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FOR MATHEMATICS

Developing Multiplicative Thinking-

Foundations of Multiplicative Thinking with Julie Adams

Welcome!

Your host

Julie Adams

Regional Consultant
Kentucky Center for Mathematics
jaadams2@moreheadstate.edu

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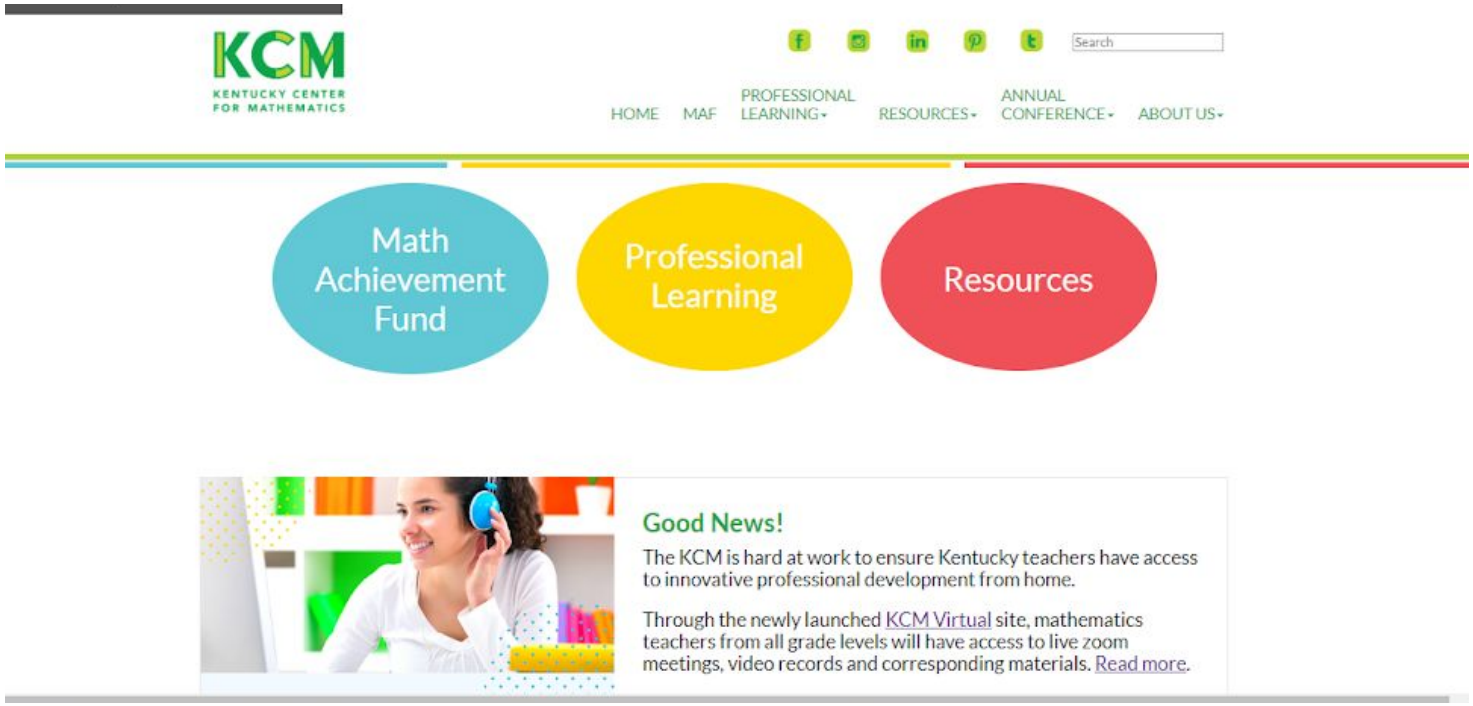

https://www.kentuckymathematics.org/

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Agenda

- A look at the Ky Academic Standards in the early grades
- What is Multiplicative thinking?
- Progression of Mathematical Reasoning
- Counting Strategies to Additive Strategies
- Activities to Support Multiplicative thinking

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Standards

Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Work with equal groups of objects to gain foundation for multiplication.

Standards	Clarifications
KY.2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members; write an equation to express an even number as a sum of two equal addends. **MP.2, MP.7**	Students understand a number can be broken apart by pairing objects to see if there are leftovers (odd) or not (even).  Coherence KY.1.OA.7→KY.2.OA.3→KY.3.OA.9
KY.2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **MP.2, MP.4**	Students model using rectangular arrays to determine the number of objects and discuss their reasoning. For example the array shows 4 + 4 + 4 + 4 + 4 =20 or 5 + 5 + 5 + 5 = 20  Coherence KY.1.OA.7→KY.2.OA.4→KY.3.OA.1

Attending to the Standards for Mathematical Practice

Students use contexts and visuals to reason about whether numbers are even or odd (**MP.2**). They notice if a number can be decomposed (broken apart) into two equal addends (16 = 8+8), then it is even, or if they group the number in twos it is even (**MP. 7**). They build on the idea of two equal sized groups to adding more equal sized groups. Students use concrete objects (counters) and pictorial representations (arrays) to explore repeated addition of equal sized groups (**MP. 5**). Students recognize in a rectangular array there are two ways to have same sized groups (rows or columns) and they can choose either way to find the total (**MP.2**).

Multiplicative Thinking: Definition

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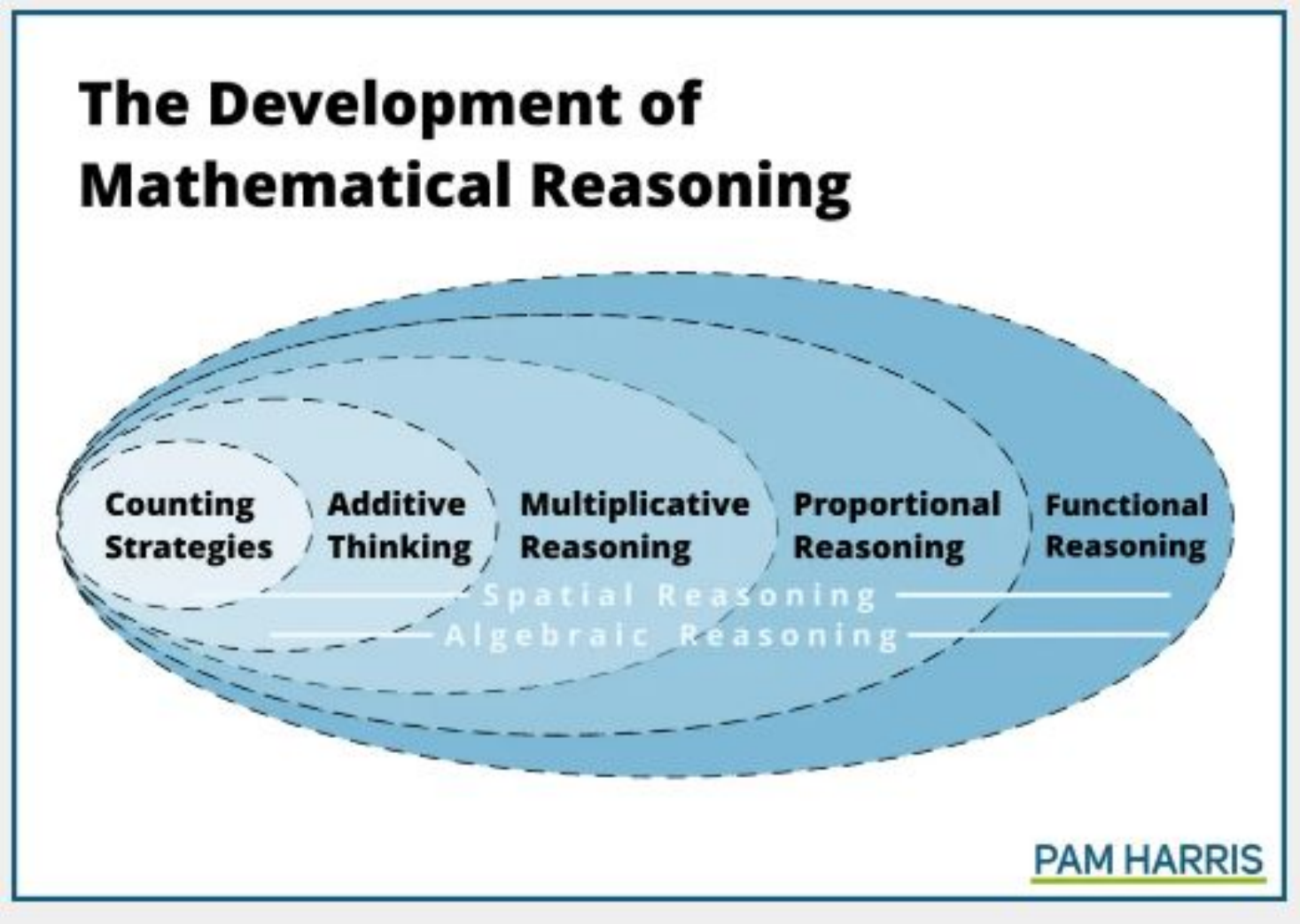
Progression of Mathematical Reasoning



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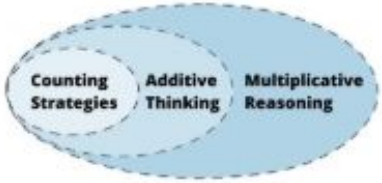
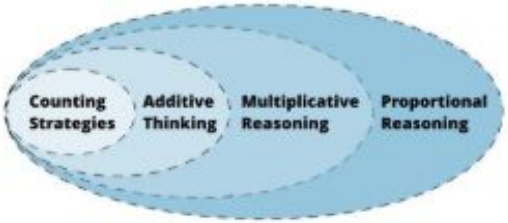
Development of Reasoning

Students need to develop each level of reasoning so that they can build on it for the next level.

It is important for students to develop counting strategies because counting is essential in the development of additive thinking.	
And additive thinking is essential for students to develop multiplicative reasoning so that they can use multiplicative strategies.	
Without multiplicative reasoning, it is impossible to develop proportional reasoning, which is the land of fractions, ratios, proportions, and percents.	

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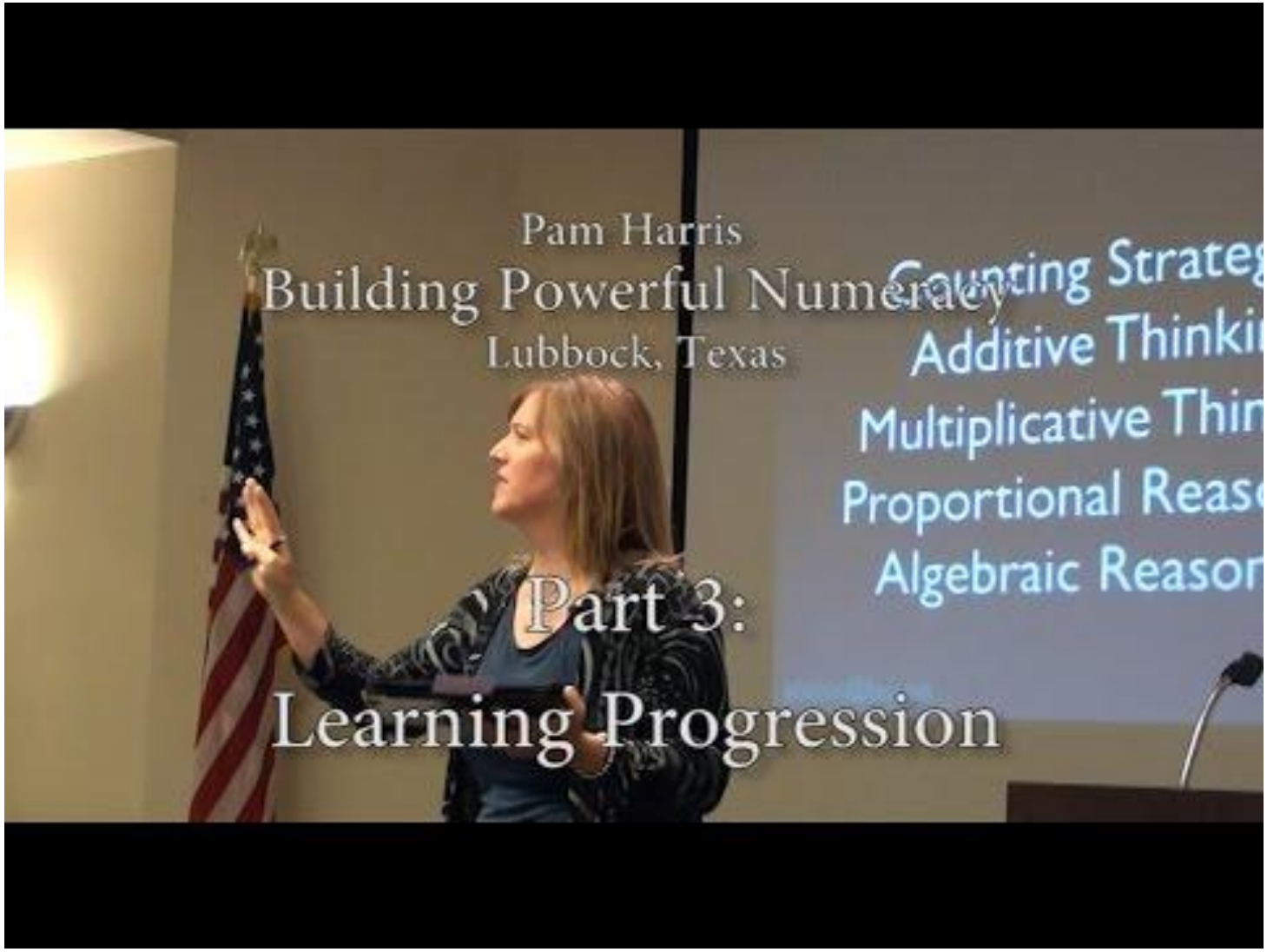
Why is it important?



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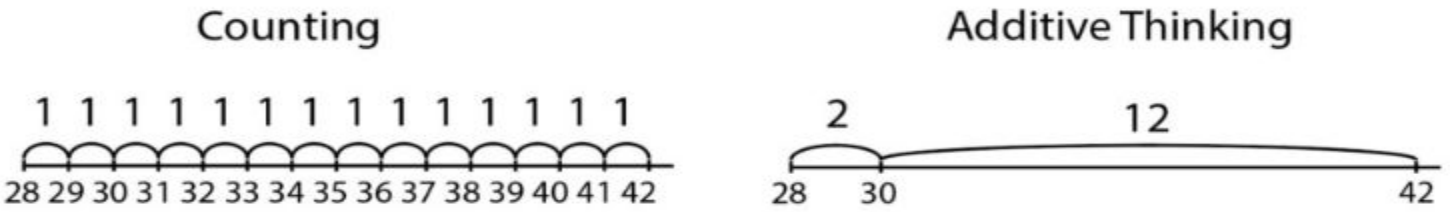
What are some consequences?

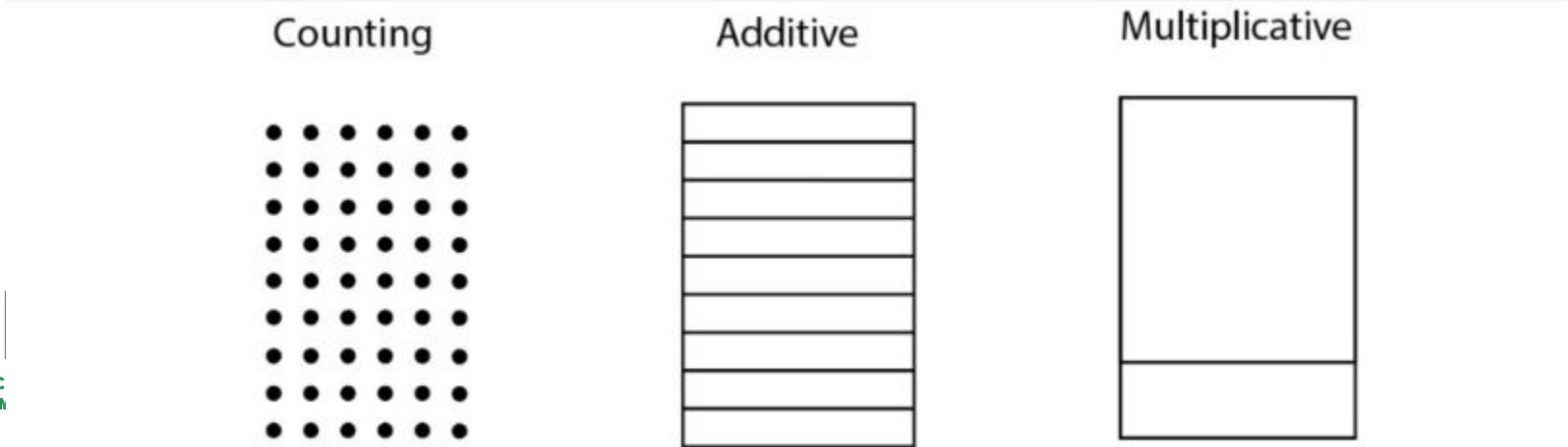
Students should be learning additive reasoning but…

28+12



Students should be learning multiplicative reasoning, but…

99 x 47



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Counting Strategies vs. Additive Strategies

27 + 8:	27, 28, 29, 30, 31, 32, 33, 34, 35. This could be done with fingers, tally marks, pictures, beads on a rack, on a number line. The part that makes it a counting strategy is the counting by 1's.
27 – 8:	Student says 27. Then puts up a finger and says 26, puts up another finger and says 25... puts up an eighth finger and says 19.
7 x 8:	Count out 8 (tallies, beans, etc.) into a pile. Then do that 6 more times so you have 7 piles, each with 8 in them. Then start at the beginning and count them all by 1's.
40 ÷ 8:	Draw 8 circles. Deal out 40 tallies, one at a time to each circle. Count the number of tallies in each circle: 1, 2, 3, 4, 5.

Additive Strategies

27 + 8:	27 + 3 is 30, then add the leftover 5, 35.
27 + 49:	27 and 50 is 77, but that's a bit too much, so I'll back up to 76.
27 – 8:	27 subtract 7 is 20, subtract 1 more is 19.
52- 27:	52 subtract 30 is 22, but I subtracted too much, so adjust back up to 25.
7 x 8:	Seven 8's. 8, 16, 24, 32, 40, 48, 56. This is called skip counting but it's not counting by 1's. It is adding one group of 8 at a time. That's additive thinking.
48 ÷ 8:	48 – 8 = 40; 40 – 8 = 32; 32 – 8 = 24; 24 – 8 = 16; 16 – 8 = 8. How many 8's? 6. This is additive thinking, repeatedly subtracting 8's and keeping track of how many 8's.

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Multiplicative Thinking

7 x 8:	I know that 7 7's is 49. But I need 8 7's so I need one more 7. 49 and 7 is 56.
7 x 59:	I'm going to think of this as 59 7's. I know that 60 7's 420. But I only need 59 7's, so 420 – 7 is 413.
48 ÷ 8:	I know that 40 ÷ 8 is 5, so 48 ÷ 8 must be 6.
1287 ÷ 13:	I know that 1,300 ÷ 13 is 100, but I need 13 less than 1,300, 1287 ÷ 13, so that's 99.

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How to assess student reasoning?

How Do You Reason?

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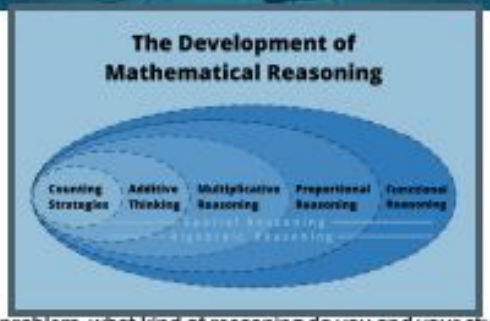
WONDERING HOW YOUR STUDENTS ARE REASONING?



When given a problem, what kind of reasoning do you and your students employ?

Answer each of the following questions. Then look at possible responses and kinds of reasoning.

What is 58 + 5?	**COUNTING STRATEGIES: counting by 1's** *Count out 58 objects (tallies, etc), count out 5. Put together, count the whole set.* *Start with 58 and count 59, 60, 61, 62, 63. Counting on.* **ADDITIVE THINKING: using jumps bigger than 1's** *From 58, add 2 to get to 60. Add the remaining 3 to get 63.*
What is 16 × 9?	**COUNTING STRATEGIES: counting by 1's** *Count out 16 groups of 9 objects (tallies, etc) or 9 groups of 16 objects, one at a time. Put together, count the whole set.* **ADDITIVE THINKING: adding one group at a time** *Skip count by 16's or 9's. (16, 32, 48, ... 144 or 9, 18, 27, ...144)* **MULTIPLICATIVE REASONING: using bigger chunks than one group at a time** *Think about 16 9's as 10 9's and 6 more 9's. 10 x 9 + 6 x 9 = 90 + 54 = 144.* *Think about 9 16's as 10 16's subtract one 16. 10 x 16 − 1 x 16 = 160 − 16 = 144.* *Think about equivalent problems by doubling/halving.* *16 × 9 = 8 × 18 = 4 × 36 = 2 × 72 = 144.*

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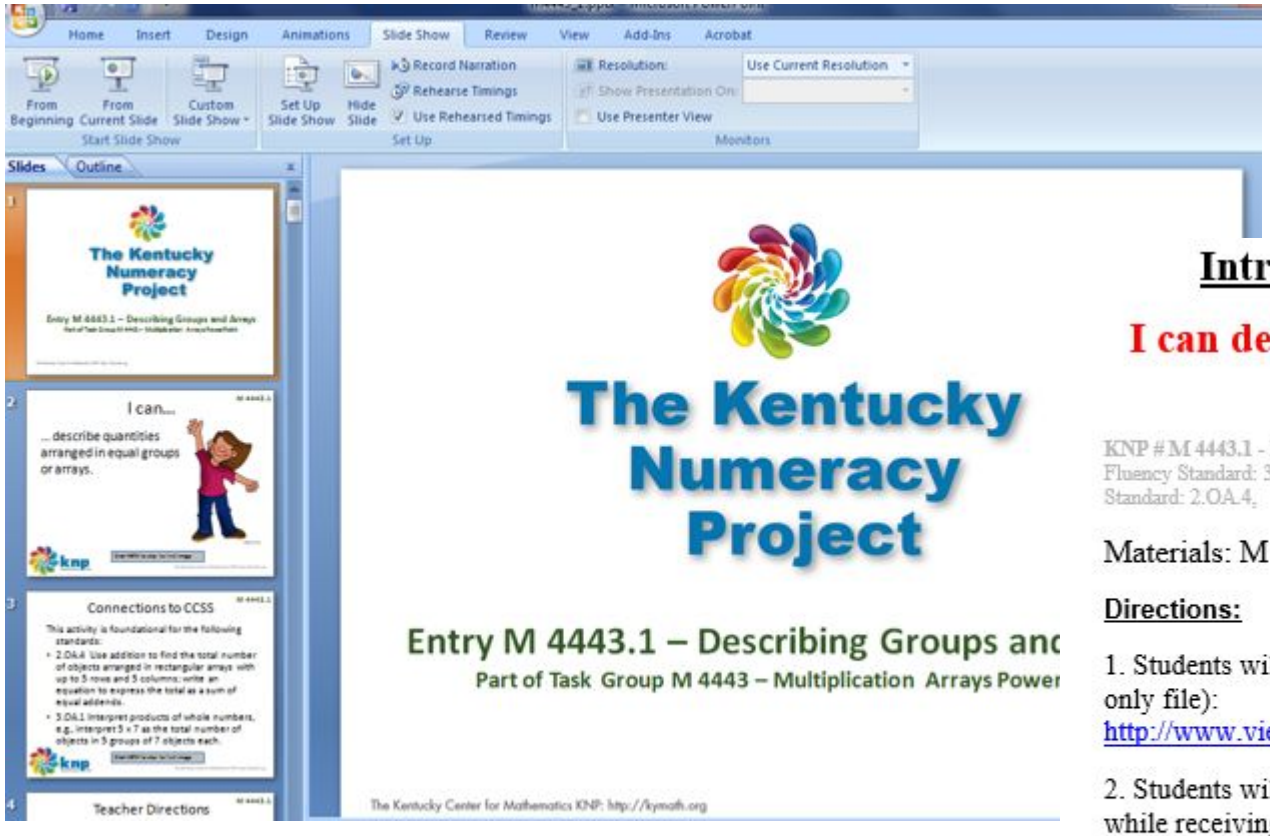
PAM HARRIS
equipping math teachers with content & pedagogy for student success

Developing Multiplicative Thinking-Introduction to Arrays



Introduction to Arrays PowerPoint

I can describe quantities arranged in equal groups or arrays.

KNP # M 4443.1 - Introduction to Arrays PowerPoint, Red
Fluency Standard: 3.OA.7
Standard: 2.OA.4,

Materials: M 4443.1 Powerpoint

Directions:

1. Students will begin the activity by watching the Virtual Demo (re
only file):
http://www.viethhosting.com/kcm/docs/M4443VirtualDemo.pptx

2. Students will next engage in the Introduction to Arrays PowerPoi
while receiving guided instructions from their teachers. The
Introduction to Arrays PowerPoint can be found under the Printable
section:
http://knp.kentuckymathematics.org/knp/uploads/printables_4443.1M.pptx

* The goal is for student to attend to and be able to describe the structure of the ima
using appropriate language. At this point, do not ask students to think about the tota
number of dots as this will likely distract students from the primary goal.

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Resources from Our Website - KNPIG

Equal Addends

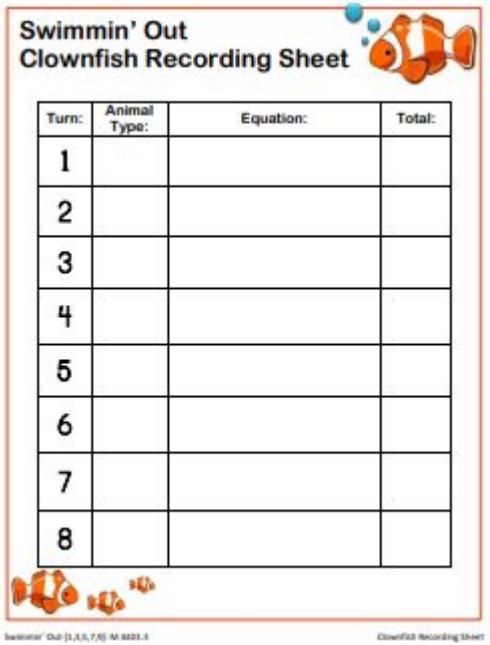
I can count the total number of hidden cubes (within 20) for two equal rows when told the number in each row and write the addition sentence showing the number in each row and the total.

KNP # M 4402.4 – Equal Addends, PURPLE
Fluency Standard: 3.OA.7
Standards: 2.OA.3

Materials: 20 snap cubes, cover, writing space

Directions:

1. Get 20 snap cubes, cover and writing material.
2. Player 1 hides his eyes.
3. Player 2 creates a block figure with two equal rows. Ex. 
4. Player 2 covers the whole figure.
5. Player 2 tells how many blocks there are in each of the two rows. Player 1 imagines the figure and finds the total number of blocks.
6. Player 1 writes the number sentence showing the addition of two equal rows.
7. Players switch places and begin again.



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Equal Groups

Composite Cookie Company

I can equally share items into a given number of groups and find the total.

KNP # M 4444.1 - Composite Cookie Company, Red
Fluency Standard: 3.OA.7
Standard: 3.OA.2, 3.OA.1, 2.OA.4,

Materials: cookies and plates cut-outs, task cards, recording sheet (many also use loose counters and paper plates instead of printables, if desired)

Directions:

1. Draw a card.
2. Use the cookies and plates cutouts to help you find the answer.
3. Record on your recording sheet and repeat until your sheet is full.

Composite Cookie Company

Get 12 cookies. Share the cookies equally on two plates. How many on each plate? Any leftovers?

Composite Cookie Company

Get five plates. Put six cookies on each plate. How many cookies?

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Building Arrays

Bead Arrays (up to 5 x 5)

I can build arrays (up to 5x5) and write a matching number sentence.

KNP # M 4449.1 - Bead Arrays (up to 5 x 5), Red
Fluency Standard: 3.OA.7
Standard: 2.OA.4,

Materials: beadrack (10 rows of 10) per player, 1 cube with labels {2 rows of, 3 rows of, 3 rows of, 4 rows of, 5 rows of, 5 rows of} and another cube with labels {2, 3, 4, 4, 5, 5}, writing space or recording sheet for each student

Directions:

1. On your turn, roll both cubes and make the array on your beadrack.

2. Write a matching addition sentence.

3. After everyone has made an array, determine who has the array with the most beads.

The player with the most beads is the winner for the round and gets a point. In case of a tie, both (or all) players get a point.

4. The first player to earn 5 points wins.

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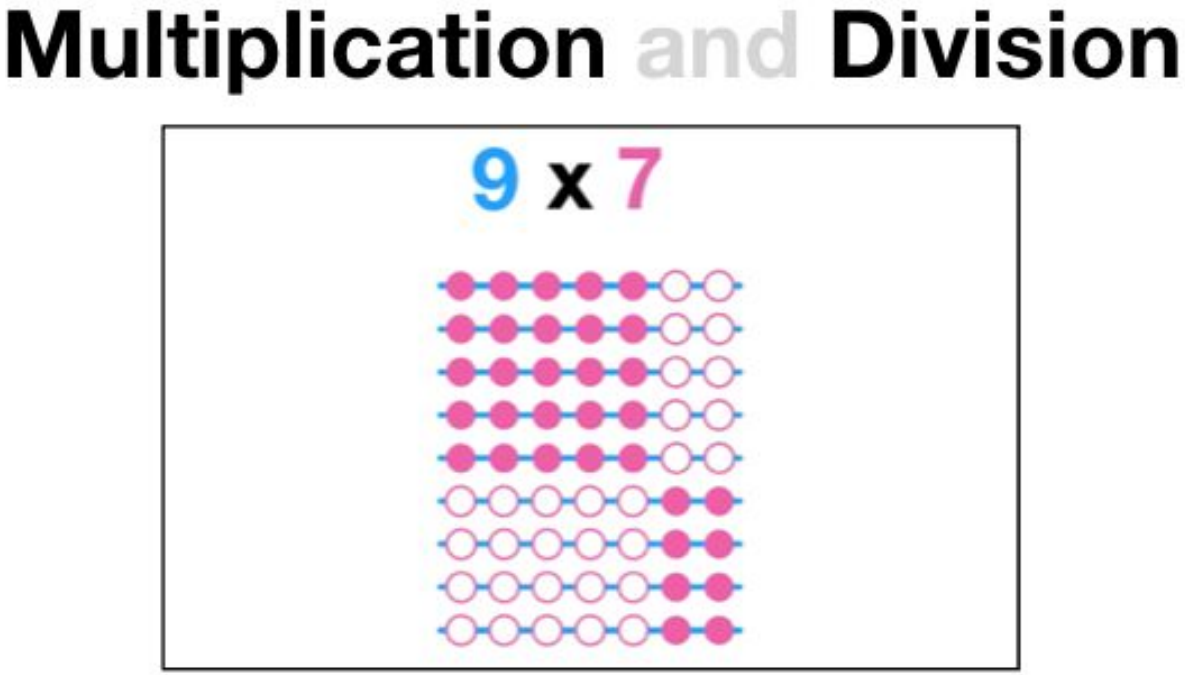
Moving from Additive to Multiplicative Thinking

Fletchy

https://drive.google.com/drive/folders/0B7ZFaFZdreqGVWpxeHVwc1lxSnc

Images to Support Multiplicative Thinking

https://mathvisuals.wordpress.com/multiplication-division/



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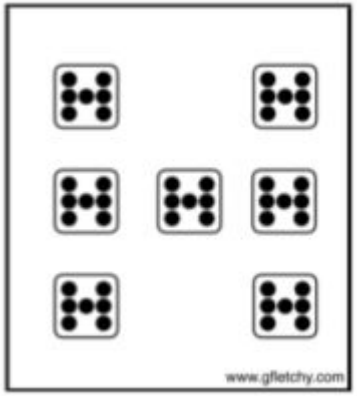
Resources to Support Multiplicative Thinking

- Subitizing to Support Multiplicative Thinking



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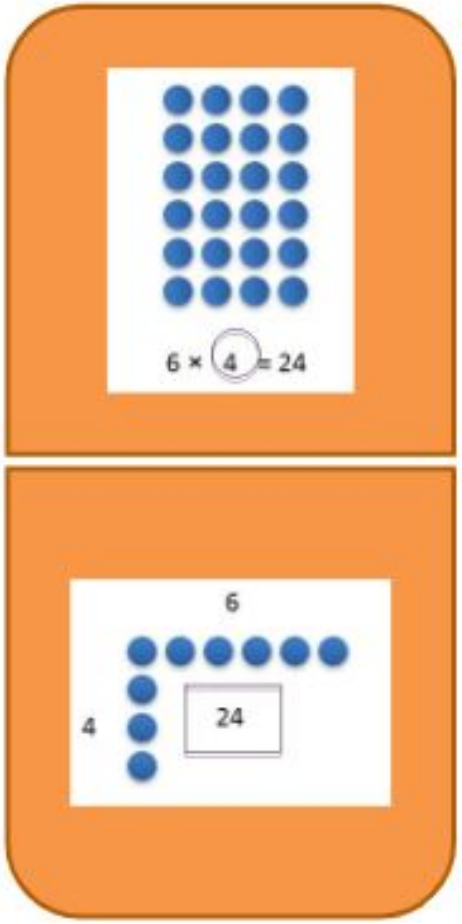
Same/Different



These are the same because

Same and Different

These are different because

https://docs.google.com/presentation/d/1S0lK-bcg-ZiiGTYKGcVMgootYv-DfgGzr-ib--N0tl0/edit#slide=id.g746caad312_0_417

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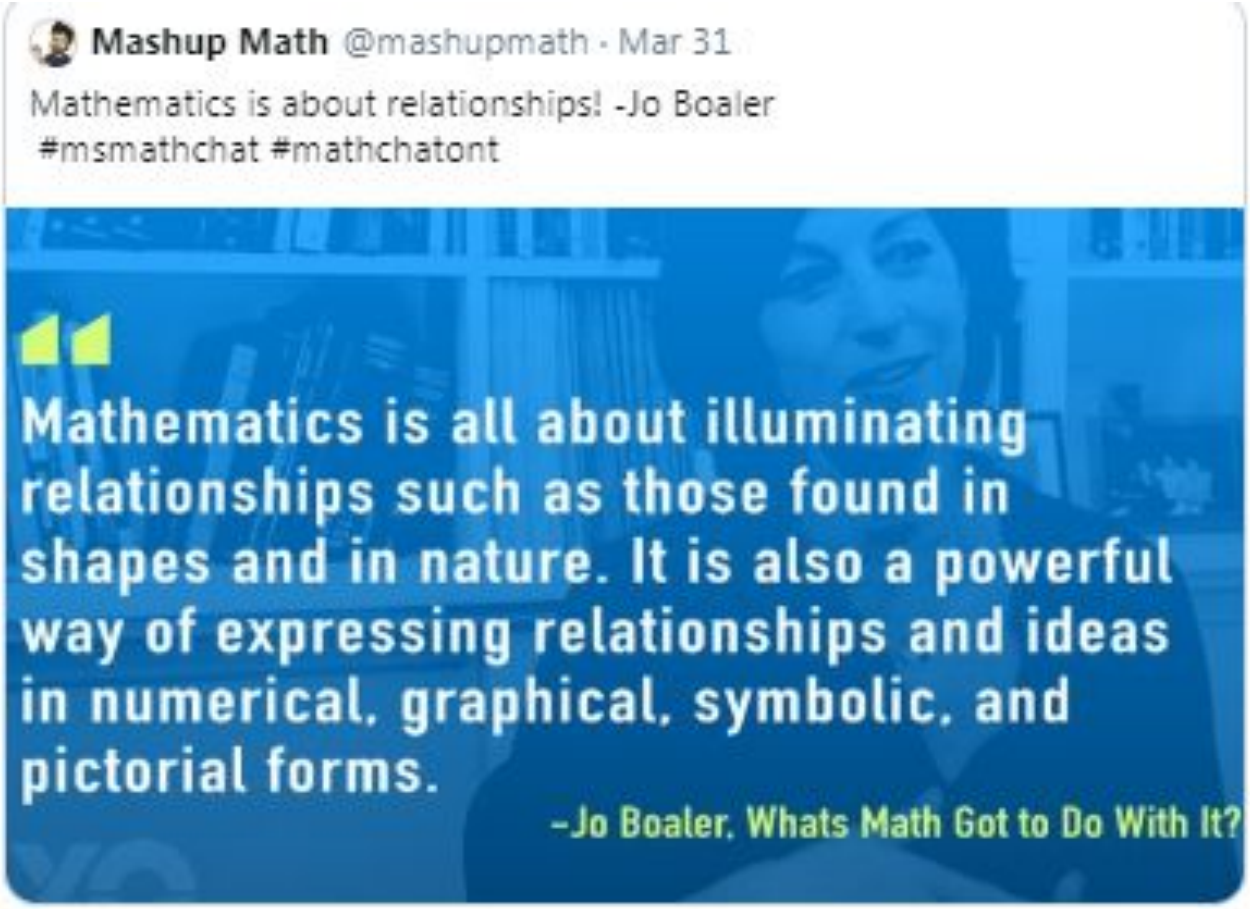
Illuminating Multiplicative Relationships found in Nature



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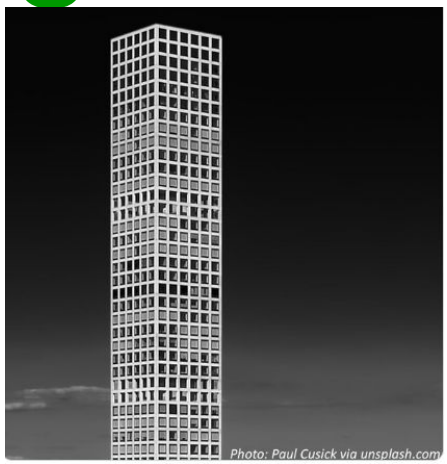
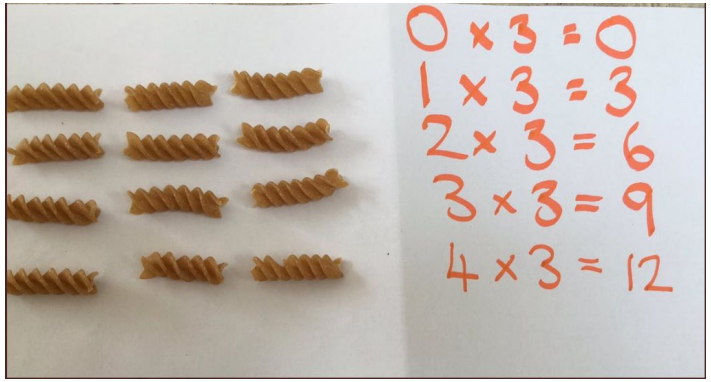
Examples of Contextual Arrays





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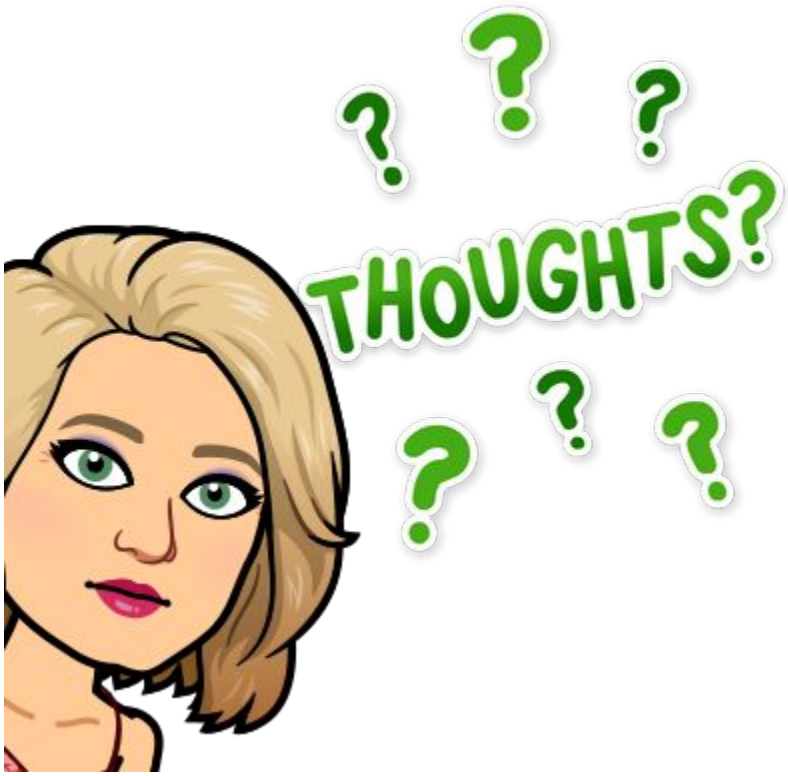
Making Decisions



Where do you fit in the development of student reasoning? How can you delay traditional algorithms? How can you emphasize reasoning over rote? How can you celebrate students wherever they are while always working to develop further?

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Upcoming Virtual Professional Learning

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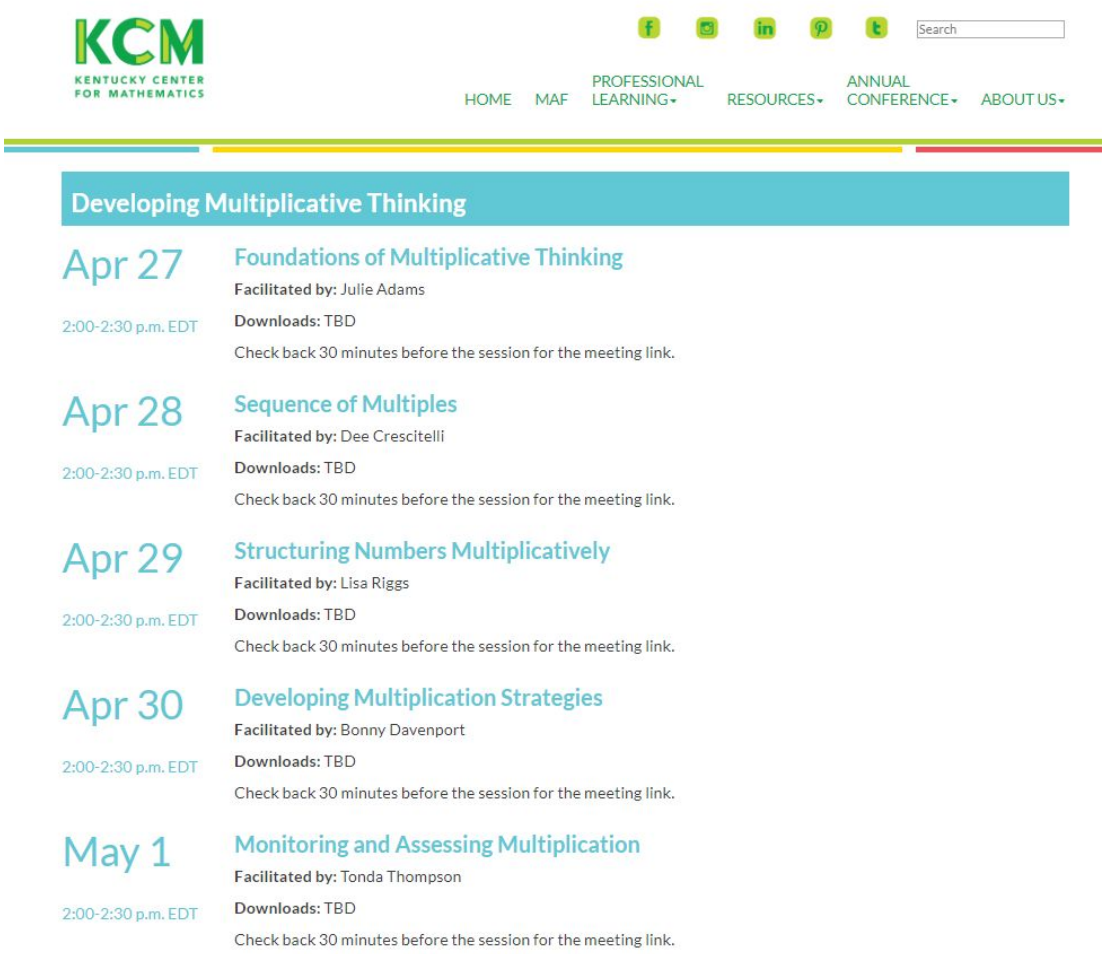
https://www.kentuckymathematics.org/kcm_virtual.php



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KCM is here to support you!

Contact me:

Julie Adams

Regional Consultant
Kentucky Center for Mathematics
jaadams2@moreheadstate.edu

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