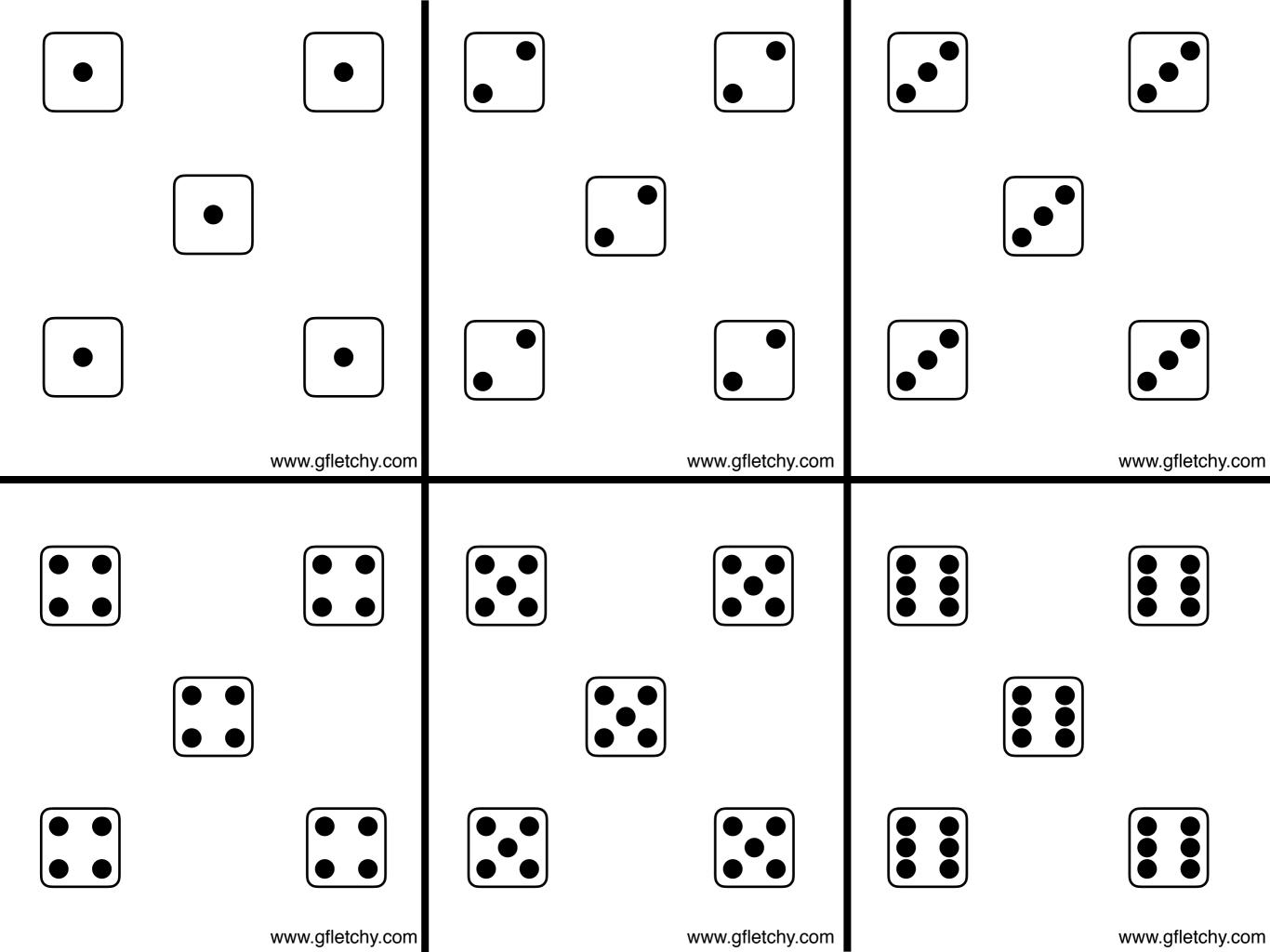


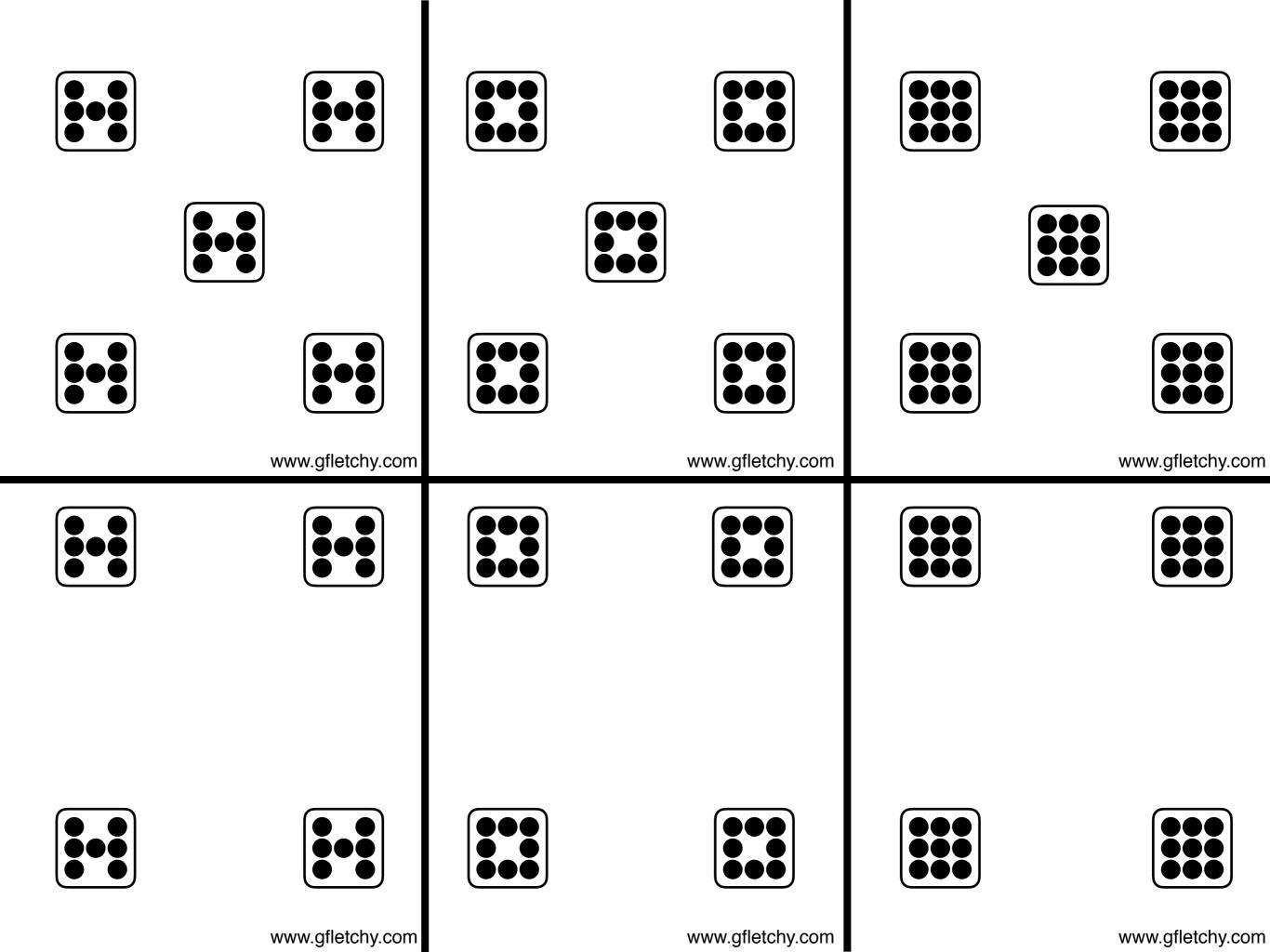


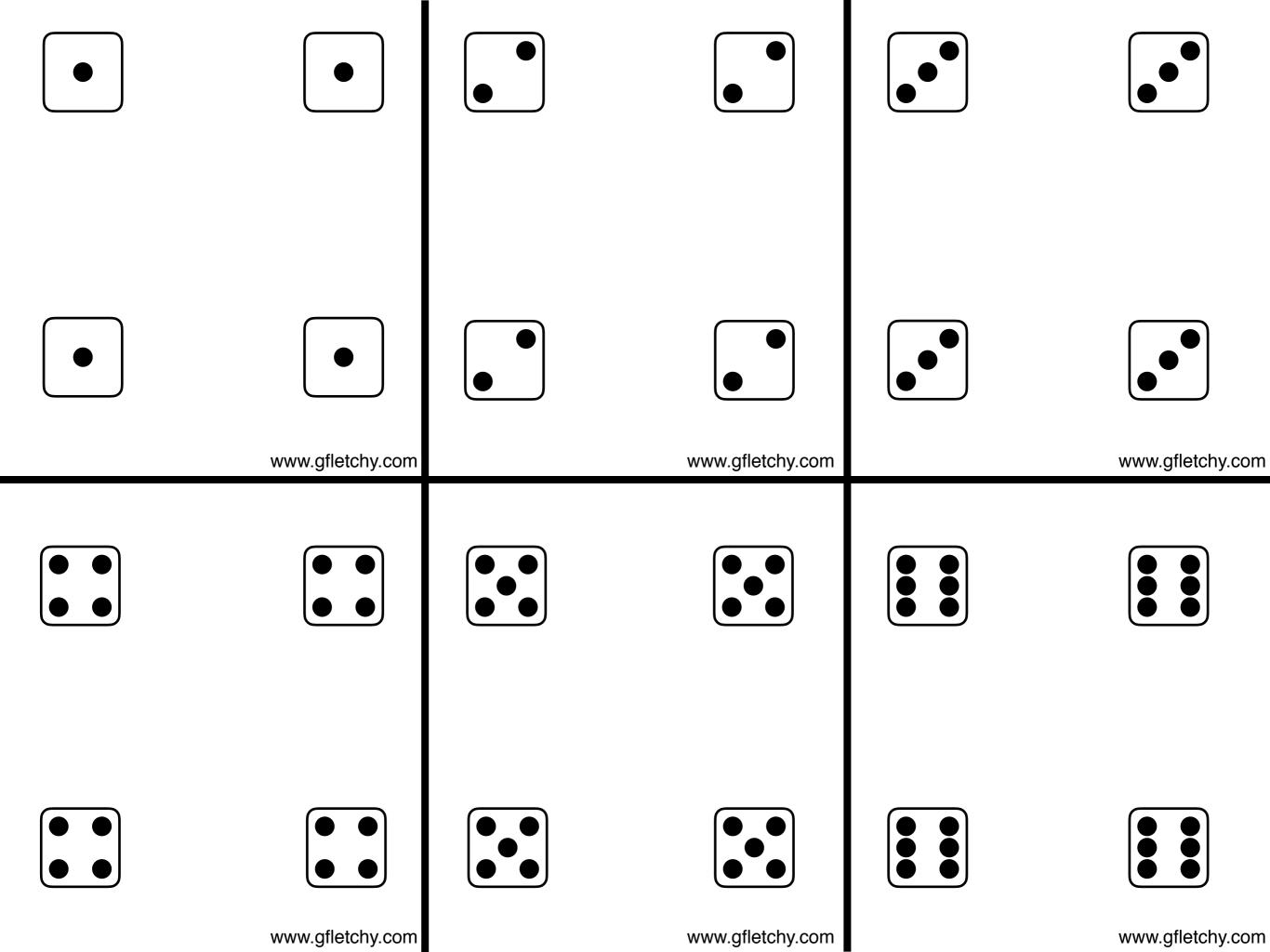


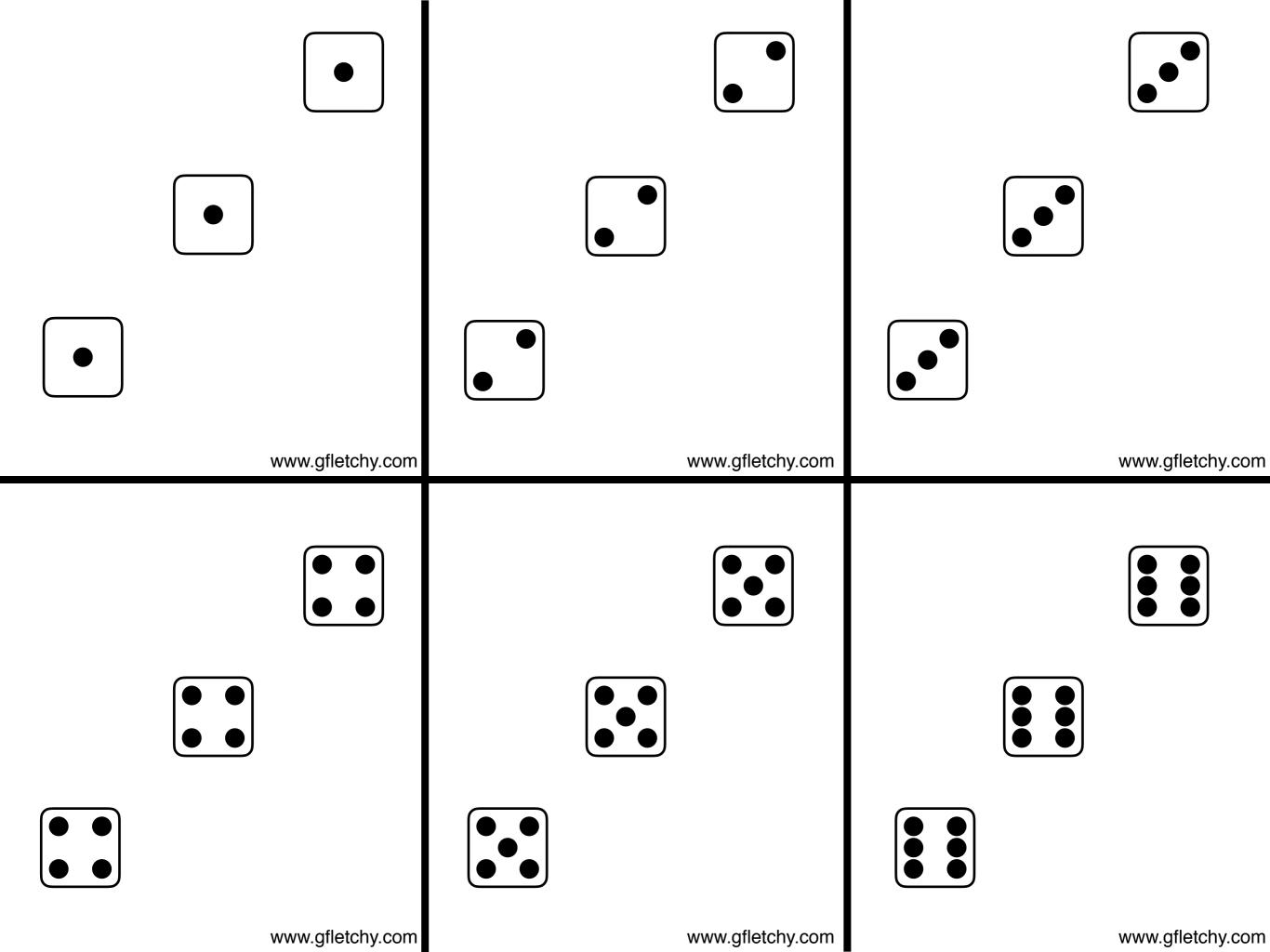


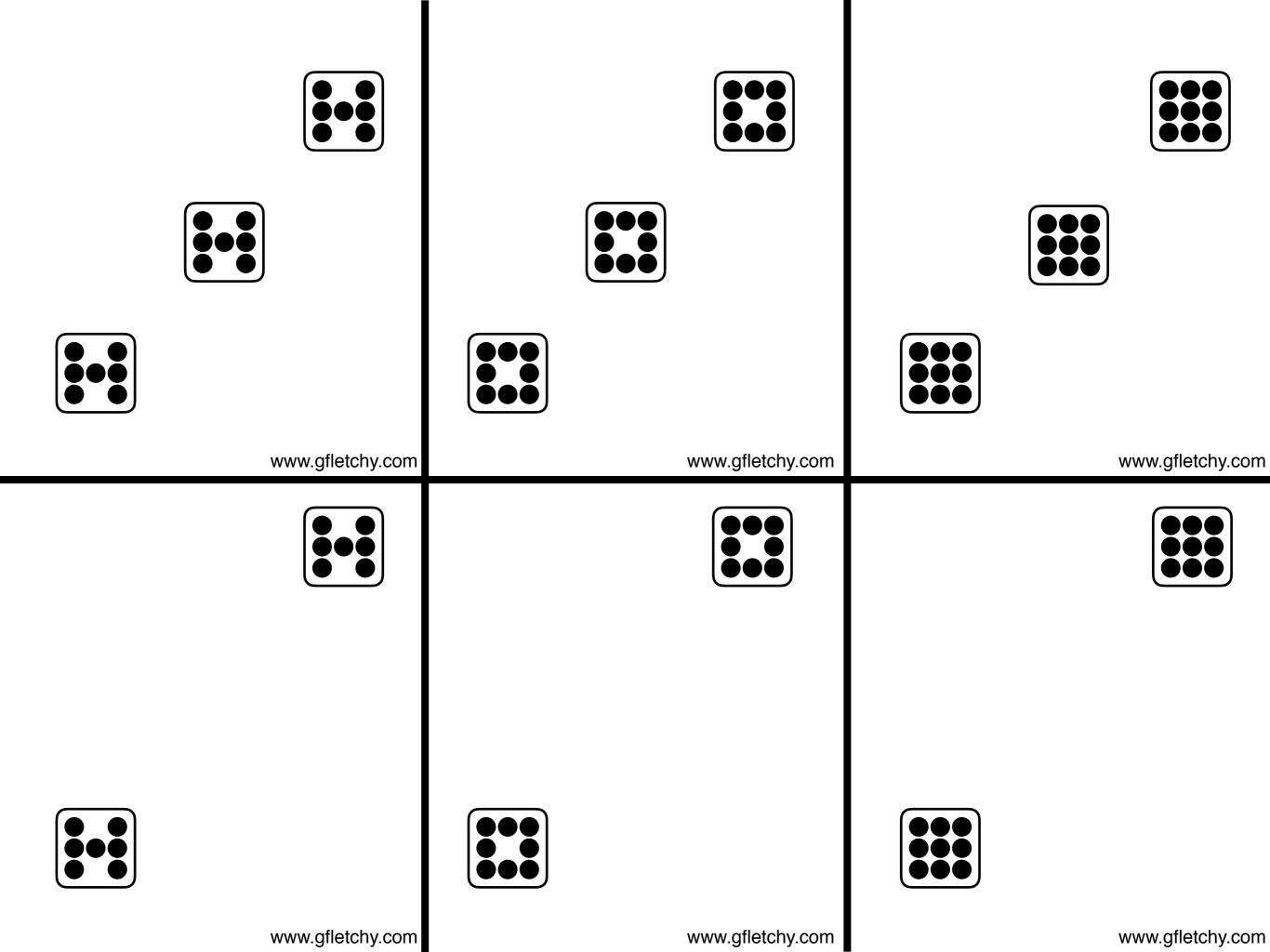


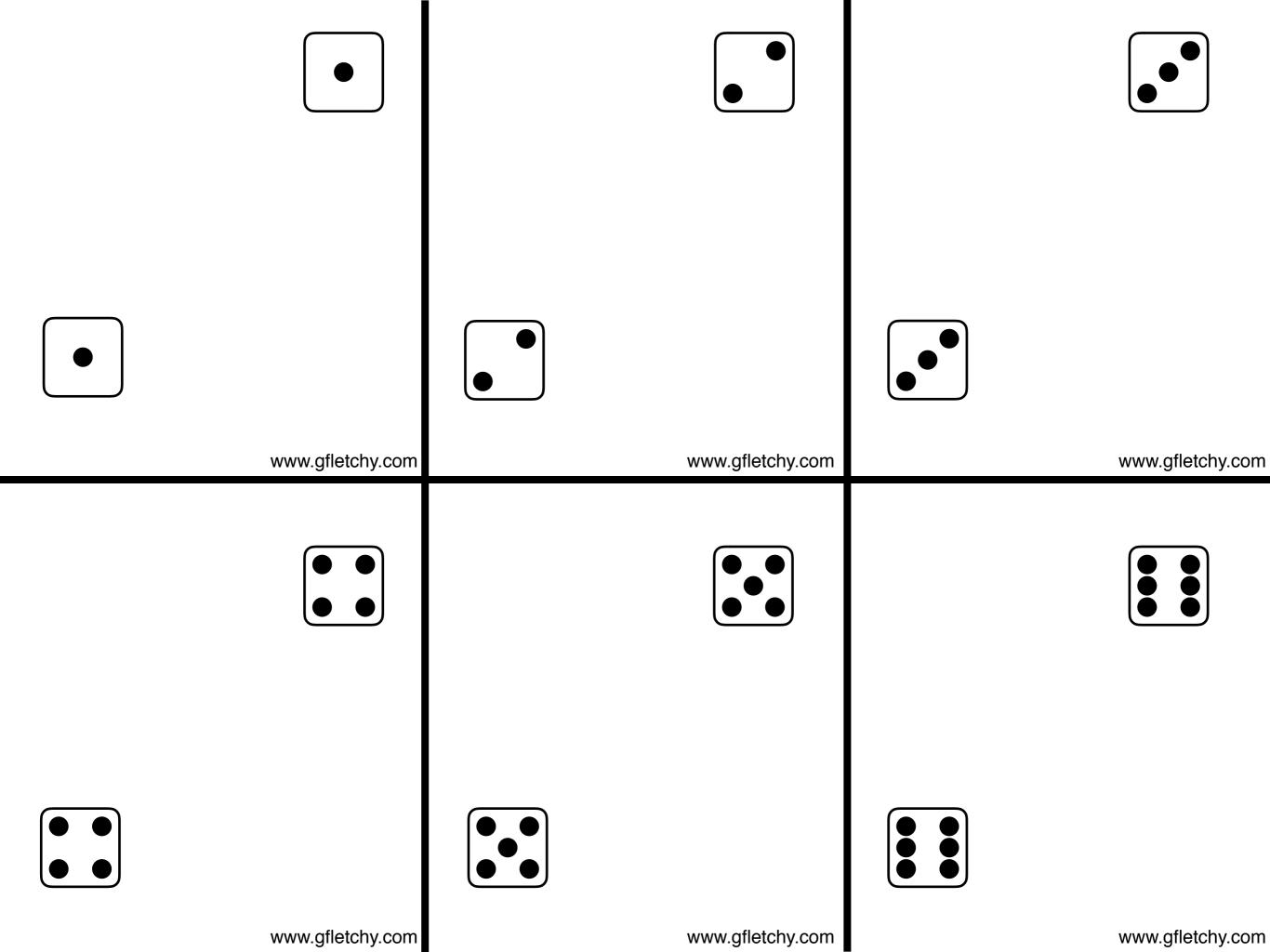




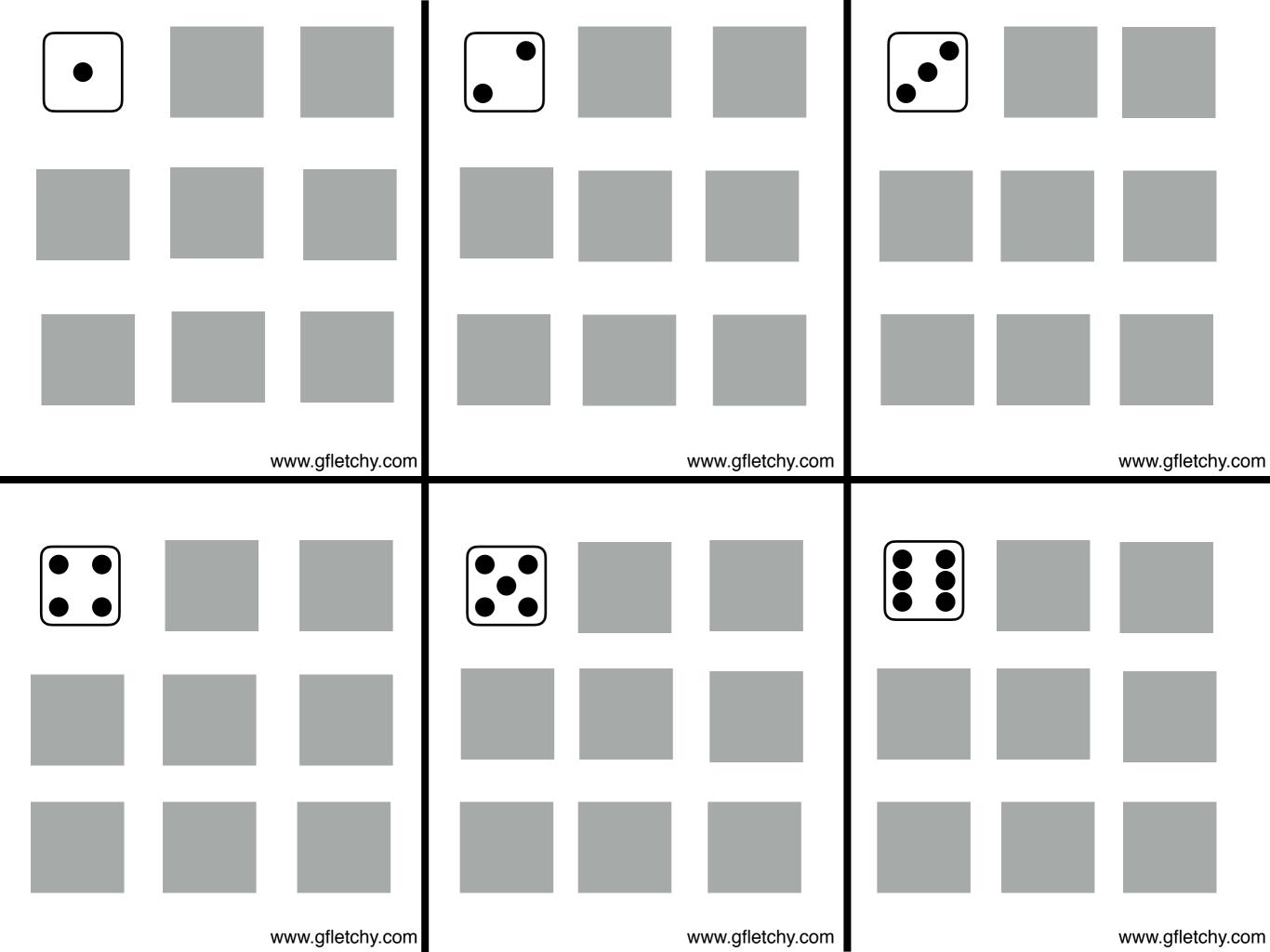


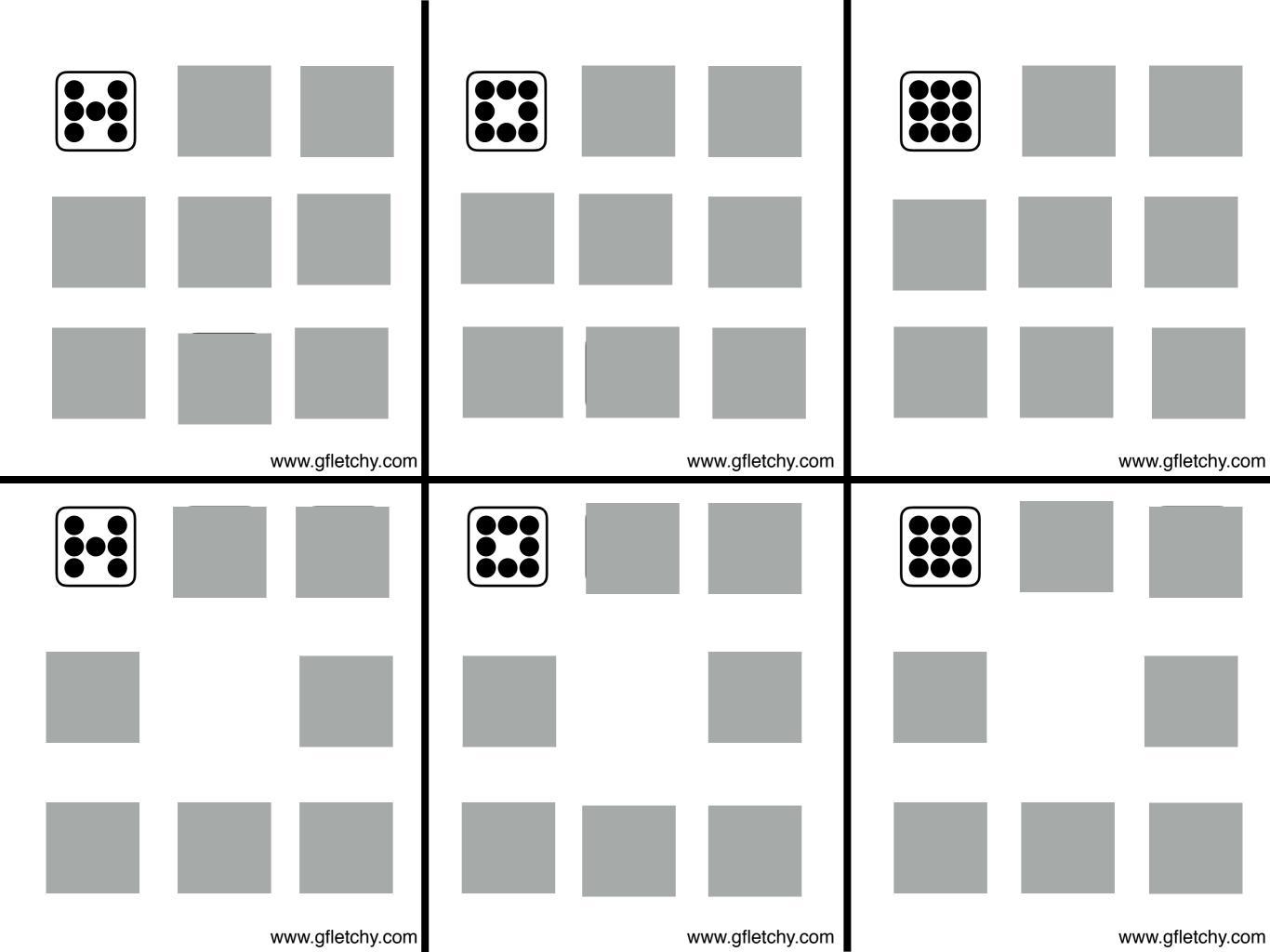


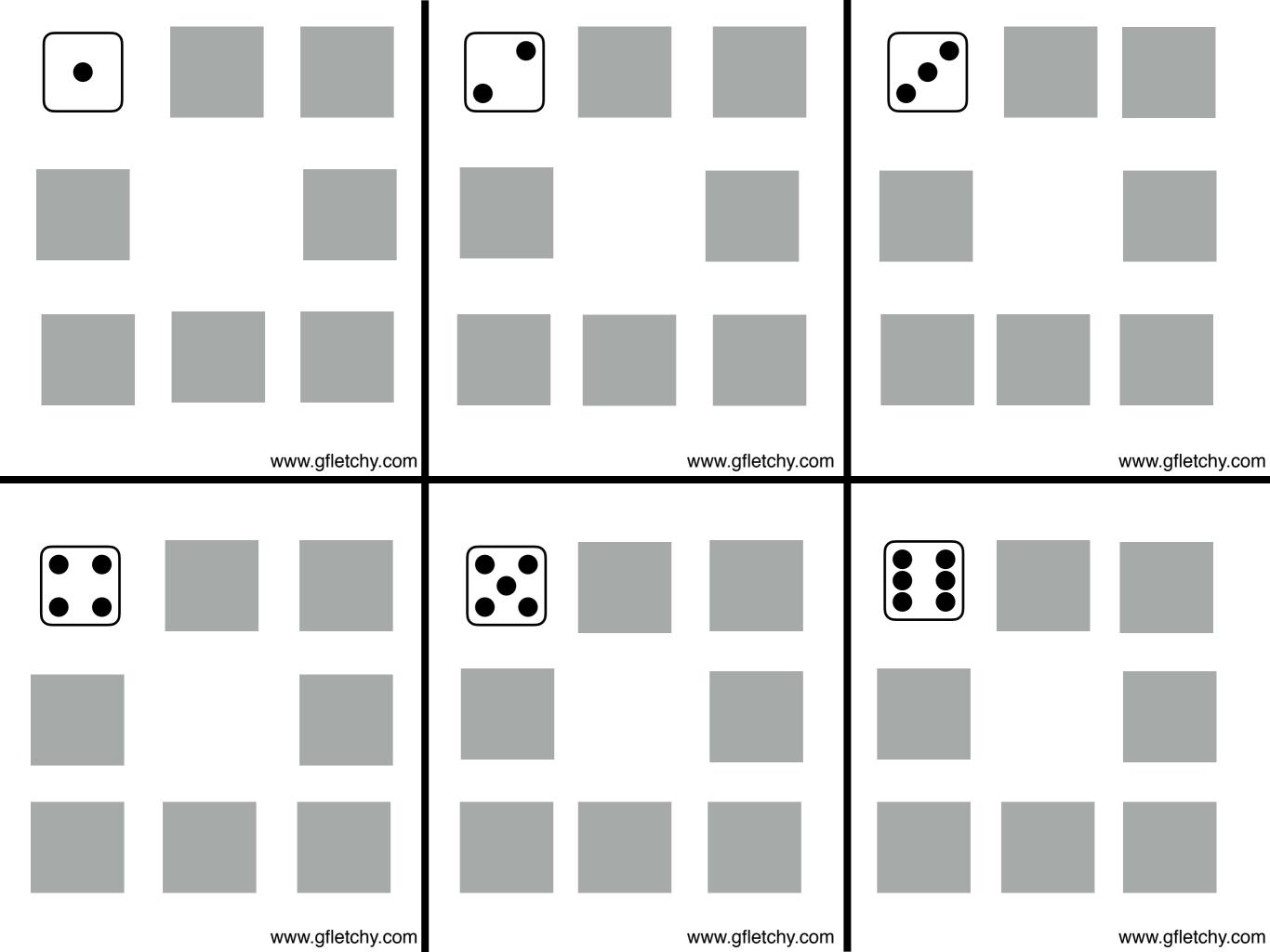


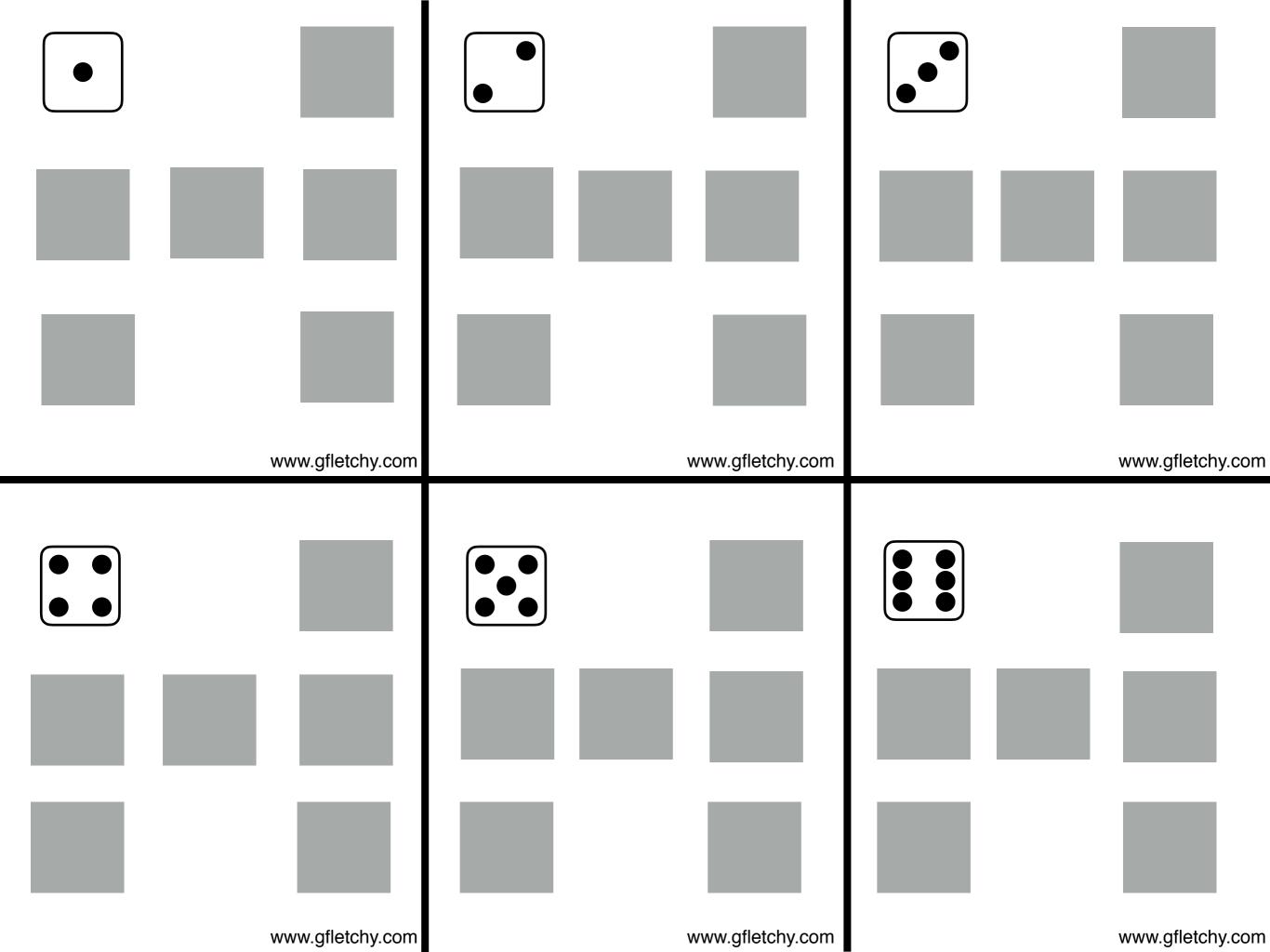


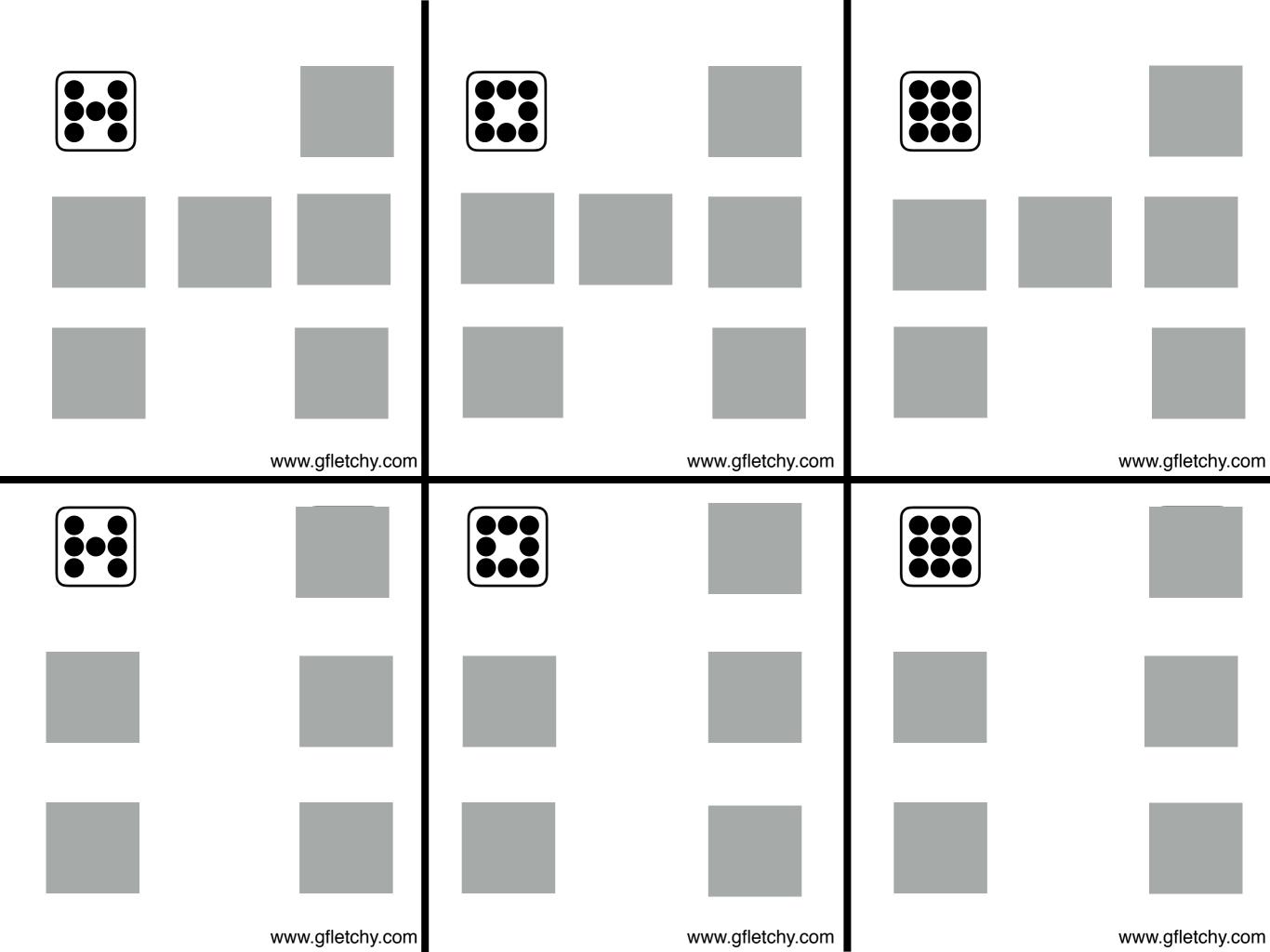
# The next set of cards is to move students away from one-to-one counting.

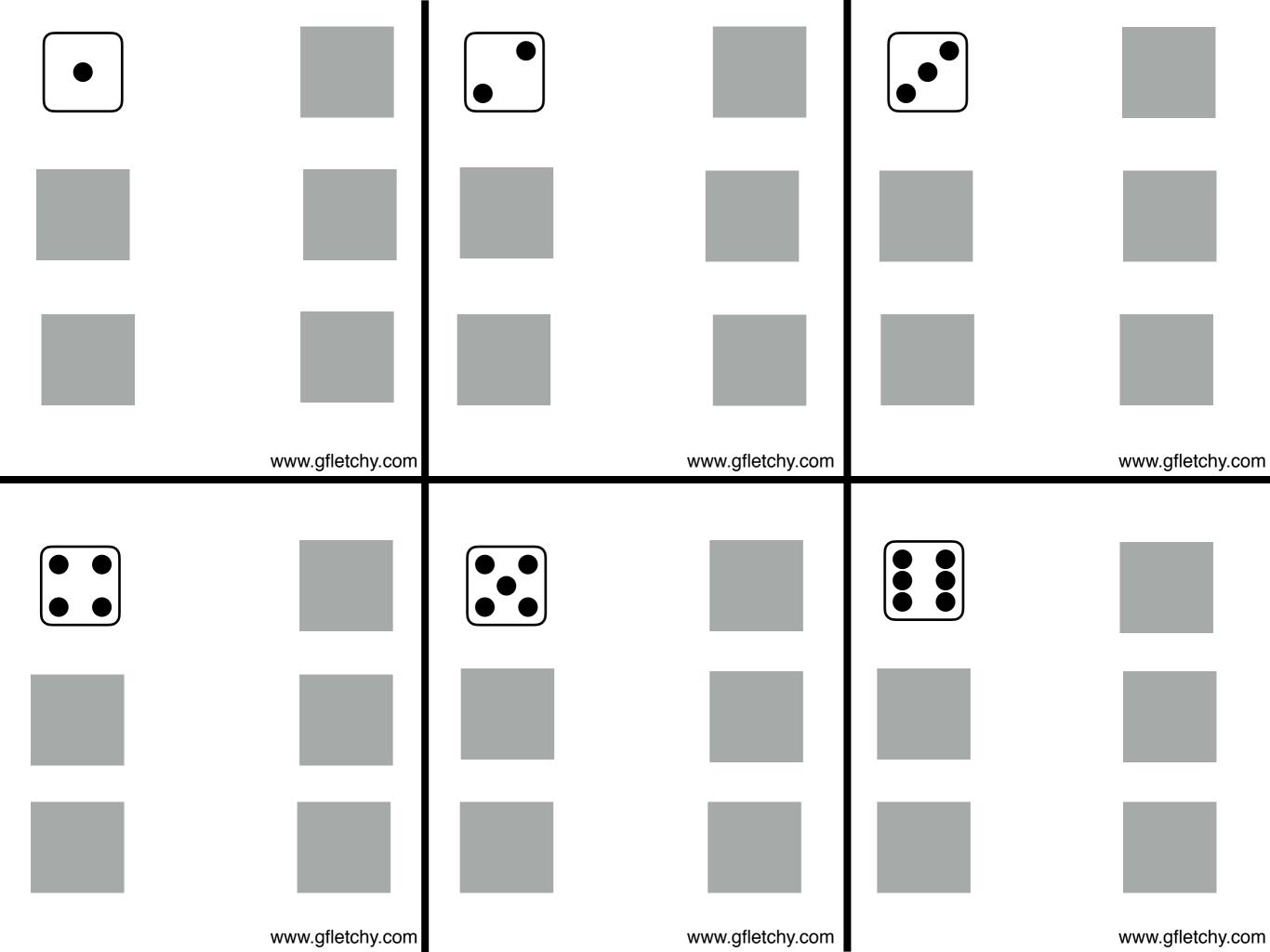


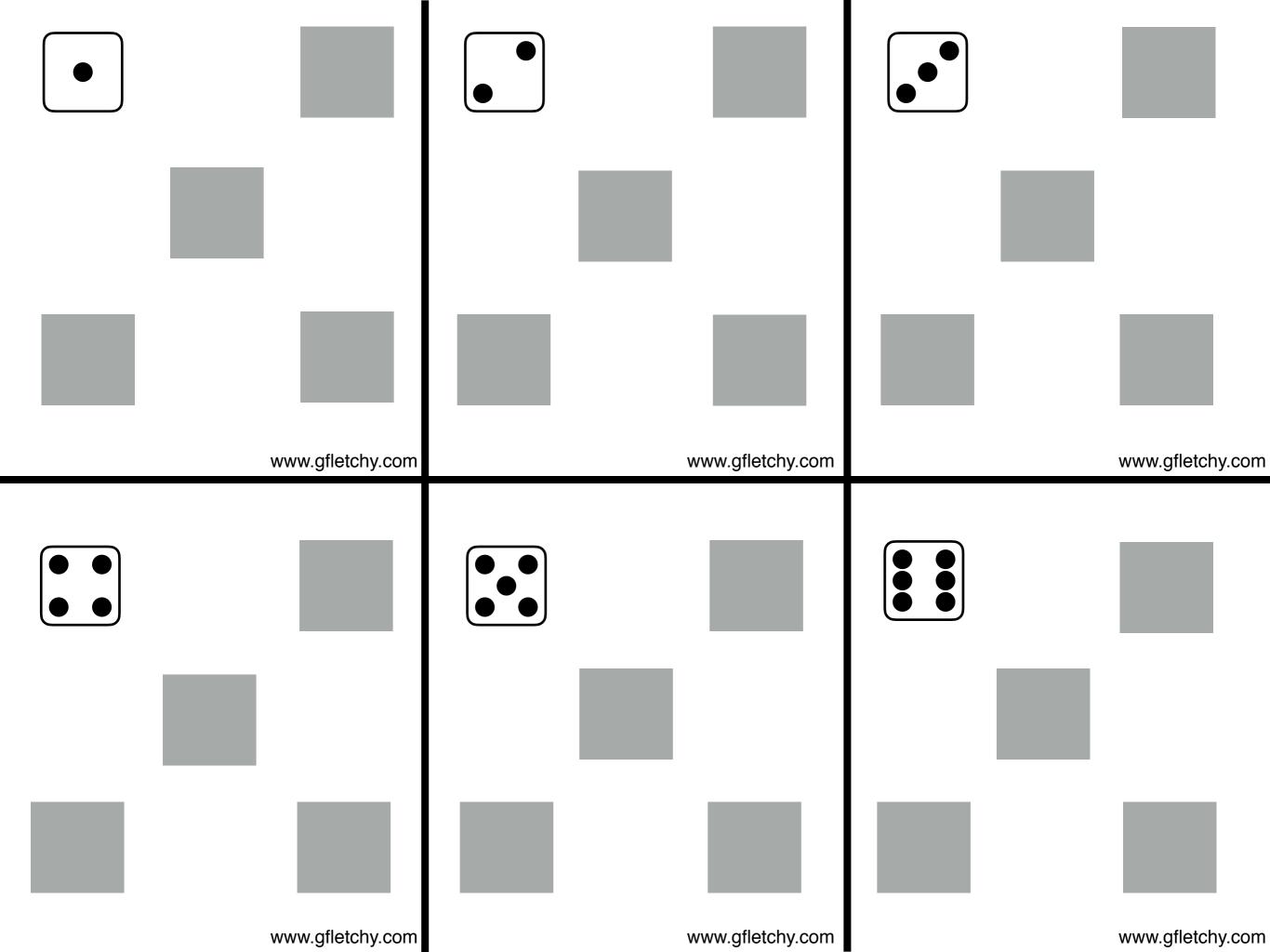


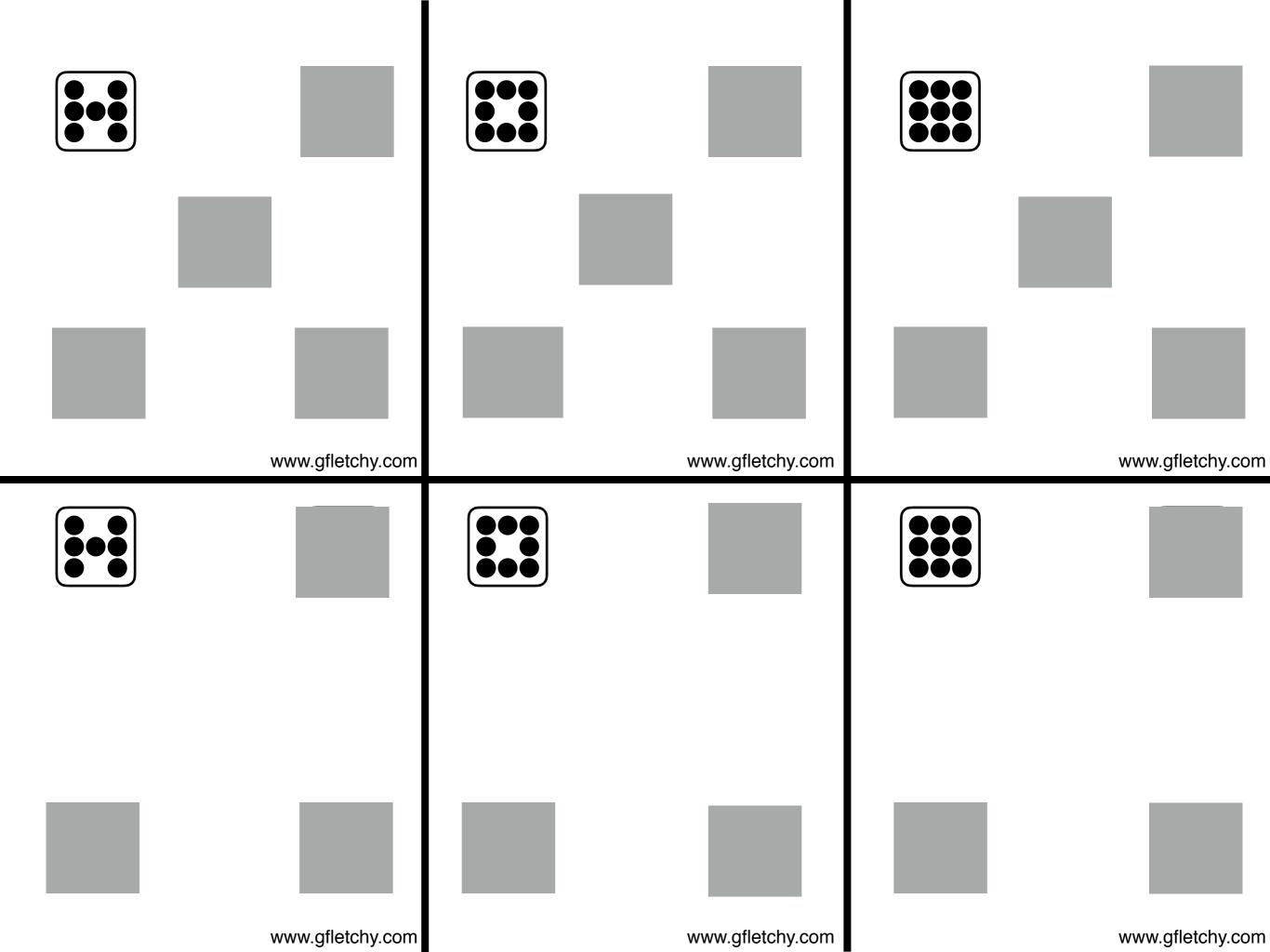


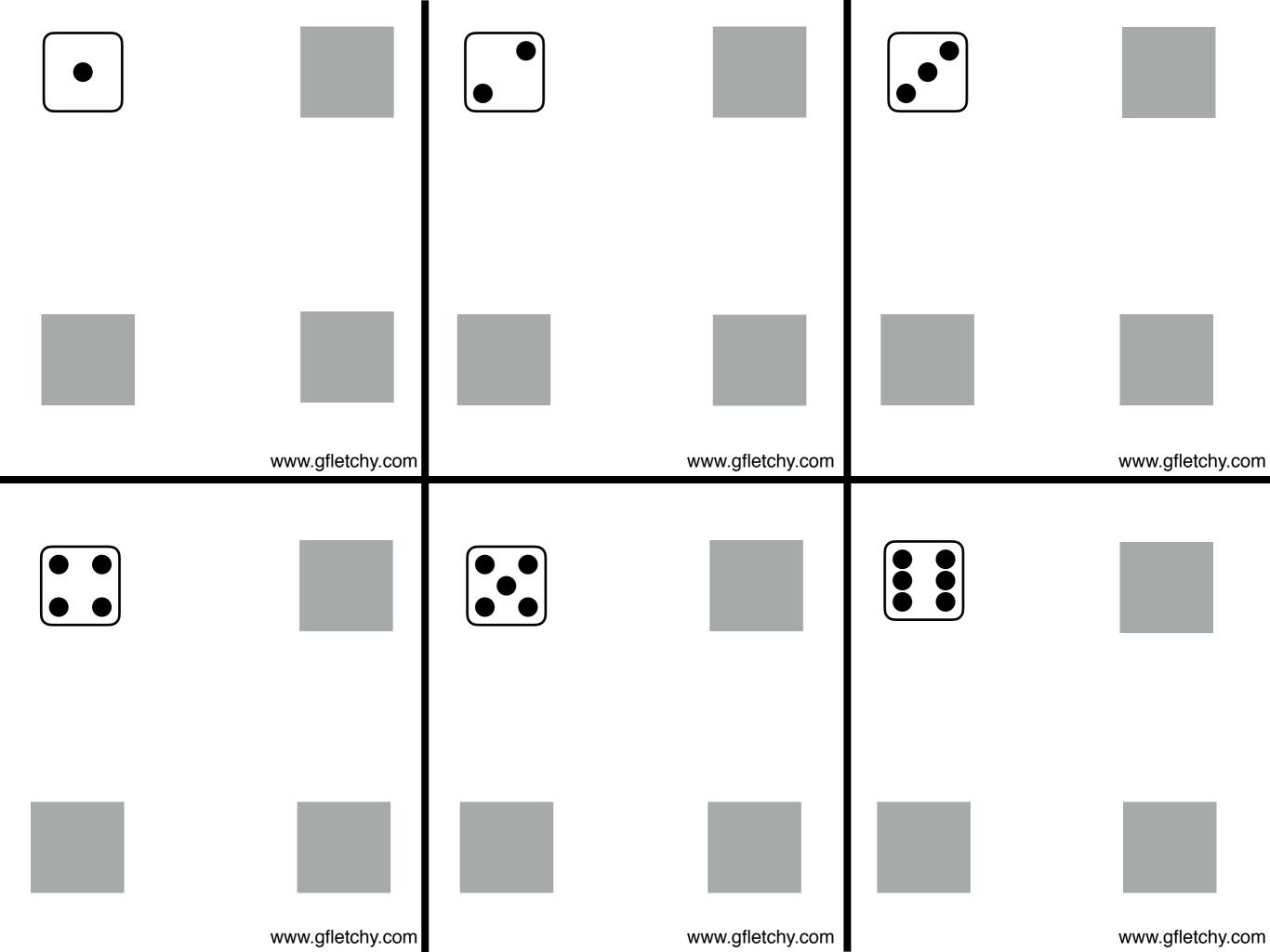


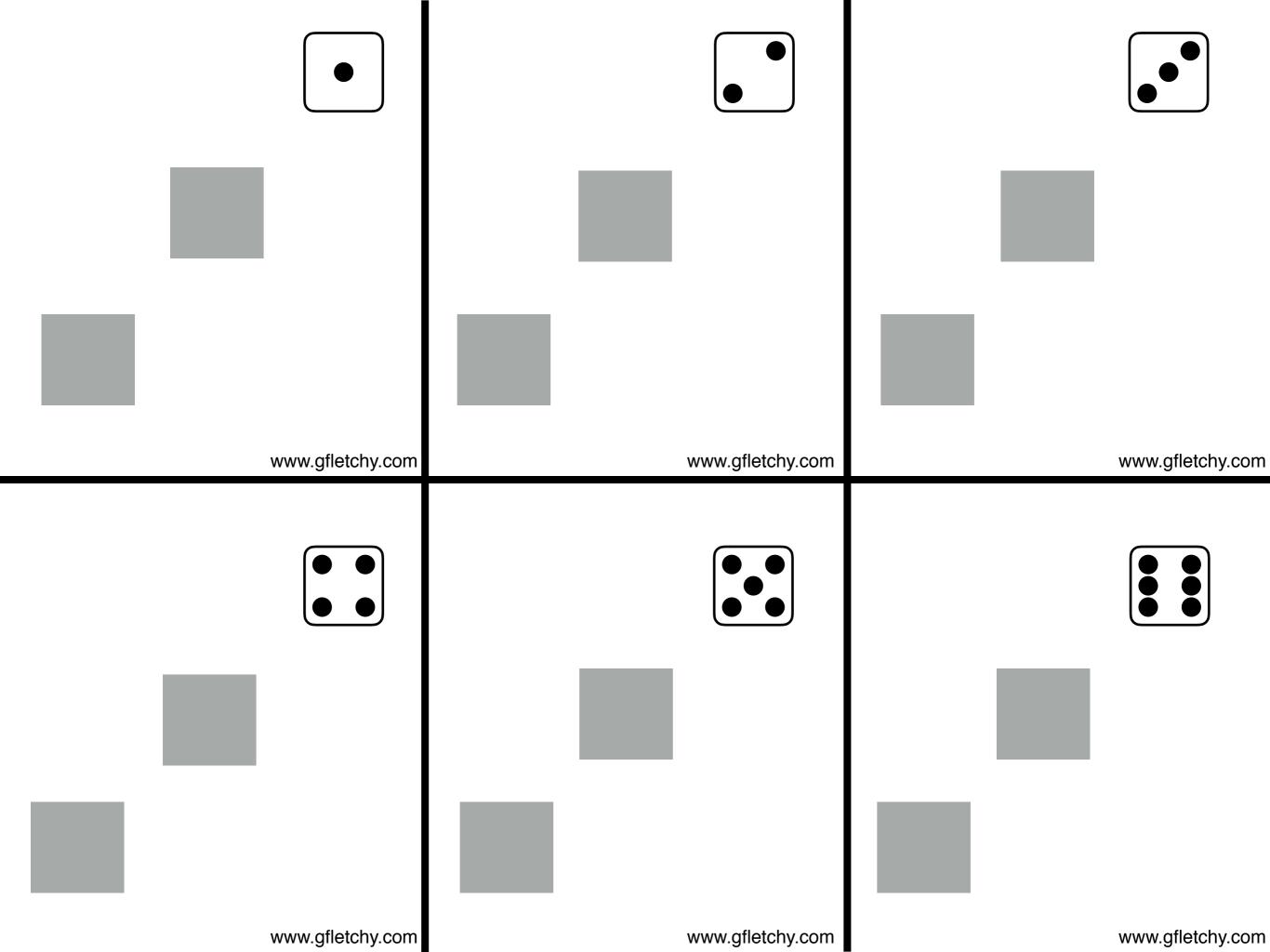


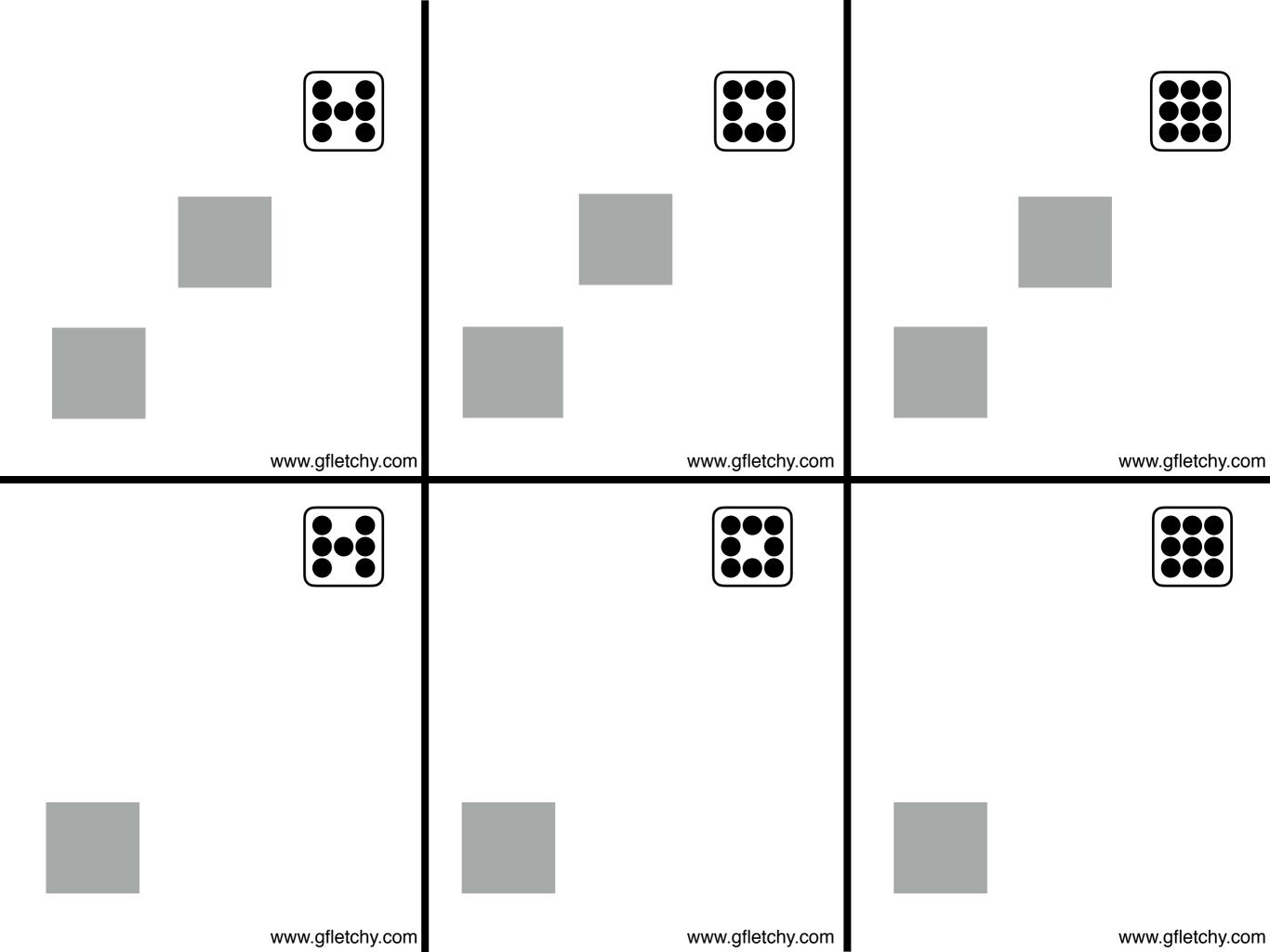


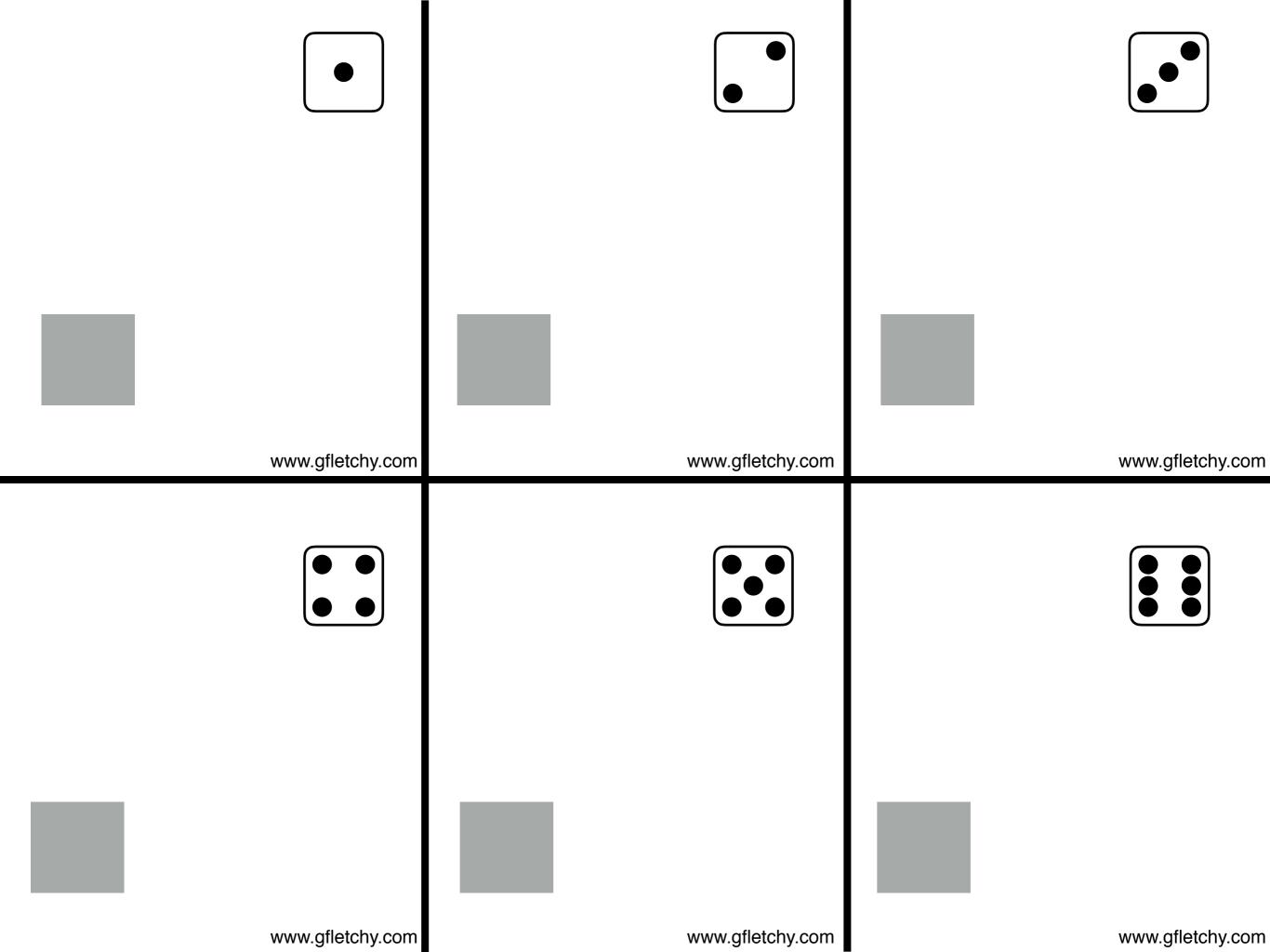






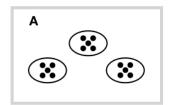


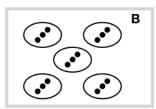




## **Math Flips: Multiplication**

2s, 5s, and 10s, Commutative Property





Version 3 - Updated May 14, 2019

 Swapped the A and B sides of pages 11-12 and 19-20 so the "easier" problem would come first.

Version 2 - Updated April 1, 2019

- Update information on first page
- New format of first page allows for folding title to same size as single card (for easy storage with deck)

#### How to Use:

Print double sided and cut along the lines. 2s Deck: pages 3-10. 5s Deck: pages 11-18. 10s Deck: pages 19-24. Also, you could mix the decks together.

With each flashcard:

- Look at side A: How many? How do you know?
- (Flip to side B) How many NOW? How do you know?

After a while, ask Generalizing and Extending questions, like:

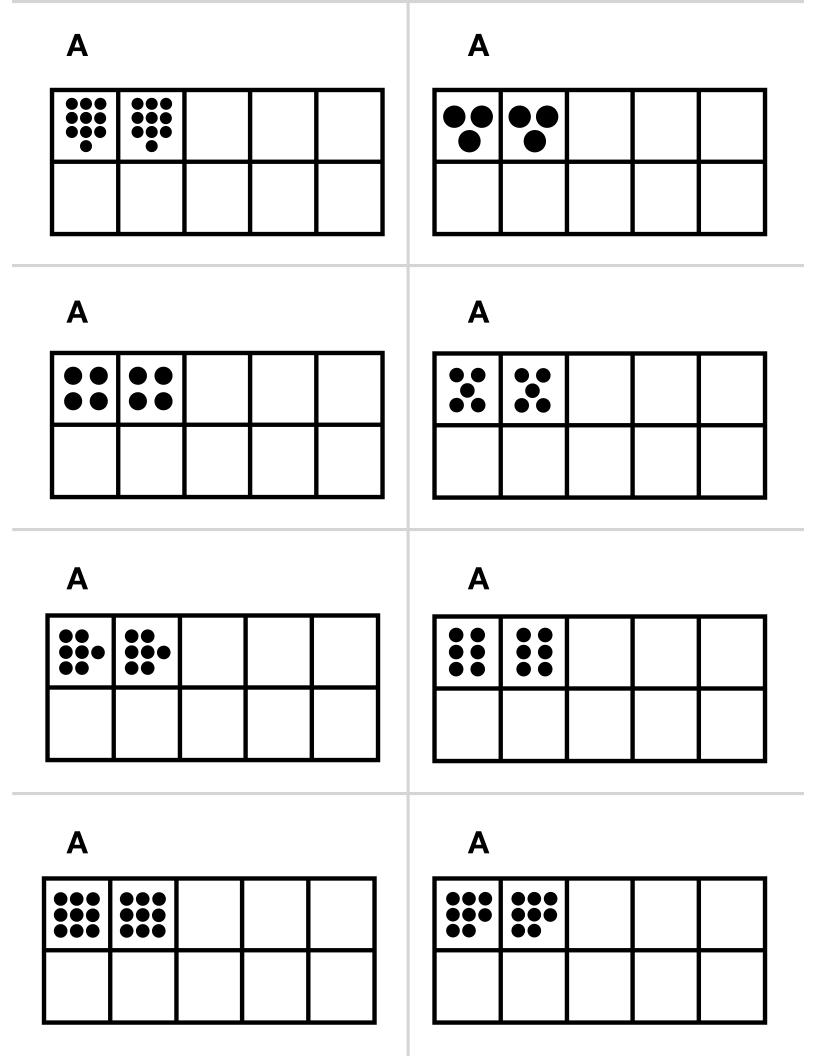
- How does side A help you with side B?
- What is the same and different about side A and side B?
- What do you notice about this deck?

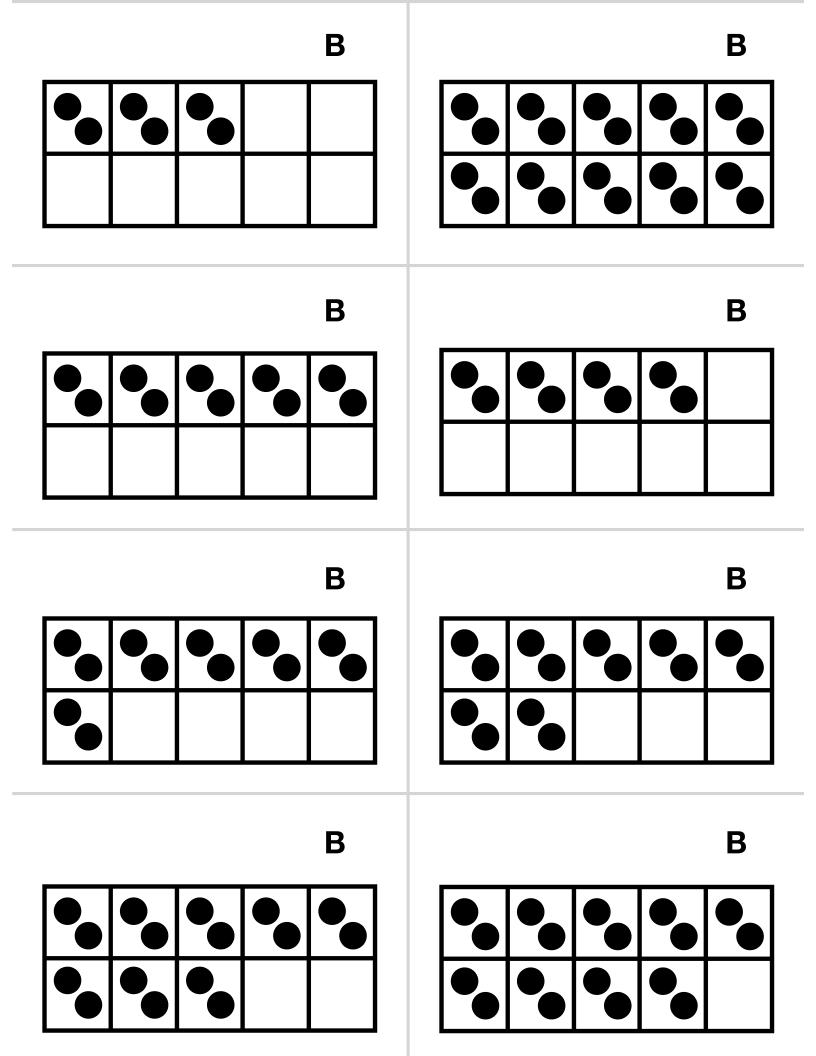
### Students Might Notice...

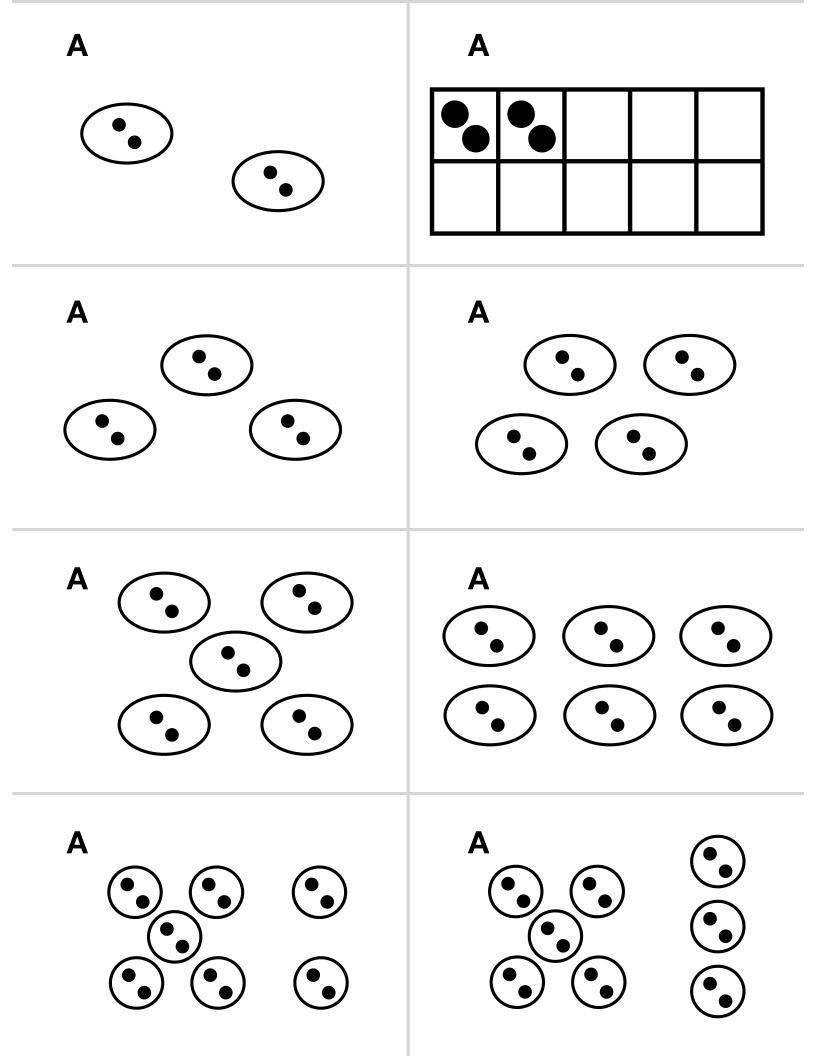
- Adding or skip counting groups of numbers is more efficient than counting by 1s, especially if
  you see the repeating groups and can add them easily or skip count by that number.
- If you know the total of 3 groups of 5, you also know the total of 5 groups of 3.
- Thinking multiplicatively is more efficient than adding or skip counting. For example: "I see 3 groups of 15, and 3 fives makes 15."

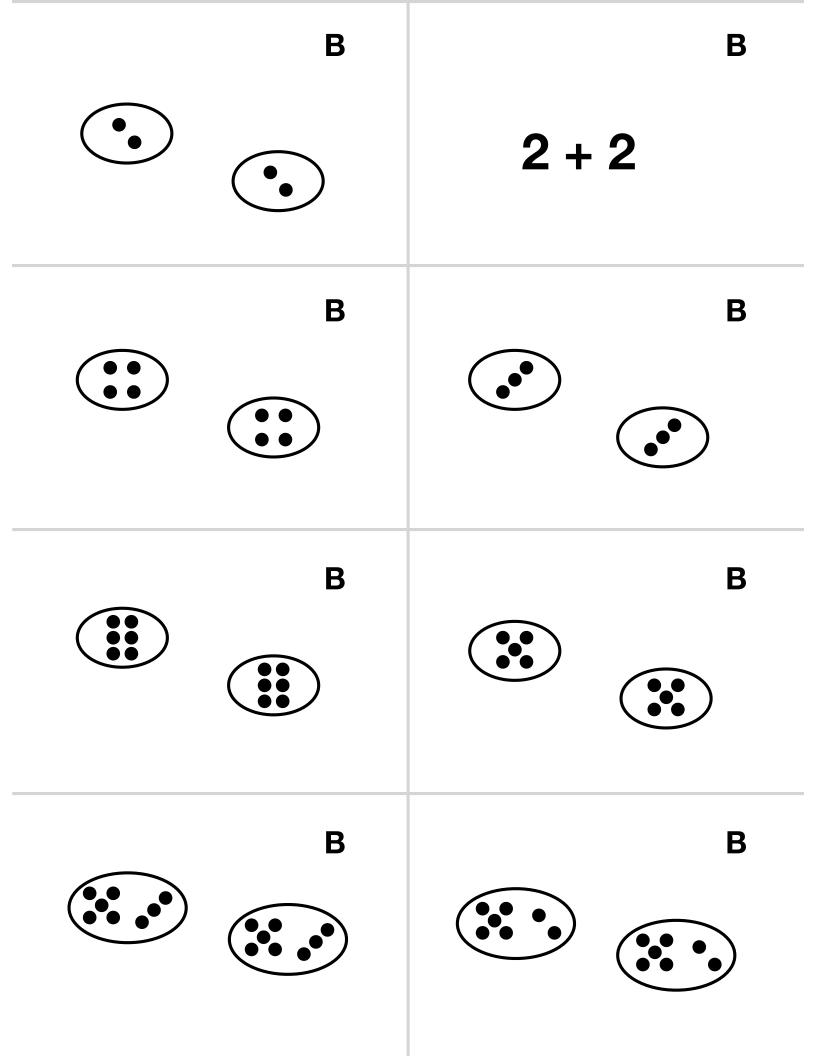
#### Keep in Mind:

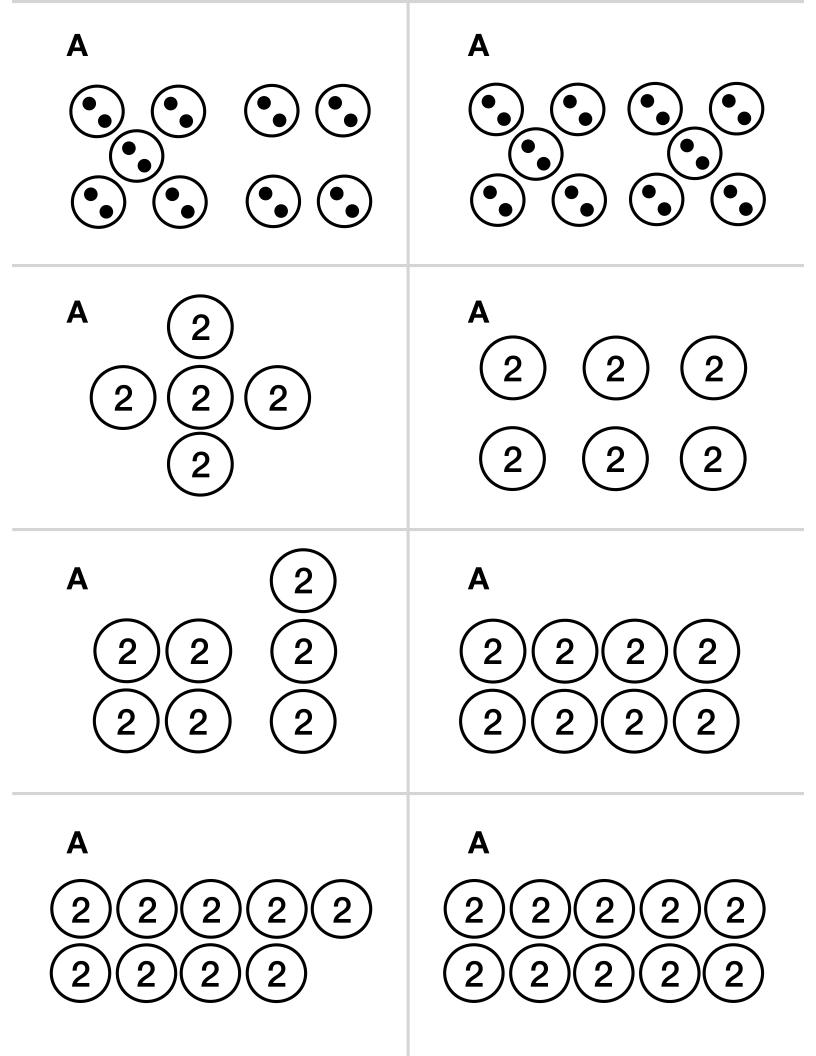
- Avoid showing students how to perform strategies. Instead, let the relationship between problems on side A and B serve as a
  conversation starter, where students can bring their own understanding and develop their own strategies!
- Learning is (and should be) messy, so the various Math Flips decks are not in any particular order. If students aren't ready to generate their own strategies with this deck, try another one and come back later!
- Although many of these cards are visual, they are more abstract than a students' own methods for acting-out/drawing/modeling a
  contextual story problem of the same concept area (like adding two single-digit numbers). Before using decks with "new" content,
  please use story problems to help students connect what they already know about the world to this new concept. Suggested
  resources:
  - Free online resources:
    - "3 Act Tasks/Lessons" various websites, start at gfletchy.com/3-act-lessons/
    - "Numberless Word Problems" various websites, start at <u>bstockus.wordpress.com/numberless-word-problems/</u>
  - · Books:
    - Children's Mathematics (Second Edition) by Thomas P Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, Susan B. Empson
    - Young Mathematicians at Work (various books by topic) by Catherine Twomey Fosnot, Maarten Dolk, William Jacob
- Other resources to develop fluency:
  - Games:
    - Tiny Polka Dot (Card Game for PreK-2nd)
    - Prime Climb (Board Game for 3rd and up)
  - Books:
    - Number Talks: Whole Number Computation, Grades K-5 by Sherry Parrish
    - Math Fact Fluency by Jennifer Bay-Williams and Gina Kling

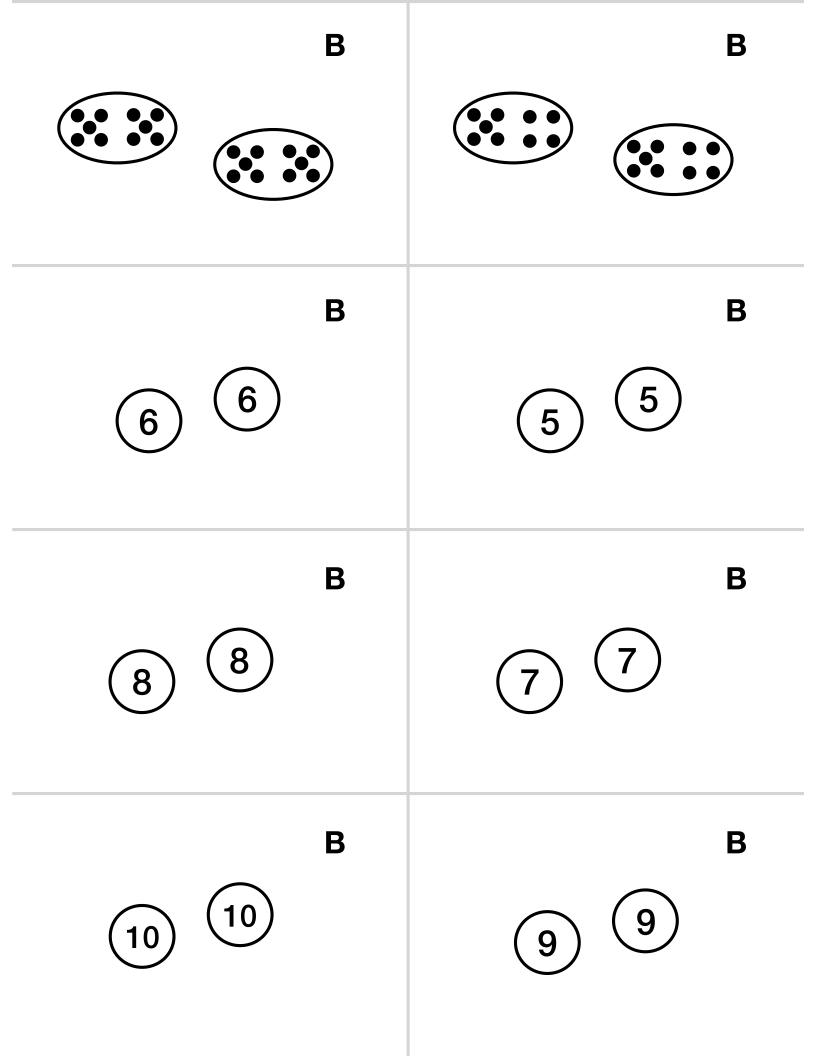








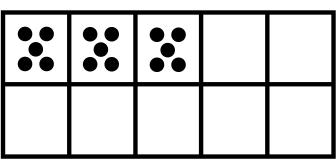




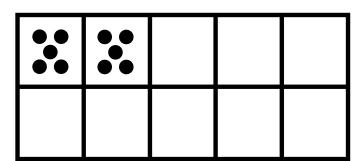
A	A
2 x 3	2 x 4
A 2 x 5	A 2 x 6
A	A
2 x 7	2 x 8
A 2 x 9	A 2 x 10

4 x 2	3 x 2
6 x 2	5 x 2
8 x 2	7 x 2
10 x 2	9 x 2

A



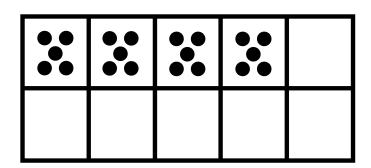
A



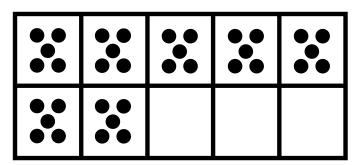
A

$$5 + 5 + 5 + 5 + 5$$

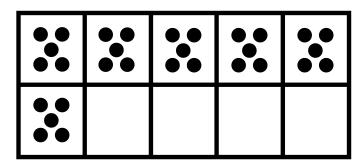
A



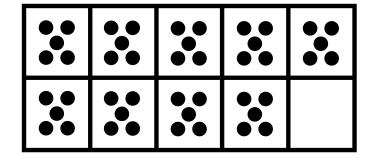
A



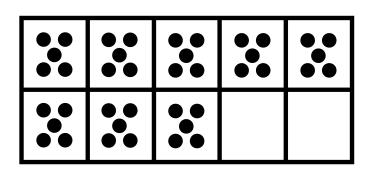
A

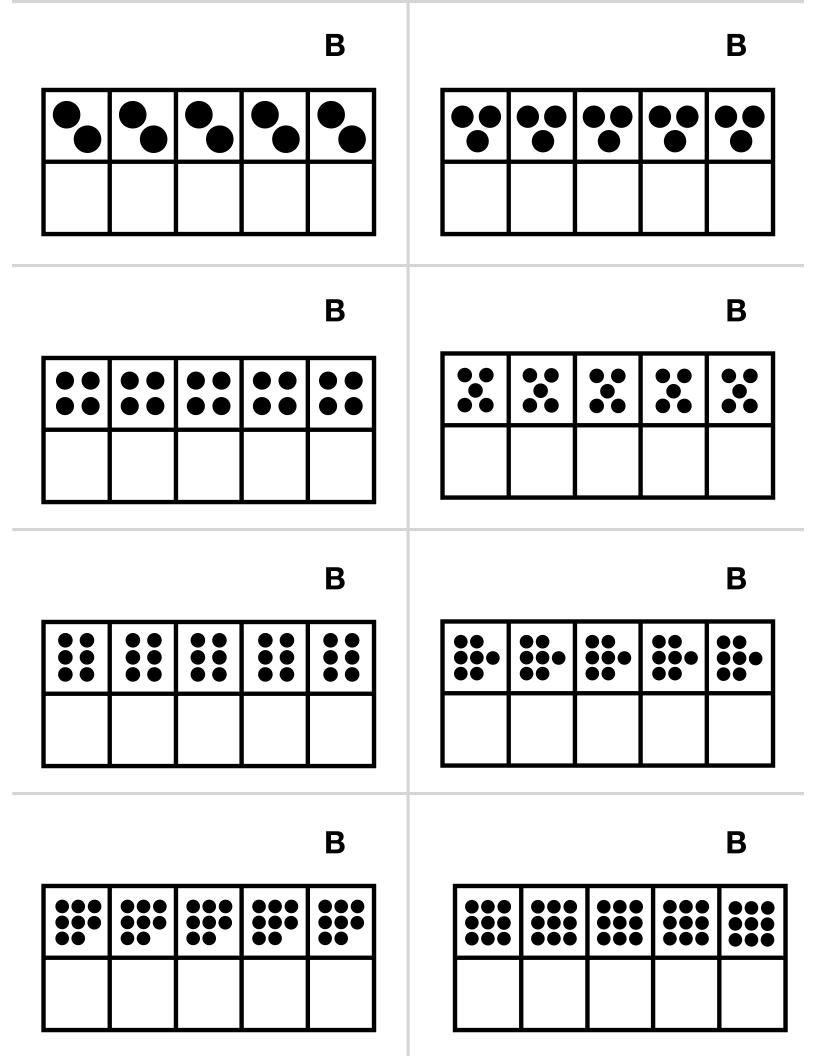


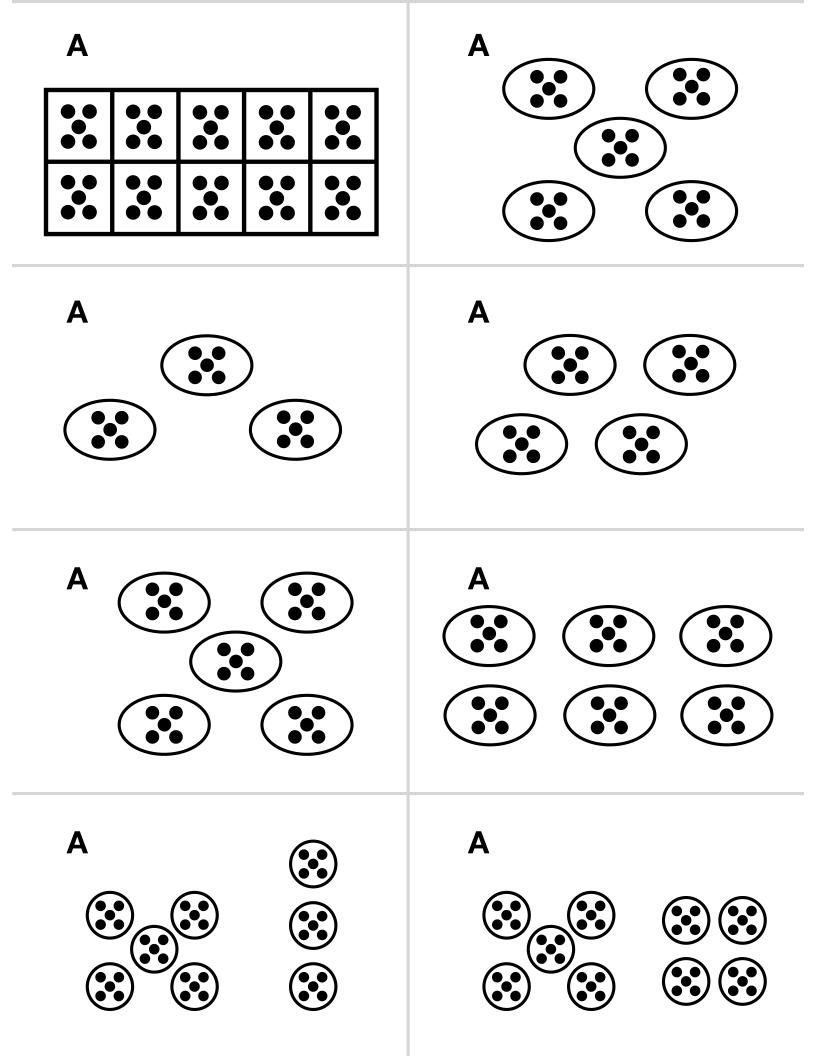
A

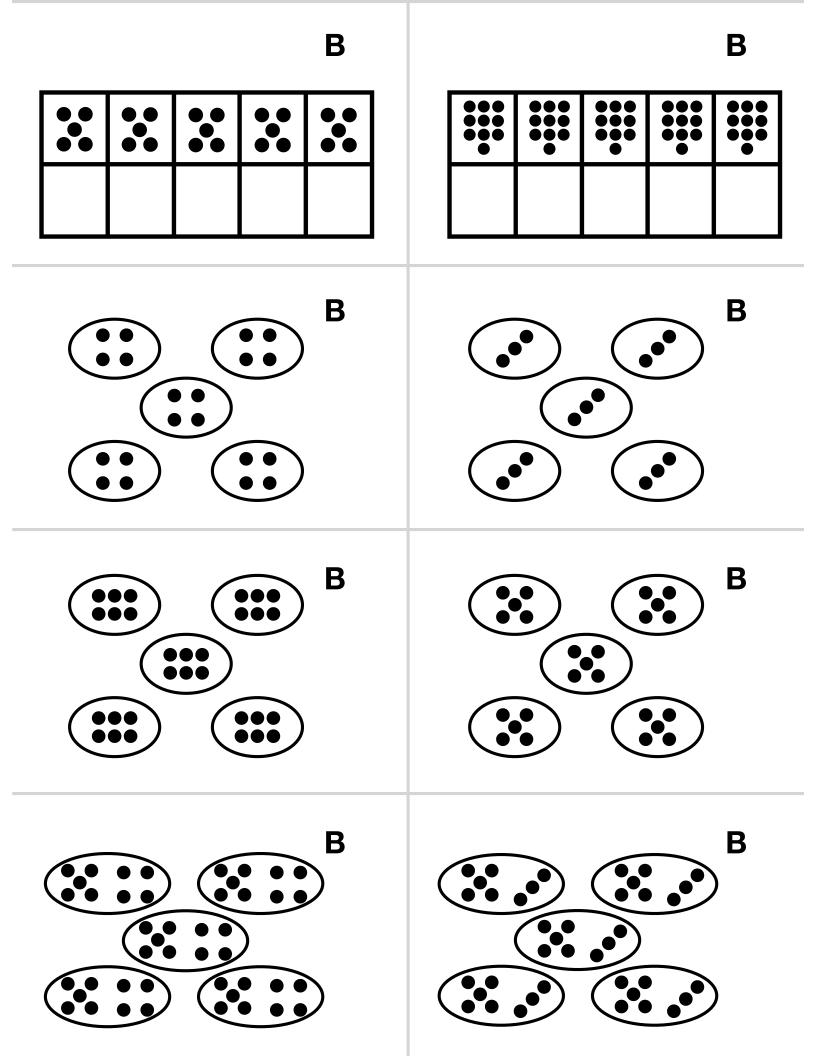


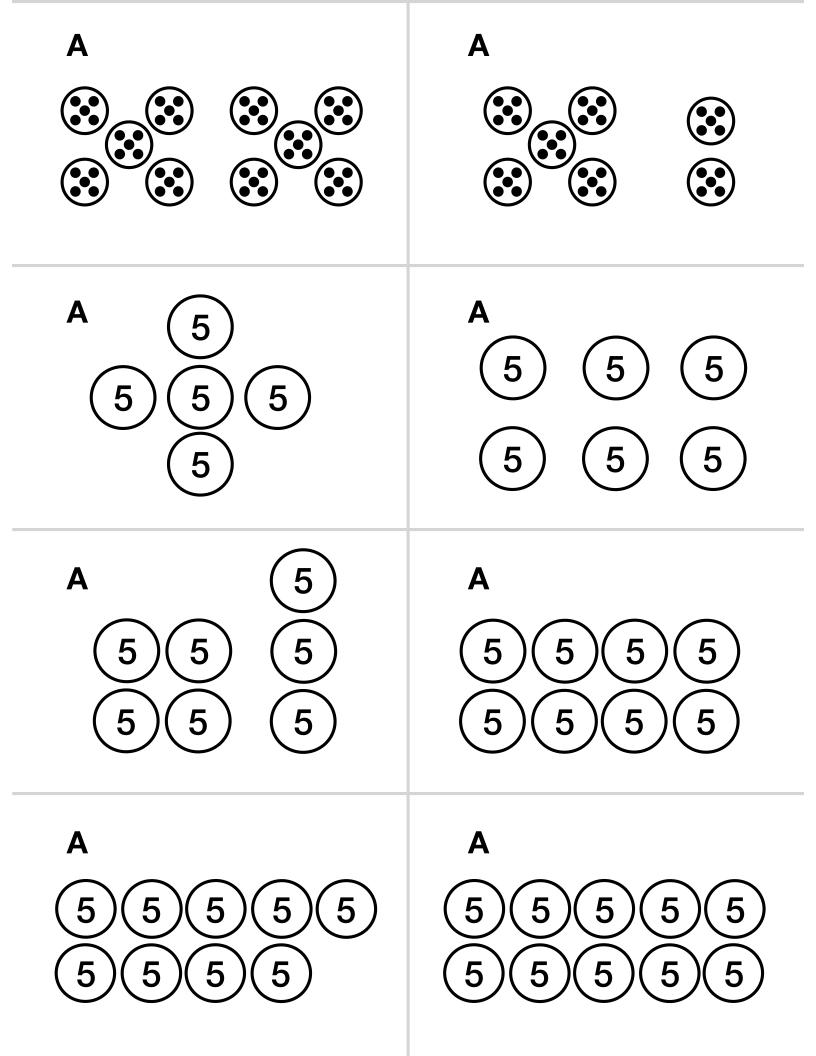
A

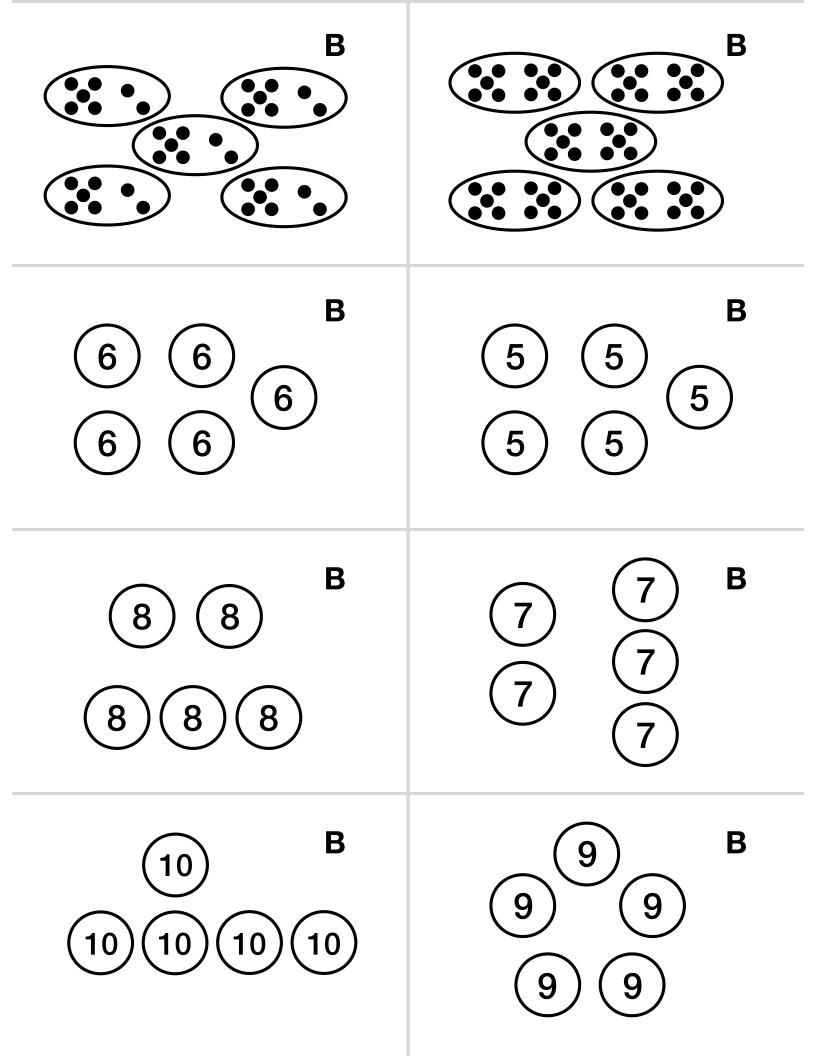






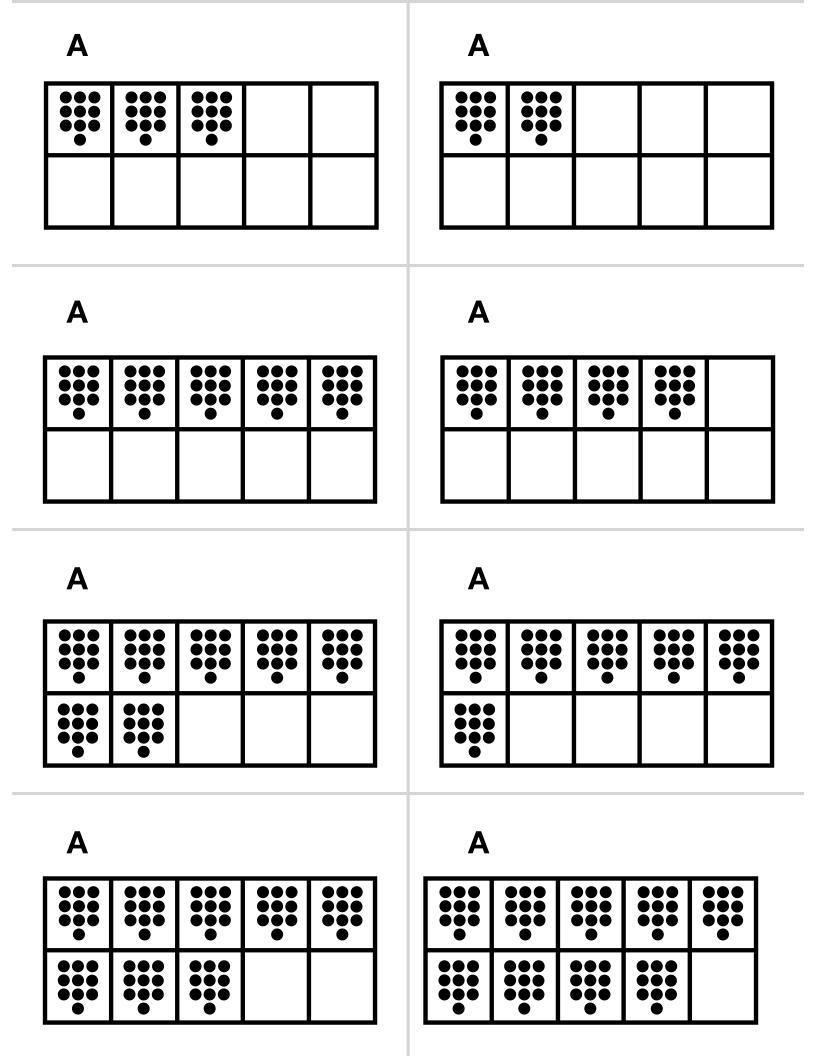


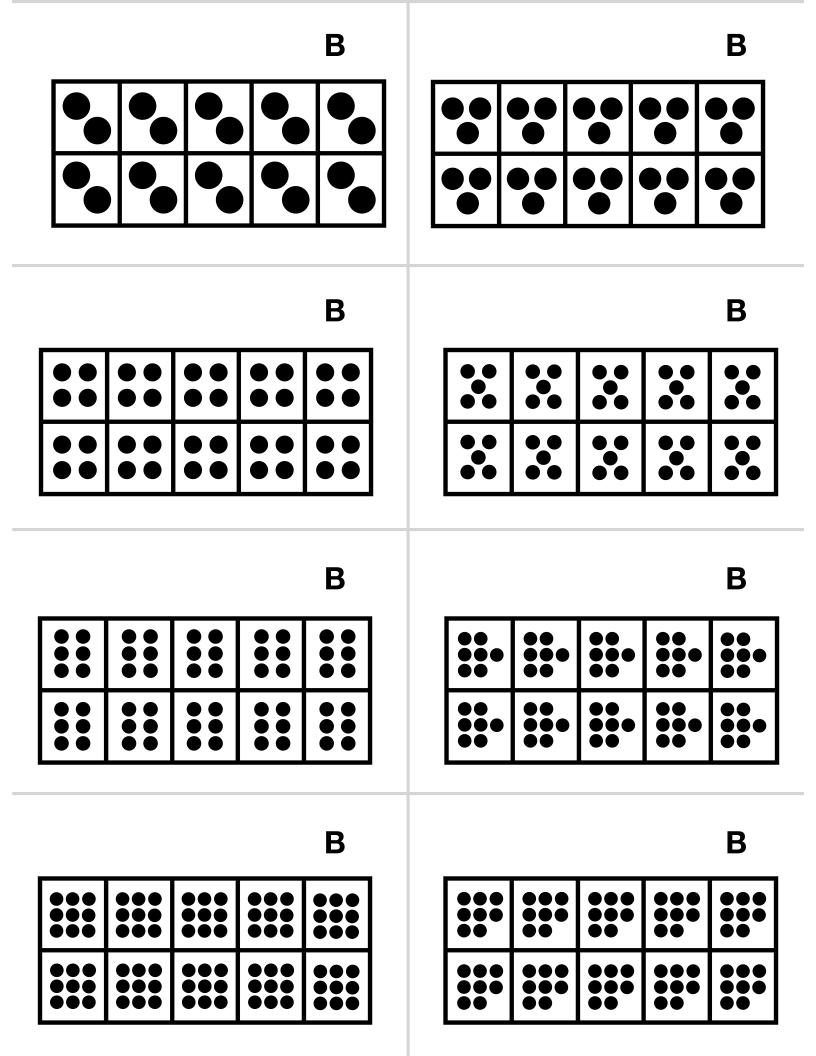


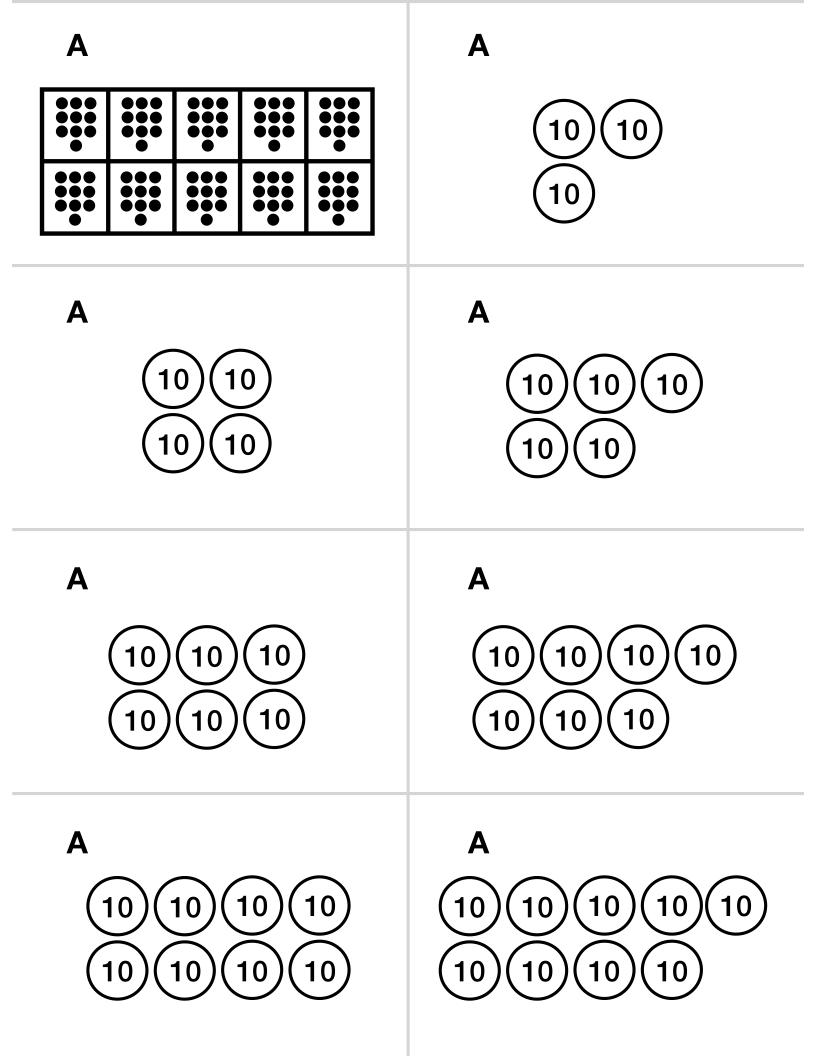


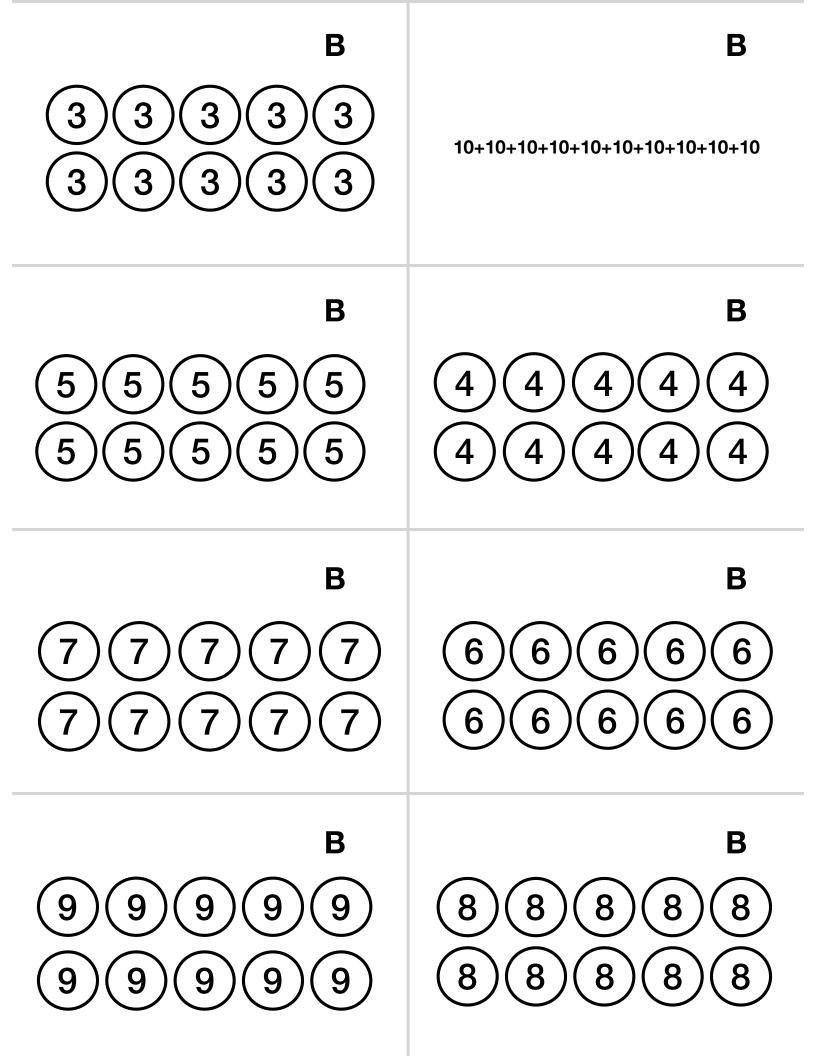
A	A
5 x 3	5 x 4
A 5 x 2	A 5 x 6
A 5 x 7	A 5 x 8
A 5 x 9	A 5 x 10

4 x 5	3 x 5
B 6 x 5	B 2 x 5
8 x 5	7 x 5
10 x 5	9 x 5









A	A
10 x 3	10 x 4
A	A
10 x 5	10 x 6
	ΙΟΧΟ
A	A
10 x 7	10 x 8
A	A
10 x 9	10 x 2

4 x 10  B  6 x 10	3 x 10 B 5 x 10
4 x 10	3 x 10
8 x 10	7 x 10
2 x 10	9 x 10

# I can equally share items into a given number of groups and find the total.

KNP # M 4444.1 - Composite Cookie Company, Red

Fluency Standard: 3.OA.7

Standard: 3.OA.2, 3.OA.1, 2.OA.4,

Materials: cookies and plates cut-outs, task cards, recording sheet (many also use loose counters and paper plates instead of printables, if desired)

#### **Directions:**

- 1. Draw a card.
- 2. Use the cookies and plates cutouts to help you find the answer.
- 3. Record on your recording sheet and repeat until your sheet is full.

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# Printables for "Composite Cookie Company"

#### KNPIG ID # M 4444.1 - RED

This file contains printables for a small group of students.

For each additional group of students print 1 new printable file.

- Dirctions for Printing.
- Task Cards 6 pages: 8 cards per page. (48 task cards in total.)
- Blank Task Cards 1 Page: 8 Cards.
- Plate Cards 2 sets: 12 cards in total.
- Cookie Cards 2 sets: 42 cards in total.
- 1 Recording Sheet

**Teacher Notes:** No formal "multiplication or division" language is used with this activity, because the purpose is simply to give students visual representations of equal, multiple groups and practice with developing strategies to find "how many" among the groups and to practice dividing items equally. Make time for students to use this activity several times, as the repetition supports multiplicative understanding.

#### Directions for Printing:

Print one copy each of the task cards and print two copies each of the plates and cookies pages. For durability, print materials on cardstock and then laminate. If printing is not available, paper plates and counters can be substituted.

Print one copy of recording sheet for each student.

Get five plates. Put four cookies on each plate.

How many cookies?

#### Composite Cookie Company

Get 16 cookies. Put the cookies on plates in groups of four. How many plates?

Any leftovers?

#### Composite Cookie Company

Get 12 cookies. Share the cookies equally on three plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get five plates. Put three cookies on each plate. How many cookies?

# Composite Cookie Company

Get 16 cookies. Put the cookies on plates in groups of five. How many plates?

Any leftovers?

# Composite Cookie Company

Get 12 cookies. Share the cookies equally on four plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get five plates. Put five cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 15 cookies. Put the cookies on plates in groups of four. How many plates?

Any leftovers?

Get 12 cookies. Share the cookies equally on two plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get five plates. Put six cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 15 cookies. Put the cookies on plates in groups of five. How many plates?

Any leftovers?

#### Composite Cookie Company

Get 15 cookies. Share the cookies equally on three plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get five plates. Put seven cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 14 cookies. Put the cookies on plates in groups of four. How many plates?

Any leftovers?

#### Composite Cookie Company

Get 10 cookies. Share the cookies equally on three plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get six plates. Put two cookies on each plate. How many cookies?

Get 13 cookies. Put the cookies in groups of four. How many plates?

Any leftovers?

## Composite Cookie Company

Get six plates. Put three cookies on each plate.
How many cookies?

#### Composite Cookie Company

Get 16 cookies. Share the cookies equally on four plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get 17 cookies. Put the cookies on plates in groups of five. How many plates?

Any leftovers?

#### Composite Cookie Company

Get 14 cookies. Share the cookies equally on three plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get 14 cookies. Put the cookies on plates in groups of two. How many plates?

Any leftovers?

#### Composite Cookie Company

Get six plates. Put four cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 16 cookies. Share the cookies equally on five plates. How many on each plate? Any leftovers?

Get six plates. Put five cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 12 cookies. Put the cookies on plates in groups of five. How many plates?

Any leftovers?

#### Composite Cookie Company

Get 20 cookies. Share the cookies equally on three plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get six plates. Put six cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 16 cookies. Put the cookies on plates in groups of three. How many plates? Any leftovers?

#### Composite Cookie Company

Get 20 cookies. Share the cookies equally on four plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get six plates. Put seven cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 10 cookies. Put the cookies on plates in groups of two. How many plates?

Any leftovers?

Get 10 cookies. Share the cookies equally on two plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get 13 cookies. Put the cookies on plates in groups of three. How many plates? Any leftovers?

#### Composite Cookie Company

Get seven plates. Put three cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 17 cookies. Share the cookies equally on three plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get seven plates. Put four cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 14 cookies. Share the cookies equally on two plates. How many on each plate? Any leftovers?

#### Composite Cookie Company

Get 16 cookies. Put the cookies on plates in groups of eight. How many plates?

Any leftovers?

#### Composite Cookie Company

Get eight plates. Put five cookies on each plate. How many cookies?

Get 9 cookies. Put the cookies on plates in groups of four. How many plates?

Any leftovers?

# Get 18 cookies. Share the cookies equally on three plates. How many on each

Composite Cookie Company

plates. How many on eac plate? Any leftovers?

#### Composite Cookie Company

Get eight plates. Put two cookies on each plate.

How many cookies?

#### Composite Cookie Company

Get 9 cookies. Put the cookies on plates in groups of three. How many plates? Any leftovers?

#### Composite Cookie Company

Get 17 cookies. Share the cookies equally on four plates. How many on each plate? Any leftovers?

### Composite Cookie Company

Get eight plates. Put three cookies on each plate. How many cookies?

#### Composite Cookie Company

Get 18 cookies. Put the cookies on plates in groups of six. How many plates?

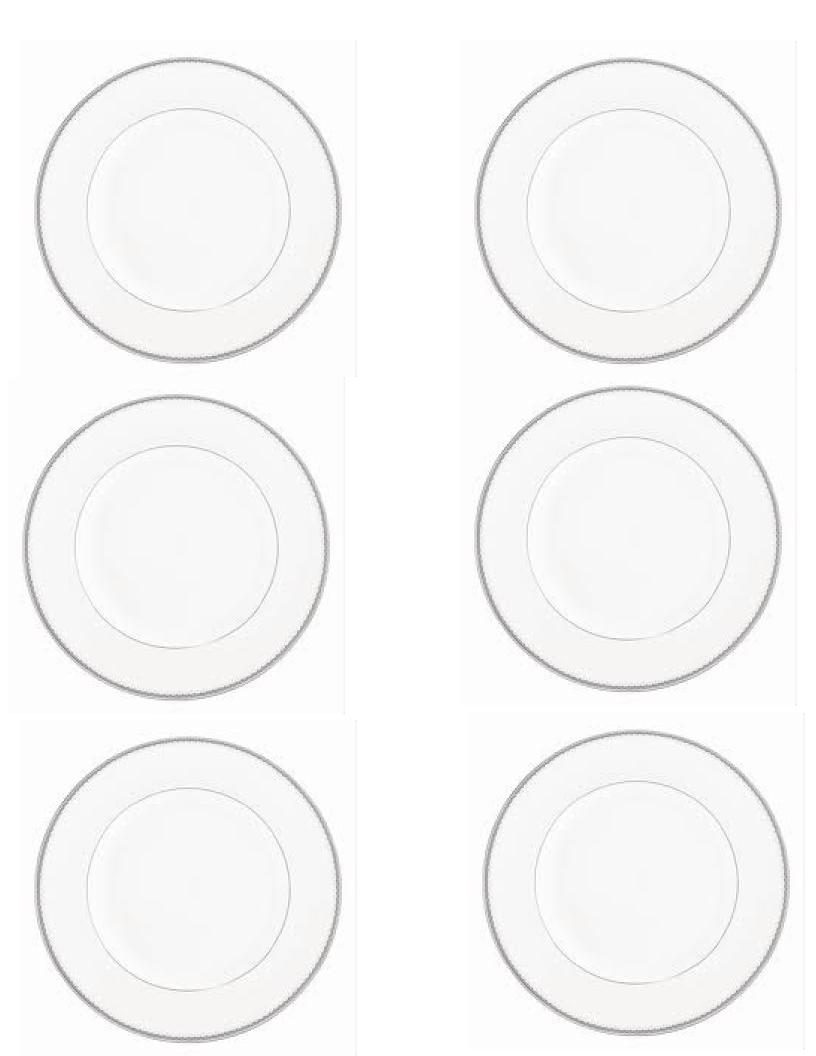
Any leftovers?

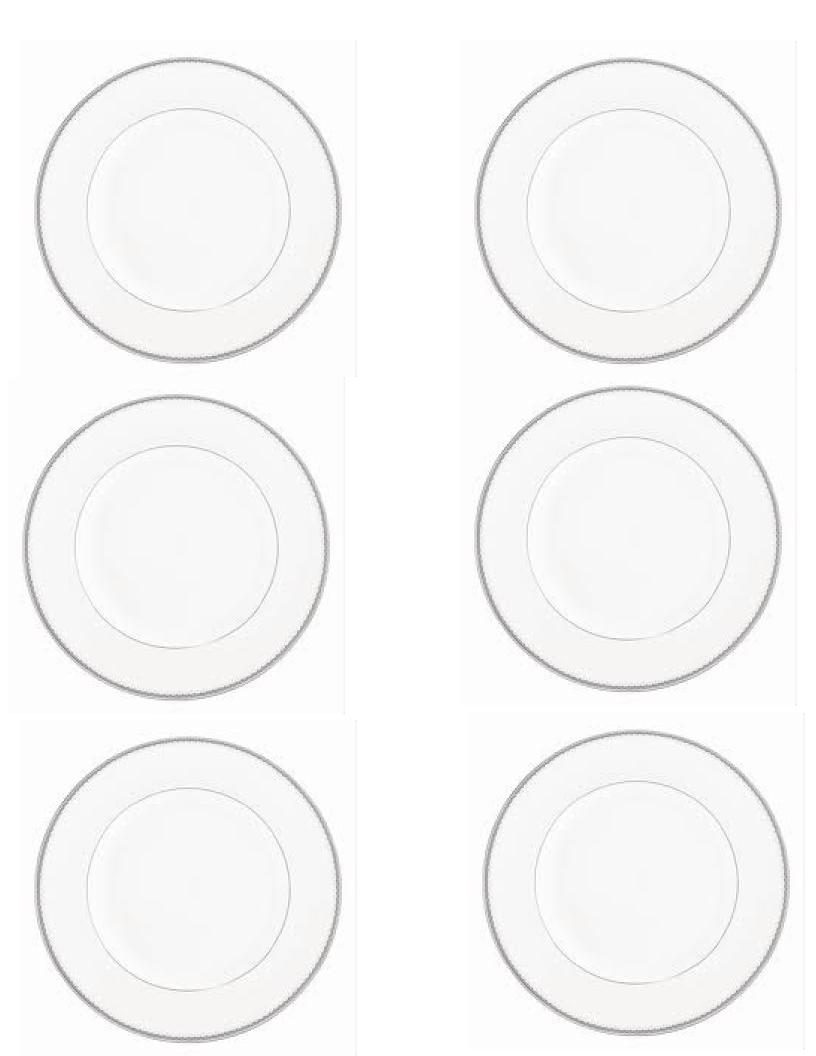
#### Composite Cookie Company

Get 7 cookies. Share the cookies equally on three plates. How many on each plate? Any leftovers?

#### M4444.1

M4444.1	
Composite Cookie Company	Composite Cookie Company
Composite Cookie Company	Composite Cookie Company
Composite Cookie Company	Composite Cookie Company
Composite Cookie Company	Composite Cookie Company









# Composite Cookie Company Recording sheet

How many plates?	How many cookies on	How many cookies	Any cookies	I found the answer by
	each plate?	altogether?	left over?	

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	<b>\(\frac{1}{2}\)</b>	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

