Powerful Problem Solving

Max Ray of the Math Forum @ Drexel

Powerful Problem Solving

Activities for **Sense Making** with the **Mathematical Practices**

HEINEMANN
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Heinemann
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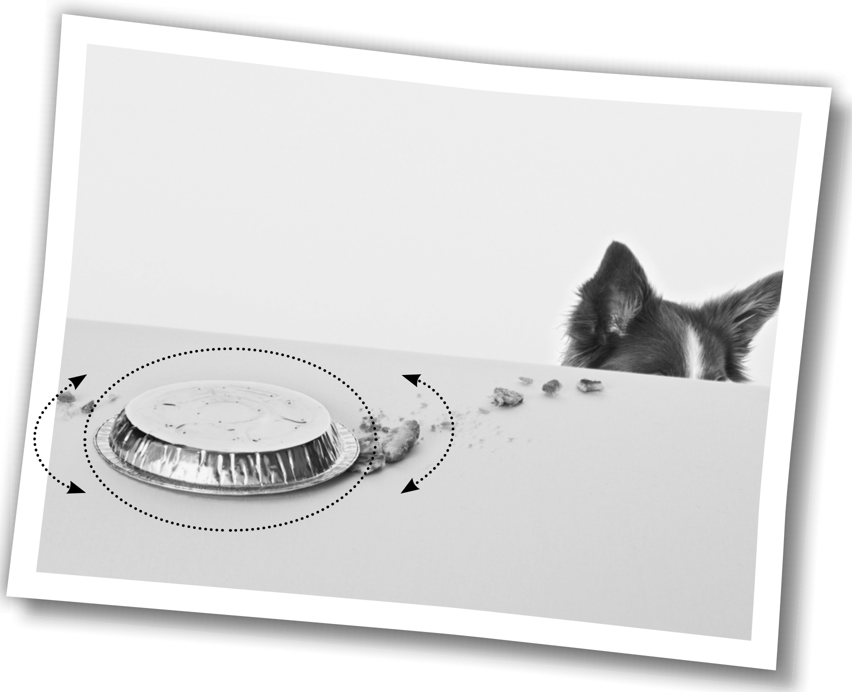

FOREWORD

Susan O'Connell

Learning mathematics is about more than memorizing facts and algorithms. Proficient mathematicians *think* mathematically. They develop mathematical reasoning and apply their computational skills and conceptual understanding to solve math problems. The Common Core State Standards place great emphasis on developing students' problem-solving skills. Standard 1 of the Common Core Standards for Mathematical Practice affirms that mathematically proficient students make sense of problems and persevere in solving them. And throughout the content standards, we repeatedly see references to students solving problems using whole numbers, fractions, measurement, or other content strands.

Have your students internalized the problem-solving process? Have they acquired a repertoire of strategies to solve varied types of problems? Have they developed enough confidence in their problem-solving skills to allow them to persevere when problems become challenging?

How do we support our students to promote the development of their problem-solving skills? How do we help them think like problem solvers? This book, *Powerful Problem Solving*, will serve as your guide. Max Ray has developed an understandable, practical, and insightful book to guide you as you explore math problems with your students. In the rich tradition of the Math Forum, which has been posing challenging problems and guiding students in the development of mathematical thinking for twenty years, this book provides rich problems and invaluable tips for building your students' problem-solving skills. The importance of math talk, the critical nature of reflection about our own thinking (metacognition), the ability to struggle while finding paths to solutions, and the benefits of exploring divergent methods for solving problems are all explored.

While standard 1 in the Common Core Standards for Mathematical Practice specifically highlights problem solving as a goal, *Powerful Problem Solving* explores the links between problem solving and the other math practice standards. Other Practice Standards (e.g., constructing viable arguments, looking for and making use of structure, using models, selecting tools, exploring repetition) are identified at the start of each chapter, and then links are made between that standard and problem solving. The interdependence of the Practice Standards, and their connection to building confident problem solvers, is clearly evident as each chapter is developed.

In addition to a clearer understanding of what makes an effective problem solver and how problem solving is linked to the Practice Standards, you will love the many practical tips and classroom activities described throughout this book. Each chapter targets a specific practice standard and highlights several activities that can be easily integrated into your math classroom and will serve to strengthen your students' skills in that critical practice. The book's companion website is filled with numerous problems that will engage and excite your students, while challenging them to think mathematically! There are also reproducibles that you can use with your students.

Our students learn to be problem solvers by solving problems. We must pose problems that challenge them to analyze, interpret, plan, check, and reflect on their approaches and solutions. We must help them develop foundational thinking skills, including modeling strategies, ways to simplify problems, and ideas for organizing data and selecting helpful tools. In addition, we must develop a classroom climate in which students feel free to talk about their efforts, listen to each other, develop arguments to support their decisions, take risks, and have enough confidence to persevere when solutions are not immediately apparent. Through this book, Max Ray shares a wealth of resources, tips, insightful anecdotes, and specific activities for developing students' skills and habits of mind. *Powerful Problem Solving* will help you envision the key components of a problem-rich classroom and guide you in creating a classroom environment that fosters the development of problem solving.

Whether you read this book from front to back (which I would recommend so you don't miss any of the important insights) or simply dive into chapters that address the needs of your students at the moment, you will find it practical, understandable, and enlightening. It provides clarity to an often-confusing and definitely complex area of mathematics, and helps us make sense of and simplify the teaching of math problem solving.

Don't wait another minute—dive into it!

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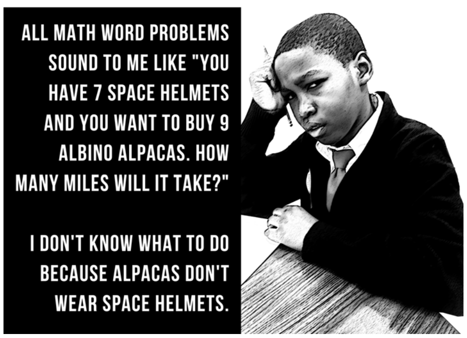
Noticing and Wondering

Focal practice:

1. Make sense of problems and persevere in solving them.



How many of our students feel like the pained-looking student in the illustration above? Too often we hear, "It's never going to make sense to me, so why should I try and understand it?" Lots of students have gotten turned off from trying to make sense of problems when too much of the math they've encountered feels like a set of arbitrary rules to memorize and apply.

Making Sense of Problems and the Common Core Practices

The very first mathematical practice identified in the Common Core is that "mathematically proficient students make sense of problems and persevere in solving them." It is easy to recognize students who aren't yet persistent problem solvers and who fail to make sense of problems: When we hand those students challenging problems, they groan, tell us, "I don't get it!," leave whole problems blank, guess at operations to do, and/or don't know when their answers are unreasonable.

One value of the Common Core State Standards for Mathematical Practice is that they emphasize that solving math problems requires habits of mind, routines that we learn to do and get better at. Although some of our students seem to naturally make sense of problems and persevere, other students seem to flounder. But habits of mind can develop over time and students can move toward becoming mathematically proficient. When our students give up, they aren't being lazy; they are showing us that they still need practice and support making sense of problems.

Even students who have come to expect that problems in math class will make as much sense as "You have 7 space helmets and you want to buy 9 albino alpacas . . ." can practice and get better at making sense of problems. We've often noticed that young elementary students participate much more enthusiastically in math challenges, and their middle school peers seem afraid to even try—the younger students have natural problem-solving instincts that they are engaging, which their middle school peers can be supported to engage, too.

How do we help students reconnect to their own sense making? How do we help them see problems the way mathematicians do, so that they can find the givens, constraints, and relationships?

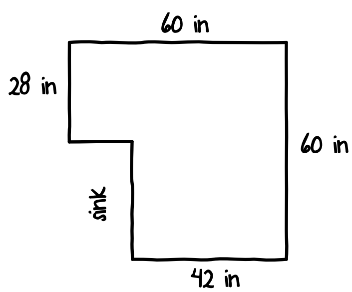
Notice and Wonder™

Often methods such as asking students to find key words, circle the important information, and cross out unnecessary information rely on students already knowing what is important in a problem—they have to understand the problem in order to use their "understand the problem" strategies! Students who feel disconnected from math often give up before they even try reading the problem, let alone circling key words. Or, they try to guess what we are looking for, blurting out answers that don't make sense.

The wonderful thing that we've found is that students are very capable, with practice, of finding important information in math problems (and stories, images, videos, etc.) and making conjectures about that information. It's a matter of helping them get started, valuing their ideas, and helping them stay connected to their own thinking. We do that using an activity we've developed called I Notice, I Wonder that was introduced by Annie Fetter, a colleague at the Math Forum, in the spring of 2007. The teacher had warned Annie, "This is my lowest-level class. Don't expect too much."

Annie started by drawing the picture in Figure 1 on the board and described to the students what they were about to do.

Figure 1



"This is a picture of Teresa's bathroom floor. We're going to list as many things as we can about the picture. I'm going to ask each of you to offer one thing that you *notice*. Anything at all." Here's what they said:

- Two sides are equal.
- Two sides are 60 inches.
- One side is 28 inches.
- They are longest.
- One side is 42 inches.
- It used to be a square.
- Your lines aren't very straight.
- The short side of the sink is 18 inches.
- The sink is a rectangle.
- The long side of the sink is 32 inches.
- You can find the area of the whole thing by making it two pieces.

She was excited—that was a great list! She picked out the items she figured were most likely to be both important in eventually solving the problem and potentially confusing and asked for volunteers to explain them to her and the rest of the class. The following responses came from many different students.

Annie: What does it mean to say that it used to be a square?

Student: The floor is like a square, but the sink is in the way.

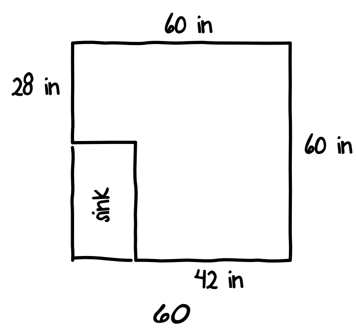
Annie: How do you know it is a square?

Student: Because all of the sides are 60 inches. That's a square.

Annie: But all the sides in the picture aren't 60 inches long. Could someone show us on the board what you mean?

Student (*drawing Figure 2*)**:** If the sink wasn't there, they would all be 60 inches.

Figure 2



Annie: Okay. How do we know that the short side of the sink is 18 inches?

Student: Because it's 60 take away 42.

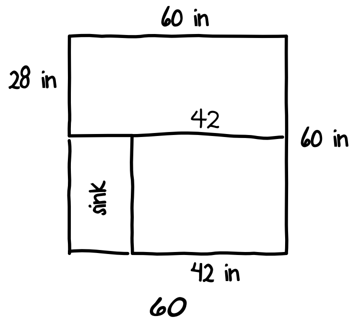
Annie: How did you know to do 60 minus 42?

Student: 'Cause 60 is all the way and I only want part of it.

Annie: Would someone like to come up and show us what that means on the board?

Student (*drawing Figure 3*)**:** That part is 42 because it is just like the bottom. So you do 60 take away 42.

Figure 3



They went through a similar process with the ideas that the long side of the sink is 32 inches and that the area can be found by splitting the floor into two pieces. Then she explained that Teresa was going to put down new tiles, and that the new tiles are squares that are 4 inches by 4 inches. She drew a small tile on the board and labeled it 4 inches on each side. Then she said, "What can we say now?"

- The tiles are smaller than the floor.
- Each tile is 16 square inches.
- It will take a lot of them!
- Seven tiles will fit across the 28-inch side.
- Fifteen tiles will fit across the top and the side.
- Tiles won't fit across the bottom.

At this point, they had reached the end of the class period. Annie gave each student a copy of the full text of the problem and told them that for tomorrow's class, they were to write down everything they remembered from the conversation. She told them that they should not worry about solving the problem. She also told them that they had done an awesome job and that she had had a lot of fun!

Annie told the teacher that she had been really impressed with the students. Turns out the teacher had been too! She was surprised by the number of things they had come up with and by how many of them had participated. A few days later Annie got an email saying that almost all of them had done their homework, and most had remembered more than half of what they talked about in class. A couple of the students had gone on to solve the problem, and when they worked on it in class the next day, many of them were engaged in the process. Annie and the teacher were excited at how well the students did, especially since their teacher said it was their first time solving a problem like this one. Annie went back to share the Notice and Wonder routine with everyone else at the Math Forum and we've been using it ever since!

You may have noticed that when Annie first asked the students to notice and wonder, she started with just a drawing of Teresa's bathroom floor, not a complete math problem with a question to answer. When students see a *question* in math class, they tend to go into "get the answer quick!" mode. Students often feel pressure and anxiety around having to get the answer, whether it's competitiveness to show their smarts or fear that they won't be able to answer that stops them from engaging. We've found that leaving off the question and just sharing an initial story/scenario/image increases participation from struggling students because there are no right or wrong answers to "What do you notice?" and "What are you wondering?" It keeps speedy students engaged in creative brainstorming rather than closed-ended problem solving. It provides a safe, welcoming opening for students who don't often feel like they have anything to say in math class—it starts to unsilence their voices! And often, students generate the very question we would have asked through their wonderings; answering a question *they* generated increases all students' engagement. Through wondering, students see that math problems come from their own thinking and they are no longer encountering problems about which they can exclaim, "I have no idea what this is talking about!"

When students see a question *in math class, they tend to go into "get the answer quick!" mode. We've found that leaving off the question and just sharing an initial story/scenario/image increases participation from struggling students.*

The I Notice, I Wonder activities in this chapter were developed out of Annie's success with supporting and engaging the supposedly "low-level" math class. The activities are designed to support students to:

- connect their own thinking to the math they are about to do
- attend to details within math problems
- feel safe (there are no right answers or more important things to notice)
- slow down and think about the problem before starting to calculate
- record information that may be useful later
- generate engaging math questions that they are interested in solving
- identify what is confusing or unclear in the problem
- conjecture about possible paths for solving the problem
- find as much math as they can in a scenario, not just the path to the answer.

Notice and Wonder Activities

I Notice, I Wonder Brainstorm

Format: Whole group.

Materials: A student-friendly handout to support students' thinking about this idea is available on the companion website: http://mathforum.org/pps/.

Step 1: Display a problem scenario (without a specific question) at the front of the room. If reading level is a concern, read the scenario to students or have a volunteer read it.

Step 2: Ask students, "What do you notice?" Pause to let as many students as possible raise their hands. Call on students and record their noticings at the front of the room.

Step 3: Ask students, "What are you wondering?" Pause to let as many students as possible raise their hands. Call on students and record their wonderings at the front of the room.

Step 4: Ask students, "Is there anything up here that you are wondering about? Anything you need clarified?" If you or the students have questions about any items, ask the students who shared them to clarify them further.

Step 5: After all students have participated and understand the scenario thoroughly, reveal a question you'd like students to work on. You might ask students to reflect quickly on whether the question clears up any of their wonderings or leads to new noticings or wonderings. Or, ask students, "If this story were the beginning of a math problem, what could the math problem be?" Then solve a problem the *students* come up with.

We often find stories and images for noticing and wondering about in textbooks, whether it's just the diagram from a picture or a story without the question from a word problem. Some texts have good fodder for noticing and wondering in the unit or lesson launch activities, and others have good scenarios in the end-of-unit and end-of-lesson projects and "challenge problems." Additional resources for finding noticing and wondering prompts are available on the companion website for this book (http://mathforum.org/pps/), as are videos of Math Forum staff implementing several different versions of I Notice, I Wonder activities.

Ideas for Differentiating Noticing and Wondering

When noticing and wondering, we've found that it's helpful for students to have the experience both of independently generating as many ideas as they can and of hearing from others what they noticed and wondered. The discipline of careful noticing requires practice and patience, time to think and write on your own. But learning to notice new kinds of things and see math situations through different, useful lenses requires hearing from others. The think-pair-share noticing and wondering activity that follows helps focus students on the individual, written work. The What Do You Hear? noticing and wondering activity (on page 48) helps students quickly hear from others and generate lots of ideas verbally, which can increase engagement and help students, especially younger students or students who have a hard time with writing, stay engaged and active during problem solving.

ACTIVITY: Think-Pair-Share: Increasing Engagement and Accountability

Format: Individual, then pairs, then whole group. Having desks prearranged so students know who their partner is helps make transitions smoother.

Step 1: Think. Give each student a recording sheet. Students brainstorm privately, recording their thinking on the sheet. When the think time is up, students draw a line below their last noticing and last wondering.

Sometimes we set a timer so that students know how long they need to hold their quiet focus.

Step 2: Pair. Students turn to the person next to them and share their list of noticings and wonderings. When one student is reading, the other is recording below the line any noticings and wonderings her partner had that she didn't.

Each pair should choose one favorite noticing or wondering they want to share—it might be something both partners had or one they felt was unique.

Step 3: Share. Quickly go around the room hearing each pair's favorite item. Then ask, "Did anyone have any other noticings or wonderings they wanted to share?" and collect those.

Students should add noticings and wonderings they didn't come up with to their own recording sheets, below the line.

Step 4: Reflect. After hearing from everyone, a chance for students to reflect on the kinds of noticings and wonderings that are easier for them to generate and the kinds they don't see on their own yet can help students set noticing and wondering goals.

Then, the bell rings and the students leave, often still talking about math and wondering about what's going to happen next!

We want to emphasize the idea of calling on all students or groups and asking them for just one thing. It implies that everyone has something to share, and that you want to hear from everyone. Just doing a show of hands and calling on kids can allow some kids to get lost or ignored or feel like someone else took all their good ideas and they didn't get "credit." If everything on a student's list has been shared by the time it's her turn, she can feel heard by telling the person making the public list to write a "Like" or a "+1" next to an idea she had that someone else has already added.

Calling on each group or many groups can feel tedious, though, especially when students have already spent time writing and talking in pairs. An alternative method that works particularly well for younger students, students who struggle with reading or writing, or when you're trying to quickly build engagement is to do the noticing and wondering completely orally at first.

ACTIVITY: What Do You Hear?

Format: Whole-class discussion.

Step 1: Tell the students, "I'm going to read you a story." Wait for students to settle down and then read the scenario from a problem (the entire story minus the question).

Step 2: Ask the students, "What did you hear?" Listen to each student who volunteers to answer. Thank or acknowledge students for participating (we might say "Uh-huh" or "Thanks") but avoid praising or revoicing what the student has said.

Step 3: Read the story again. Ask students "Did what you heard the first time match what you heard this time or do you want to change anything?" Again listen without praising or revoicing.

Suzanne likes to use an I Notice, I Wonder brainstorm or What Do You Hear? activity during the very last five minutes of a lesson. She'll read a story or display a scenario (definitely not using a specific question to solve) and have students notice and wonder. Then, the bell rings and the students leave, often still talking about math and wondering about what's going to happen next! The activity might preview the next day's lesson or be part of a problem-solving task that she will gradually build up over the course of a week or two. Not every noticing and wondering session has to immediately lead into doing math; sometimes it's great to leave them wondering. If you'd like to read more about how Suzanne would build a problem-solving task in five-minute chunks over several lessons, visit the companion website to read her article, "Think You Don't Have Time to Use Problems of the Week?" (http://mathforum.org/pps/).

Noticing and Wondering Tips

Valuing and Unsilencing Students' Voices

If your students don't seem to enjoy noticing and wondering, we've found it's often because the teacher is doing lots of restating. For students to value and own the process, they must recognize that their own ideas in their own words are the beginning of the problem-solving process. As you hear from students about what they notice and wonder, thank or acknowledge each student equally. Record all student suggestions. Avoid praising, restating, clarifying, or asking questions until everyone's noticings have been recorded. Then you might ask, "Is there anything on this list you are wondering about?"

I Notice Versus I Know

Another key to keeping the environment as welcoming as possible and helping students focus on generating as much math as they can rather than getting to one specific conclusion is really just a matter of a single syllable. Asking students "What do you *notice*?" instead of "What do you *know*?" changes how students participate. "What do you know?" can too often tie into students' existing beliefs about whether they know any math—students who feel confident want to rattle off a lot of knowledge, and students who lack confidence already think, "I don't know anything about math, what can I say?" Asking "What do you notice?" focuses students on this specific scenario and welcomes them to start fresh, just saying what they saw or heard. We've had multiple teachers share their observations that they get more students engaged and a better quality of results when they consistently ask "What do you notice?" instead of "What do you know?"

Building Student Independence: Moving from Noticing and Wondering Toward Seeing the World Mathematically

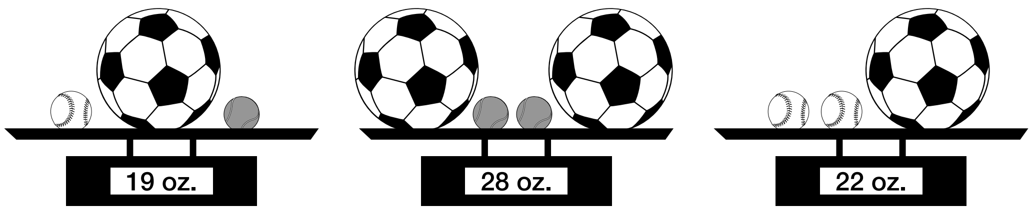
Noticing Mathematically: Finding Quantities and Relationships

Noticing and wondering activities are very open-ended, and at first can lead to noticings and wonderings that are off-topic and even silly. The initial process of writing noticing and wondering lists can take a long time, and students will notice details that they won't end up using as they solve the problem. In short, noticing and wondering is something students get better at over time: more focused, more relevant, more efficient, and more automatic.

How do we support students to move from noticing "The graph is red" to noticing "The graph goes down" to noticing "The graph shows that as time goes by, the car gets closer to home," for example? It is a process that takes time and patience, as the teacher first supports students to notice and wonder at all, and then to be thorough and generate lots of ideas, and finally to begin to think about narrowing down on the kinds of noticing and wondering that mathematicians might do. Once students have become prolific noticers and wonderers, one simple prompt we've found to be very helpful in focusing students is simply asking,

> "Which of these noticings have to do with math?"
>
> "Which of these wonderings could we use math to help us answer/prove?"

Following is an example from Philadelphia fifth graders who had been noticing and wondering for several weeks. First they listed what they noticed and wondered, and then they used asterisks to mark the entries that they thought answered the question, "Which of these could we use math to prove or did we use math to come up with"?



What do you notice?

Teacher model: I notice that there are 3 different groupings.

Kaya: I notice the middle one weighs the most.

Kyla: I notice there are 10 balls all together.

Safiya: I notice the middle one has more balls.

Jaquir: I notice there are 4 soccer balls and 3 baseballs and 3 tennis balls.

Omar: I notice the weight of all 3 adds up to 69 ounces.

Duwan: I notice each group doesn't weigh the same.

What do you wonder?

Teacher model: I wonder why she grouped them this way.

Kyla: I wonder what the question is.

Sue: I wonder if it is a pattern.**

Safiya: I wonder why the last one weighs more than the first one.**

Michael: I wonder if the soccer balls are 11 pounds [sic].**

Sienna: I wonder why are there balls and what is she doing.

Kaya: I wonder why the middle has too many soccer balls.

Zenaia: I wonder if we have to add or divide.**

Jaquir: I wonder why . . . I don't know, I forgot.

Bria: I wonder why she picked soccer balls, baseballs, and tennis balls.

Students almost always identify as mathematical the same set of noticings and wonderings we would, saying that those items had to do with numbers, math words, measuring, how many, and so on. In middle school, it's appropriate to help students realize that the "math-y" items are (almost always) about *quantities*: things you can count or measure to find the value of. Once students have begun to focus on noticings and wonderings that they feel are mathematical, we start to ask students to challenge themselves to notice as many quantities as they can in a mathematical situation. For example, in the fifth-grade students' noticings and wonderings above, they noticed these quantities:

- the number of groupings (3)
- the weight of each grouping
- the total number of balls (10)
- the number of balls in each grouping
- the number of each type of ball
- the total weight
- the weight of an individual soccer ball.

Math problems are explored by finding relationships among quantities. Whether the relationship itself is the answer (e.g., problems that ask students to find a formula or to tell how many times bigger something is) or a relationship is used to find a value that satisfies the constraints of the problem, good noticers seek out relationships among quantities. For example, the fifth graders noticed and wondered about these relationships:

- The middle group weighs the most.
- The middle one has more balls.
- The middle group has an extra soccer ball.
- Each group doesn't weigh the same.

- The groupings can be combined to get total weight and total number of balls.
- The last one weighs more than the first one.

These relationships involve many of the quantities that students previously noticed. It's interesting that the students haven't yet explicitly connected the different weights to the different types of balls, but perhaps as they explore questions like "Why does the last one weigh more than the first?" they will notice it has a different combination of balls, and even that the baseballs must be heavier than the tennis balls.

Noticing and describing relationships can be challenging for students, but once they are good at noticing all of the quantities in a problem, we push them to try to name any relationships that involve those quantities. As you can see in the example, relationships of comparison (things that are equal, more than, less than) often are easiest to notice, as well as relationships made by doing basic mathematical operations (e.g., how many there are all together). Students have a harder time seeing relationships that are about change or about how two or more quantities are connected (for example, how the type of ball is related to the weight on the scales, how swapping a tennis ball for a baseball changes the weight, etc.). Eventually, students will use relationships based on comparison, mathematical operations, and change/connection relationships to build guess and check tables or mathematical models, identifying constraints, unknowns, and writing relationships mathematically. But just being able to notice and describe relationships in their own language is an important and difficult first step!

Wondering Strategically

What other guidelines might we put on noticing and wondering to help students develop the practice of making sense of problems? In addition to noticing *quantities* and *relationships*, we can look to the standards for more guidance.

The description of "Make sense of problems and persevere in solving them" continues, "They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem."[1]

Some of the strategic behaviors described here bring to mind key problem-solving strategies. The wonderings that follow can help guide students toward strategic behaviors (I've included the related strategies in parentheses after each wondering):

- What does this problem remind me of? Have I ever seen a problem like this before? (Solve a Simpler Problem, Change the Representation)
- What makes this problem hard? What would make it simpler to solve? (Solve a Simpler Problem)
- What must be true? What can't be true? What might be true? (Use Logical Reasoning)
- What am I trying to figure out? (Make a Plan)
- What needs to be organized? (Make a Table)
- What patterns can I see? (Make a Table)
- How is this information displayed (represented)? How else can I represent (display) this information? (Change the Representation)
- What guess can I make? (Guess and Check)
- How would I check if my answer were wrong? (Guess and Check)

Each of the wonderings above can be a launch into the problem-solving strategy listed next to it (we expand more on this idea in Chapter 9). We tend to introduce the questions using a three-phase introduction, releasing responsibility for asking the question from the teacher to the problem solver.

1. The teacher poses the question to the whole group as a noticing and wondering prompt. For example, after noticing and wondering about a problem, the teacher might say, "Now let's sort these into three lists. What *must* be true, what *can't* be true, and what *might* be true? What else can we put in each list?"
2. Once the students have worked as a whole group to address one of these strategy-specific wonderings, the teacher then starts using the questions to support students working in independent groups or individually. The teacher also listens to learn if students working together have begun to ask each other these questions.
3. As students make use of these questions or ways of thinking to solve problems, the teacher supports them to tell other students how these questions are helpful. Students might make a poster listing things they wonder when they solve problems that includes any of the wonderings they find helpful. At this point when students are stuck, the teacher asks, "What can you ask yourself to help you look at the problem strategically?"

Noticing and Wondering to Get Unstuck

Noticing and wondering can be a great way to help students get themselves unstuck. If students have noticed and wondered their way into a problem, those noticings and wonderings can serve as reminders for different ideas that can help students get unstuck. Good ways to use noticing and wondering to get unstuck include:

- Make a public record of noticings and wonderings and *keep them on display throughout the problem-solving process*. Check back on them when you feel stuck.
- Use noticing and wondering as a way to step back when you're stuck:
  - Notice and wonder about the problem text/images again. Look for new things to add to your noticing and wondering list.
  - Notice and wonder about your own work—read through what you've done so far and let your mind freely come up with noticings and wonderings.
  - Ask a partner to notice and wonder about your work. Just let them say what jumps out at them and what it makes them wonder.
- Use wondering as a way to say the things that are in the back of your mind. Try to wonder at least five things like:
  - I wonder if it would help to . . .
  - I wonder if it's worth it to try . . .
  - I wonder what would happen if . . .
  - I wonder if ________ would work . . .
  - I wonder why they wrote/drew ________ this way . . .

Using noticing and wondering as a tool for getting unstuck helps students realize that it's more than just an activity—it's a skill that helps them throughout the problem-solving process.

Conclusion

Everyone can notice something, and everyone has something they wonder.

No single other activity in this book has been more popular (or, as far as we know, produced more dramatic changes in math classrooms) than the simple I Notice, I Wonder Brainstorm. The teacher whose fifth-grade students provided the noticing and wondering example discussed earlier credits noticing and wondering about, and then solving, one word problem a week with raising her class' benchmark test scores from 3 percent to 80 percent over their fifth-grade year. Her principal confirmed that during testing, the fifth-grade students had much more "stamina"—they were able to persist on problems that they would have shied away from before.

The magic of noticing and wondering, we think, is that it invites every student to participate mathematically. Everyone can notice something, and everyone has something they wonder. It builds momentum in thinking. It generates content that students record and can then use to generate other ideas. It encourages the process of thinking about math and connecting ideas and wondering. Learning depends on mulling, connecting, wondering, and repeatedly thinking about, and noticing and wondering enables this to take hold and blossom. When we invite everyone to share in math class, and we see how each student's contribution builds toward a complete mathematical understanding of the problem at hand, we invite students to think of themselves as mathematicians. Students' confidence increases, and they have a real tool for beginning the task of making sense of problems and persevering in solving them.

Adding especially mathematical noticing and wondering skills (noticing quantities and relationships, wondering strategically) to students' repertoire increases the usefulness of noticing and wondering. As students get better at targeted, mathematical noticing and wondering, and as they begin to notice and wonder automatically (as mathematicians do), they may find that all of the other problem-solving strategies become easier to learn as well.